Gravitation and Spacetime, Third Edition

The third edition of this classic textbook is a quantitative introduction for advanced undergraduates and graduate students. It gently guides students from Newton's gravitational theory to special relativity, then to the approximate linearized relativistic theory of gravitation, and finally to the full nonlinear theory of general relativity. This book views general relativity from several perspectives: as a theory constructed by analogy with Maxwell's electrodynamics, as a relativistic generalization of Newton's theory, and as a theory of curved spacetime. The authors provide a concise overview of the important concepts and formulas, coupled with the experimental results underpinning the latest research in the field. Numerous exercises scattered throughout the chapters help students master essential concepts for advanced work in general relativity and give them practice in the mathematics needed, while abundant spacetime diagrams encourage them to think in terms of four-dimensional spacetime geometry. Featuring comprehensive reviews of recent experimental and observational data, the text concludes with chapters on current developments in cosmology and the physics of the Big Bang and inflation.

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Hans C. Ohanian and Remo Ruffini
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In memory of John Archibald Wheeler (1911–2008),
who showed us the way

Gravitation and Spacetime

Third Edition

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Preface

Einstein discovered his theory of gravitation in 1916. By rights, this theory should not have been discovered until 20 years later, when physicists had acquired a clear understanding of relativistic field theory and of gauge invariance. Einstein's profound and premature insights into the nature of gravitation had more to do with intuition than with logic. In contrast to the admirably clear and precise operational foundations on which he based his theory of special relativity, the foundations on which he based general relativity were vague and obscure. As has been emphasized by Synge and by Fock, even the very name of the theory indicates a misconception: There is no such thing as a relativity more general than special relativity. But whatever murky roads he may have taken, in the end Einstein's intuition led him to create a theory of dazzling beauty. If, using Arthur Koestler's image, we regard Copernicus, Kepler, and Newton as sleepwalkers who knew where they wanted to go and managed to get there without quite knowing how, then Einstein was the greatest sleepwalker of them all.

The aim of this book is to develop gravitational theory in the simplest and most straightforward way – in the way it probably would have developed without Einstein's intervention. This means that we begin with the linear approximation and regard gravitation as the theory of a second-rank tensor field in a flat spacetime background, analogous to electrodynamics. The geometrical interpretation and the nonlinear Einstein equations gradually emerge from this tensor theory as we attempt to understand and improve the equations of the linear approximation. This approach is not new: Gupta, Feynman, Thirring, and Weinberg have presented it from somewhat different points of view and with varying amounts of detail. One advantage of this approach is that it gives a clearer understanding of how and why gravitation is geometry. Another advantage is that the linear theory permits us to delve immediately into the physics: Gravitational redshift, light deflection, lensing, time delay, Lense-Thirring precession, and gravitational radiation can be treated directly in the context of the linear approximation, without any lengthy preliminary digressions on the mathematics of Riemannian spacetime geometry.

After a full exploitation of the results accessible via the linear approximation (Chapters 1–5), we redevelop the gravitation field equations via the geometrical approach pioneered by Einstein (Chapters 6–9). This may seem to be a duplication of effort, but it helps students attain a deeper grasp of the principles. In our exploration of multiple lines of approach, from different perspectives, we are following the example set by Lorentz in his celebrated "Monday lectures" at Leiden, where he would "turn the subject round and round and over and over" to achieve new insights.

As in earlier editions of the book, we enliven the theoretical treatment by presenting relevant experimental and observational results. In its early years, general relativity acquired the reputation of an abstract, highly mathematical theory, with a limited experimental basis. But since the 1960s, general relativity has enjoyed a harmonious and invigorating synergy of theory and experiment, with theory motivating experiments and experiments supporting and confirming theory. The last 30 years have yielded a rich harvest of experimental and observational results, and we try to make the presentation of this information as complete and up to date as possible. As in earlier editions, we include extensive tables of repetitions of experiments and observations, because testing and retesting are what make experimental results credible (*provando e riprovando*, as says the motto of the Academia dei Lincei, of which Galileo was a founding member). However, in contrast to the earlier editions, which aimed to include all repetitions of a given experiment or observation, limitations of space compelled us to make some judicious selections, so the entries in our tables are now restricted to the most recent, most precise, and most memorable results.

In this third edition of the book we retain the organization of the second edition, with various shifts of emphasis, additions, and updates – mostly motivated by new experimental measurements and sometimes by improvements in the theoretical treatment. The following list summarizes the changes relative to the second edition, apart from corrections of various unfortunate misprints.

Chapter 1: New results of measurements of G, new tests of the short-range behavior of the inverse-square law, new data on the quadrupole moment of the Sun from solar oscillations, and new measurements of tidal forces with the Gravity Field and Ocean Circulation Explorer spacecraft. But we deleted most of the previous material on the fifth force, which can now be regarded as refuted.

Chapter 2: Expanded treatment of special relativity, which now goes beyond the mathematical formalism and provides a brief, self-contained introduction to the theory, with concise derivations of the invariance of the spacetime interval, the energy-momentum of particles and of systems, and the energy-mass relation.

Chapter 3: Improved explanations of the connection between the equation of motion of particles and the field equation, and the connection between the equation of motion and the geometric interpretation of gravity.

Chapter 4: Updated experimental and observational results on the gravitational redshift, deflection, and time delay of light and radio waves. Updates on observations of gravitational lensing (especially with the Hubble Space Telescope) and applications to investigations of dark mass and microlensing. Expanded discussion of the orbital and spin precession according to the Lense-Thirring effect and the Laser Geodynamics Satellite results.

Chapter 5: Explicit discussion of the relationship between the polarization states (or spin states) of gravitational waves, their gauge invariance, and the conservation law for the energy-momentum tensor. New data on the Hulse-Taylor pulsar and other binary pulsars and the implications for gravitational radiation. A fuller discussion of sensitivity of LIGO gravitational wave detectors, with omission of most of the previous discussion of the sensitivity of resonant quadrupole detectors, which have now fallen out of favor.

Chapter 6: Geometric interpretation of the Bianchi identities in terms of parallel transport around a parallelepiped and physical interpretation of the Riemann tensor in terms of measurements within small regions; for instance, measurements of small volumes or areas. Also, a full treatment of Fermi coordinates and Fermi-Walker transport.

Chapter 7: Clearer explanation of the motivation underlying the general-invariance symmetry. Elimination of the separate treatment of the Birkhoff theorem, which is now incorporated directly into the Schwarzschild solution. Discussion of the long-awaited final results of the Gravity Probe B experiment on the measurement of the geodetic and Lense-Thirring precession effects.

Chapter 8: Examination of the turning points for motion in the equatorial plane of the Kerr geometry and characterization of the possible circular orbits. Fuller discussion of the irreducible mass when the black hole includes electric charge, as well as implications of the Cauchy horizon for the maximal Kerr geometry. Description of recent calculations of the equilibrium configuration of neutron stars and the critical mass limit according to the novel method of Ruffini et al. based on the gravitational Fermi-Thomas model. Update on the observational evidence for black holes.

Chapter 9: Recent determinations of the Hubble constant, the age of the universe, and the conclusions about the acceleration of the universe extracted from observations of type Ia supernovas by Riess et al. and Perlmutter et al. In accord with the observational evidence, this chapter now emphasizes the spatially flat Friedmann-Lemaître model of the universe with a positive cosmological constant.

Chapter 10: Update of the information on helium abundance and a more detailed treatment – on the basis of the Jeans mass – of the growth of perturbations in the early universe. Also an improved discussion of inflation, especially in regard to the flatness puzzle and the Grand Unified Theory (GUT) phase transition, and an examination of the implications of small-scale anisotropies in the cosmic background radiation detected by the Wilkinson Microwave Anisotropy Probe satellite, leading to the discovery of baryon acoustic oscillations.

Appendix: Direct derivation of the conservation of the energy-momentum tensor from the general-invariance symmetry, and addition of a new section with the general-relativistic theoretical proof of the equality of inertial and gravitational mass.

In this new edition, we retain the exercises that are scattered throughout the chapters as an integral part of the text; they amplify discussions or supply proofs, and they are intended to be done while the book is being read. Only a fanatic will find the time to do them all; readers are invited to consider these exercises as challenges that should not always be refused. We expanded the collection of problems at the ends of the chapters, mostly by the addition of problems from examinations that were given to students at Rensselaer Polytechnic Institute and at the University of Vermont. However, we deleted the extensive, annotated Further Reading sections at the ends of the chapters in the earlier editions, because inclusion of the numerous recent publications would have made these sections too long and unwieldy. Online searches on the Web are a more efficient way to survey the literature today, and there are excellent resources available that give updates on the latest progress in theoretical and experimental relativity; for instance, *Living Reviews in Relativity* published online by the Max Planck Institute (relativity.livingreviews.org), the *Resource Letters* in the *American Journal of Physics*, and a concise section on general relativity and cosmology in the *Review of Particle Physics* published in even-numbered years by the Particle Data Group (pdg.lbl.gov).

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H. C. O. and R. R. September 2012

Constants

Fundamental constants

Speed of light	$c = 3.00 \times 10^{10}$ cm/s
Planck's constant	$\hbar = 1.05 \times 10^{-27}$ erg · s $= 6.58 \times 10^{-22}$ MeV · s
Gravitational constant	$G = 6.67 \times 10^{-8}$ cm^3 g^{-1} s^{-2}
	$\kappa = (16\pi G/c^4)^{1/2} = 2.04 \times 10^{-24}$ (cm · g)$^{-1/2}$ s
Planck length	$l_{Pl} = (\hbar G/c^3)^{1/2} = 1.62 \times 10^{-33}$ cm
Planck time	$t_{Pl} = (\hbar G/c^5)^{1/2} = 5.39 \times 10^{-44}$ s
Planck mass	$m_{Pl} = (\hbar c/G)^{1/2} = 2.18 \times 10^{-5}$ g
	$= 1.22 \times 10^{19}$ GeV/c^2
Electron mass	$m_e = 0.911 \times 10^{-27}$ g $= 0.511$ MeV/c^2
Proton mass	$m_p = 1.67 \times 10^{-24}$ g $= 938$ MeV/c^2
Neutron mass	$m_n = m_p + 2.31 \times 10^{-27}$ g $= m_p + 1.29$ MeV/c^2
Proton charge	$e = 4.80 \times 10^{-10}$ esu
Fine structure constant	$\alpha = e^2/\hbar c = 1/137.0$
Compton wavelength	$\lambda\!\!\!^{-}_C = \hbar/m_e c = 3.86 \times 10^{-11}$ cm
Bohr radius	$a_0 = \hbar^2/m_e e^2 = 0.529 \times 10^{-8}$ cm
Boltzmann constant	$k = 1.38 \times 10^{-16}$ erg/K $= 8.62 \times 10^{-5}$ eV/K
Stefan-Boltzmann constant	$\sigma = \pi^2 k^4/60\hbar^3 c^2 = 5.67 \times 10^{-5}$ g · s^{-3} K^{-4}

Conversion constants

1 year (y) $= 3.16 \times 10^7$ s
1 astronomical unit (A.U.) $= 1.50 \times 10^{13}$ cm
1 light year (ly) $= 0.946 \times 10^{18}$ cm
1 parsec (pc) $= 3.26$ ly $= 3.09 \times 10^{18}$ cm
1 second of arc (arcsec) $= 4.85 \times 10^{-6}$ radian
1 electron volt (eV) $= 1.6 \times 10^{-12}$ erg

Astronomical constants

Sun	
mass	$M_\odot = 1.99 \times 10^{33}$ g
radius	$R_\odot = 6.96 \times 10^{10}$ cm
surface gravity	$g_\odot = 2.74 \times 10^{4}$ cm/s^2
luminosity	$L_\odot = 3.8 \times 10^{33}$ erg/s
Earth	
mass	$M_E = 5.98 \times 10^{27}$ g
equatorial radius	$R_E = 6.38 \times 10^{8}$ cm
polar radius	$R'_E = R_E - 2.15 \times 10^{6}$ cm
surface gravity	$g = 9.81 \times 10^{2}$ cm/s^2
moment of inertia, polar axis	$I^{33} = 0.331\, M_E R_E^2$
moment of inertia, equatorial axis	$I^{22} = I^{11} = 0.329\, M_E R_E^2$
period of rotation 1 sidereal day	$= 8.62 \times 10^{4}$ s
mean distance to Sun, 1 A.U.	$= 1.50 \times 10^{13}$ cm
orbital period, 1 year	$= 3.16 \times 10^{7}$ s
orbital speed	$= 29.8$ km/s
Moon	
mass	$M_☾ = 7.35 \times 10^{25}$ g
radius	$R_☾ = 1.74 \times 10^{8}$ cm
mean distance from Earth	$= 3.84 \times 10^{10}$ cm
orbital period, 1 sidereal month	$= 27.3$ days
Universe	
Hubble constant	$H_0 = 100h$ km/(s $\cdot$ Mpc), with $h \cong 0.70$
critical density	$3{H_0}^2/8\pi G = 1.88 \times 10^{-29}\, h^2$ g/cm^3 $\cong 0.92 \times 10^{-29}$ g/cm^3
density parameters	$\Omega_{m,0} = 0.27,\ \Omega_{\Lambda,0} = 0.73$

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Notation

The components of a 3-D vector **A** with respect to 3-D rectangular coordinates will be indicated by superscripts with the values 1, 2, 3:

$$A^1 = A_x\,, \quad A^2 = A_y, \quad A^3 = A_z$$

For the position vector **x**, the components are

$$x^1 = x, \quad x^2 = y, \quad x^3 = z$$

The symbol A^k, where the Latin superscript k takes on the values $k = 1, 2, 3$, then stands for the kth component of the vector. If no particular value of k is specified, the symbol A^k will also stand for the set (A^1, A^2, A^3) of all the components taken together; in the latter case, A^k represents the entire vector **A**.

The Einstein summation convention applies: when a repeated Latin index appears in a term in an equation, a summation is to be carried out over the values 1, 2, 3 of that index, for example

$$A^n B^n \equiv \sum_{n=1}^{3} A^n B^n = A^1 B^1 + A^2 B^2 + A^3 B^3$$

The 3-D Kronecker delta will be written as

$$\delta^n_m = \begin{cases} 1 & \text{if } m = n \\ 0 & \text{if } m \neq n \end{cases}$$

Integration over a 3-D volume will be written as

$$\int f(\mathbf{x}) d^3x \equiv \iiint f(\mathbf{x}) dx\,dy\,dz$$

The components of a 4-D vector will be indicated by superscripts with the values 0, 1, 2, 3. In flat 4-D spacetime, with rectangular coordinates ct, x, y, z,

$$A^0 = A_t, \quad A^1 = A_x, \quad A^2 = A_y, \quad A^3 = A_z$$

and

$$x^0 = ct, \quad x^1 = x, \quad x^2 = y, \quad x^3 = z$$

The symbol A^μ, where the Greek superscript takes on the values $\mu = 0, 1, 2, 3$, stands for the μth component of the vector; it also stands for the set (A^0, A^1, A^2, A^3), and in the latter case represents the entire 4-D vector.

The definition of the 4-D Kronecker delta is the same as in the 3-D case:

$$\delta^\nu_\mu = \begin{cases} 1 & \text{if } \mu = \nu \\ 0 & \text{if } \mu \neq \nu \end{cases}$$

When a repeated Greek index appears in a term of an equation, a summation is to be carried out over the values 0, 1, 2, 3 of that index, for example,

$$\eta_{\mu\nu}A^{\nu} \equiv \sum_{\nu=0}^{3} \eta_{\mu\nu}A^{\nu} = \eta_{\mu 0}A^{0} + \eta_{\mu 1}A^{1} + \eta_{\mu 2}A^{2} + \eta_{\mu 3}A^{3}$$

The Minkowski metric tensor of flat spacetime is taken as

$$\eta_{\mu\nu} = \begin{pmatrix} \eta_{00} & \eta_{01} & \eta_{02} & \eta_{03} \\ \eta_{10} & \eta_{11} & \eta_{12} & \eta_{13} \\ \eta_{20} & \eta_{21} & \eta_{22} & \eta_{23} \\ \eta_{30} & \eta_{31} & \eta_{32} & \eta_{33} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

and the spacetime interval of flat spacetime is

$$ds^2 = \eta_{\mu\nu}dx^{\mu}dx^{\nu} = \sum_{\mu=0}^{3}\sum_{\nu=0}^{3} \eta_{\mu\nu}dx^{\mu}dx^{\nu} = (cdt)^2 - (dx)^2 - (dy)^2 - (dz)^2$$

In the last expression, the parentheses will often be omitted, so the right side becomes $c^2dt^2 - dx^2 - dy^2 - dz^2$ (this somewhat careless notation for a second-order differential imitates what is routinely done in second derivatives, for instance $\frac{d}{dz}\frac{d}{dz}f = \frac{d^2}{dz^2}f$).

The spacetime interval of curved spacetime is

$$\begin{aligned} ds^2 = g_{\mu\nu}dx^{\mu}dx^{\nu} &= \sum_{\mu=0}^{3}\sum_{\nu=0}^{3} g_{\mu\nu}dx^{\mu}dx^{\nu} \\ &= g_{00}c^2dt^2 + g_{01}cdtdx + g_{10}cdxdt + \cdots + g_{33}dz^2 \end{aligned}$$

If x^0 is the time coordinate, then $g_{00} > 0$ (this is called the timelike sign convention).

In general, indices are raised and lowered with the metric tensor of curved spacetime, for instance,

$$A_{\mu} = g_{\mu\nu}A^{\nu}$$

However, in all equations that are written in the linear approximation, the indices are raised and lowered with the Minkowski metric tensor $\eta_{\mu\nu}$.

Partial derivatives are indicated by a comma or by the differential operator ∂,

$$\frac{\partial f}{\partial x^{\mu}} = f_{,\mu} = \partial_{\mu} f$$

A dot over a variable indicates a derivative with respect to time (for example, $\dot{z} = dz/dt$ in Chapter 5 and in the Appendix) or a derivative with respect to proper time (for example, $\dot{r} = dr/d\tau$ in Chapter 8) or a derivative with respect to a "time parameter" (for example, $\dot{a} = da/d\eta$ in Chapter 9).