Index

Airy stress functions formulation of 60-1 table of 426 alternator operator 419 Brown's formula 255 Burgers equation 264-5 Christoffel stiffness tensor 34 "corresponding" elastic fields 25-7 curvature tensor, κ_{ij} 347 relationship to state of dislocation tensor, $\alpha_{ii}, 347-8$ cylindrical curvilinear coordinate system 411 defect core regions, description of 3-4 defect source of stress in homogeneous body, interactions with stress force on defect by use of energy-momentum tensor when applied stress 107 when image stress 107 when internal stress 107 force on defect due to change in interaction energy when displaced basic formulation of 103 when applied stress 112 when image stress 112 when internal stress 112 force on defect due to change in total system energy when displaced basic formulation of 103 when applied stress 108-9 when image stress 110-11 interaction energy of defect with image stress 100-2 interaction energy of defect with imposed applied stress when defect represented by transformation strain 99 - 100defect represented by its elastic field 98-9 defect represented by point body forces 100 interaction energy of defect with imposed internal stress when

defect represented by a transformation strain 96-7 defect represented by its elastic field 95-6 defect represented by point body forces 97-8 see also point defect, interaction with stress; dislocation, force on (due to stress); dislocation in finite region, image effects; inclusion, interaction with imposed stress defects, classification of interactions with various types of stress 93 defects, types considered in book 1 del operator \bigtriangledown , basic relationships involving \bigtriangledown expressed in cylindrical coordinates 411 expressed in spherical coordinates 411-12 delta function, properties of 417-18 deviatoric stress and strain in isotropic system 427 dislocation-dislocation interactions force between loops 397 rational differential segments 396-7 straight segments 397 force in isotropic system between differential segments 398 long straight parallel edge dislocations 409-10 loops 398 straight segments 398, 408-9 interaction energy between loops 391 rational differential segments 388-91 straight non-parallel segments 392 straight parallel segments 391-2 interaction energy in isotropic system between long straight parallel dislocations, 406-7 loops 393-4, 407-8 straight non-parallel segments 395-6 straight parallel segments 395 dislocation fan description of 325 role as source of image stress 325 stress field of 330 dislocation, force on (due to stress) force due to image stress 307-9 forces on long straight edge and screw dislocations 335-6

435

Index

net force on loop 337 Peach-Koehler force equation 304-7, 336-7 dislocation, geometrical features of Burgers circuit 231 FS/RH rule for 231 SF/RH rule for 231 Burgers vector 229, 231 edge component of 232 screw component of 232 edge type 229-30 mixed type 230 screw type left-handed 230 right-handed 230 \sum cut and displacement rule 232 \sum cut surface 229 unit normal vector to \sum cut surface, \hat{n} 229, 232 direction of \hat{n} rule 232 unit tangent vector 229 dislocation in finite region, image effects infinitesimal loop near planar surface of isotropic half-space 334-5 interaction with image stress, general formulation of force 307-9 interaction energy 307 loop near planar interface between dissimilar half-spaces 335 loop near planar surface of half-space 333-4 loop near planar surface of isotropic half-space 334 straight long dislocation impinging on planar surface of half-space edge dislocation at normal incidence in isotropic system 332-3 general inclined dislocation 325-31 inclined dislocations in isotropic system 333 screw dislocation at normal incidence in isotropic system 331-2 straight long dislocation parallel to free surface edge dislocation along axis of isotropic cylinder 315-17 edge dislocation in isotropic half-space with planar surface 314-15, 338-9 general dislocation in half-space with planar surface 309-12 screw dislocation along axis of isotropic cylinder 313-14 screw dislocation in isotropic half-space with planar surface 312-13 screw dislocation parallel to axis of isotropic cylinder 337-8 straight long dislocation parallel to planar interface between dissimilar half-spaces edge dislocation in isotropic system 324 general dislocation 317-23 screw dislocation in isotropic system 323-4

straight long screw dislocation parallel to surfaces of plate 338 dislocation in interface see interfacial dislocation dislocation-inclusion interaction force exerted on dislocation by spherical inhomogeneous inclusion with $\varepsilon_{ii}^{T} = \varepsilon^{T} \delta_{ii}$ in isotropic system 406 general formulation of 405-6 dislocation, production of (by cut and displacement) 229 rule for directions of displacements at cut surface 232 rule for direction of positive unit vector to cut surface 232 dislocation loop, infinitesimal characterization of 260 elastic field of 260-2 elastic field of (in isotropic system) 268-9 dislocation loop, smoothly curved elastic field of, by use of hairpin dislocations and Brown's formula 252-7, 301 infinitesimal loops 262 modified Burgers equation 249-51 Mura equation 247-9, 302-3 rational differential dislocation segments 257-60 Volterra equation 245-6 strain energy of 263 dislocation loop, smoothly curved in isotropic system elastic field of, by use of Burgers equation 264-5 infinitesimal dislocation loops 268-9, 301 multi-straight segment approximation 282 Peach-Koehler equation 265-8 strain energy of 270, 299-301 dislocation-point defect interaction energy see point defect-dislocation interaction energy dislocation, segmented structure elastic field of angular dislocation 275 planar dislocation node 274-5 planar polygonal loop 275-7 straight segment by use of Brown's formula 271-2 straight segment by use of Mura equation 272-4 three-dimensional multi-segment structure 277 - 8strain energy of multi-segment structure 279 straight segment 278-9 dislocation, segmented structure in isotropic system elastic field of angular dislocation 282-3

436	Index					
	dislocation, segmented structure in isotropic	Lamé constants 28				
	system (cont.)	Poisson's ratio 28				
	straight segment 279-82	shear modulus 28				
	three-dimensional multi-segment structure	Young's modulus 29				
	by use of triangular loops 283–7	elastic displacement 5				
	strain energy of	elastic distortion 7				
	multi-segment structure 288–91	elastic stiffness tensor 21				
	straight segment 287-8	contracted notation for 23				
	dislocation, straight and infinitely long	for cubic crystal 23				
	elastic field of (by use of integral formalism) 233–7	relation to elastic compliance tensor 24–5				
	displacement field 236, 292	symmetry of 21-2, 23				
	distortion field 236	transformation of components 22				
	stress field 236	elastic strain tensor 7				
	traction on plane containing dislocation	compatibility condition for 13-15				
	line and field point 237	cubical dilatation 13				
	elastic field of (by use of Volterra equation)	eigenvalues 11				
	246-7	eigenvectors 11				
	strain energy of (by use of integral formalism)	principal coordinate system for 10-1				
	238–40	principal directions 10				
	core energy parameter, α 240	principal strains 10				
	strain energy factor 240	strain ellipsoid 12-13				
	dislocation, straight and infinitely long in	strains, normal 7				
	isotropic system	strains, shear 8				
	elastic field of edge dislocation by use of	trace of 13				
	Airy stress function 296	transformation of components 9-10				
	integral formalism in Cartesian, polar	equilibrium, equation of 18				
	and cylindrical coordinates 240-1	Hooke's law, for				
	Mura equation 294–5	general system 21, 24				
	straight segment 297-8	isotropic system 27				
	transformation strain formalism 242-3	incompatibility tensor 15				
	Volterra equation 292, 293	linear superposition, principle of 3				
	elastic field of mixed dislocation 244	rigid body rotation 8–9				
	elastic field of screw dislocation by use of	rotation matrix 8				
	integral formalism in Cartesian, polar and	rotation vector 8				
	cylindrical coordinates 243-4	strain energy				
	transformation strain formalism 295-6	general system 29–31				
	triangular loop 298–9	isotropic system 31				
	strain energy of	physical interpretation of 29–30				
	edge dislocation 244, 298	stress tensor 16				
	mixed dislocation 244	contracted notation for 23				
	screw dislocation 244	normal stress 16				
	divergence (Gauss') theorem 413	principal coordinate system for 20 shear stress 16				
	elasticity, basic elements of linear theory of	symmetry of 19				
	body force density distribution 18	trace of 20				
	body forces 15, 18	transformation of components 19–20				
	"corresponding" elastic fields 25–7	traction vector 15–17				
	deviatoric stress and strain 427	elasticity, basic elements of linear theory				

deviatoric stress and strain 427 elastic compliance tensor 24 contracted notation for 24 for cubic crystal 24 relation to elastic stiffness tensor 24–5 symmetry of 24 transformation of components 24 elastic constants for isotropic system 27–9

bulk modulus 29

of (when transformation strain present) 55–9 compatibility condition 57 planar elastic stiffness tensor 58–9, 63 relationship between Fourier amplitudes of transformation strain and associated stress 58 total displacement 57

437

Index

total strain 57 see also elasticity theory for defects, basic methods of (for solving problems) elasticity, selected equations of linear theory of (for isotropic medium) expressed in cylindrical coordinates 421-2 expressed in spherical coordinates 423 elasticity theory for defects, basic methods of (for solving problems) by use of Fourier transforms, when body forces present 34 transformation strains present 59 by use of Green's functions, when body forces present 36 transformation strains present 59-60 by use of image stresses when interfaces present 61-3 by use of integral formalism when two-dimensional 38, 52-5 by use of sextic formalism when two-dimensional 38-52 by use of stress functions 60 Airy stress functions 60-1 by use of transformation strains 55-9 elasticity theories, types of linear 3 non-linear 4 size-dependent 4 elasto-mechanical energy, definition of 95 energy-momentum tensor 106 formulation of 103-6 "engineering" shear strain 8 equivalent homogeneous inclusion method for inhomogeneities 188-9 for inhomogeneous inclusions 126-8 Eshelby tensor for homogeneous ellipsoids of revolution in isotropic systems 424-5 needle 425 sphere 424 thin-disk 425 form of 123-4 form of (in isotropic system) 136-7 symmetry properties of 136-7 field point, description of 35 force multipole elastic field of 210 elastic field of (in isotropic system) 208-10 force dipole moment tensor force dipole moment approximation 210 - 11forms for different crystal symmetry systems, table of 211-12

produced by body forces mimicking point defect 204–8 produced by surface tractions 225

relationship to corresponding force density distribution 212-13 force octopole moment tensor 204 force quadrupole moment tensor 204 effect of inversion symmetry 206 types of combinations of double forces 206 elementary double force 205 further examples 206-8 volume change due to 214 see also point defect forces on defects see defect source of stress in homogeneous body, interactions with stress; energymomentum tensor; inhomogeneity, interaction with imposed stress Fourier transform, definition of three-dimensional 420 two-dimensional 65 Fourier transform methods for determining elastic displacements 34, 59 for determining point force Green's functions 66-9, 72-8 see also elasticity theory for defects, basic methods of (for solving problems) Frank-Bilby equation application of 351-2, 359-60, 373, 374, 381-2 derivation of 348-51 form of 351 Gauss' (divergence) theorem see divergence (Gauss') theorem Green's function method for electrostatic problems 34-6 Green's functions for unit point force in half-space joined to dissimilar half-space 75-8

half-space joined to dissimilar half-space in isotropic system 85 half-space with planar interface 72–5 half-space with planar interface in isotropic

system 86–7 infinite isotropic region 85–6, 88–9 infinite region 66–9, 89–90 basic equation for 36–7 Fourier transform of 37 spatial derivatives of 70–2, 91–2 *see also* elasticity theory for defects, basic methods of (for solving problems)

hairpin dislocation description of 252 stress field of 252 harmonic function, definition of 81 hetero-elastic interface *see* interface, hetero-elastic

438

image stresses, basic method of 61-3 impotent dislocation array 352 inclusion, characterization of 116-17 inclusion, coherent -> incoherent transition of inhomogeneous inclusion with uniform ε_{ii}^{T} in isotropic system results for ellipsoidal inclusion 149-51 treatment of 147-9 inclusion-dislocation interaction force see dislocation-inclusion interaction force inclusion, elastic field in isotropic system when coherent, homogeneous, arbitrary shape, uniform ε_{ij}^{T} 130–3, 153 coherent, homogeneous, ellipsoidal, uniform ε_{ii}^{T} 133-40, 155 coherent, homogeneous, spherical, $\varepsilon_{ij}^{T} = \delta_{ij}\varepsilon^{T}$, 151 - 2coherent, inhomogeneous, ellipsoidal, uniform ε_{ij}^{T} 140–1 spherical, $\varepsilon_{ij}^{\mathrm{T}} = \delta_{ij}\varepsilon^{\mathrm{T}}$ 141–2, 155–7 incoherent, inhomogeneous, ellipsoidal, uniform ε_{ij}^{T} needle 150 sphere 149-50 thin-disk 150, 158 inclusion, elastic field of (when coherent and homogeneous) arbitrary shape and ε_{ij}^{T} 118–19 ellipsoidal, arbitrary ε_{ij}^{T} 119–22 ellipsoidal, non-uniform ε_{ii}^{T} represented by polynomial 124-6 ellipsoidal, uniform ε_{ij}^{T} 123–4 inclusion, elastic field of (when coherent and inhomogeneous) ellipsoidal, non-uniform ε_{ii}^{T} represented by polynomial 128 ellipsoidal, uniform ε_{ii}^{T} 126–8 inclusion in finite region, image effects elastic field and force when homogeneous arbitrary shape, uniform ε_{ii}^{T} , near interface 174-5 spherical, $\varepsilon_{ij}^{T} = \delta_{ij}\varepsilon^{T}$, isotropic system, near interface 177–8, 180–1 image field when homogeneous, spherical, $\varepsilon_{ii}^{\mathrm{T}} = \delta_{ij}\varepsilon^{\mathrm{T}}$, isotropic system, far from interface 171-2 image force when homogeneous 175-7 strain energy of 179, 184-5 volume change due to image field 172-4, 180, 182-3, 185-6 inclusion-inclusion interaction energy between homogeneous inclusions 399-401 between inhomogeneous inclusions 401 inclusion, interaction with imposed stress homogeneous case force when spherical, $\varepsilon_{ij}^{T} = \delta_{ij}\varepsilon^{T}$, isotropic system 160-2, 167

interaction energy 159-60, 167-8 inhomogeneous and ellipsoidal case interaction energy and force when spherical, $\varepsilon_{ii}^{\rm T} = \delta_{ij}\varepsilon^{\rm T}$, isotropic system 165–6, 168–9 interaction energy, formulation of 163-5 perturbation of imposed stress field 163, 169 - 70inclusion, model for point defect 213 inclusion-point defect interaction energy between point defect and spherical inhomogeneous inclusion with $\varepsilon_{ii}^{T} = \varepsilon^{T} \delta_{ii}$ in isotropic system 405 general formulation of 404 inclusion, strain energy when coherent, homogeneous, ellipsoidal, $\varepsilon_{ii}^{T} = \varepsilon^{T} \delta_{ij}$, isotropic system 154-5 coherent, homogeneous, ellipsoidal, uniform $\varepsilon_{ij}^{\mathrm{T}}$, isotropic system needle 143 sphere 143 thin-disk 143 coherent, homogeneous or inhomogeneous, arbitrary shape and ε_{ij}^{T} 128–9, 153–4 coherent, inhomogeneous, ellipsoidal, $\varepsilon_{ij}^{\mathrm{T}} = \varepsilon^{\mathrm{T}} \delta_{ij}$, isotropic system sphere 144, 157 thin-disk 144-5 coherent, inhomogeneous, ellipsoidal, uniform ε_{ij}^{T} , isotropic system needle 146–7 sphere 146-7 thin-disk 146-7 incoherent, inhomogeneous, ellipsoidal, uniform $\varepsilon_{ii}^{\mathrm{T}}$, isotropic system needle 150 sphere 149-50 thin-disk 150, 158 inhomogeneity, interaction with imposed stress force, when non-uniform inhomogeneity and imposed stress 196-8 uniform spherical inhomogeneity and hydrostatic stress 195-6 force, basic formulation when attributed to change in interaction energy 113 attributed to change in total system energy 113 obtained by use of energy-momentum tensor 113-14 interaction energy basic formulation of 113 when uniform ellipsoidal inhomogeneity and imposed stress 189-92 when uniform ellipsoidal inhomogeneity and imposed stress in isotropic system 193-4

Index 439

when uniform spherical inhomogeneity and hydrostatic stress 194, 200 perturbation of imposed stress, when uniform ellipsoidal inhomogeneity and imposed stress 188-9 uniform ellipsoidal inhomogeneity and imposed stress in isotropic system 192-3 uniform spherical inhomogeneity and hydrostatic stress 198-200 integral formalism for two-dimensional elasticity problems 52-5 [Q], [S] and [B] matrices 52-4 [Q], [S] and [B] matrices for isotropic system 54 - 5interaction strain energy between internal and external stress 99 interface, force on energy-momentum tensor" force 378-80 "interfacial dislocation" force 378 when heterophase interface 384-5 when large-angle homophase interface 383-4 when small-angle asymmetric tilt interface 377-8, 381-3, 385 when small-angle symmetric tilt interface 377-8, 380-1 interface, hetero-elastic elastic field of, when array of parallel dislocations in interface 370 - 3array of parallel dislocations in interface, isotropic system 362-6 single dislocation in interface 366-9 single dislocation in interface (isotropic system) 360-2 strain energy of single dislocation in interface 369 - 70interface, iso-elastic elastic field when array of parallel dislocations in interface 353-5 array of parallel dislocations in interface, isotropic system 355-7 strain energy (interface energy) when array of parallel dislocations in interface, isotropic system 357-8 interfaces, classification and description of geometrical features macroscopic degrees of freedom 341 procedure for creating planar interface in bicrystal 341, 342, 374-5 hetero-elastic definition of 340 epitaxial type 359 heterophase interface 341 homophase 341 iso-elastic definition of 340

large-angle homophase (vicinal) 343-5 small-angle homophase (tilt, twist, and mixed) 342-3 interfacial dislocation elastic field of 360-2, 366-9 strain energy of 369-70 terminology disconnection 383 dislocation with step character 383 transformation dislocation 383 iso-elastic interface see interfaces, iso-elastic lambda tensor, λ 221–2 line of force, straight and infinitely long displacement field of 238 displacement field of (in isotropic system) 365, 376 distortion field of 238 modified Burgers equation 249-51 Mura equation 248 Navier equation 33 nuclei of strain 269 Papkovitch functions for point force in half-space joined to dissimilar half-space in isotropic system point force normal to interface 80-4 point force parallel to interface 84 for point force in half-space with planar surface 86 for point force in infinite homogeneous body 85 general formulation of 79-80 Peach-Koehler equations for force on dislocation 306, 336-7 for stress field of dislocation loop 267-8 planar elastic stiffness tensor 58-9, 63 point defect force multipole model for 203-5 small inclusion model for 213 structure and symmetry of 201-3 inversion symmetry 206 see also force multipole point defect-dislocation interaction energy between point defect and screw dislocation in isotropic system 402-4 general formulation of 401-2 point defect-inclusion interaction energy see inclusion-point defect interaction energy point defect, interaction with stress force 216-17 interaction energy 215-16 see also defect source of stress in homogeneous body, interactions with stress

440	Index				

point defect-point defect interaction energy between two point defects in isotropic system 387-8 general formulation of 386-7 point defects in finite bodies, image effects for single point defect volume change of body 217-18, 224-7 for statistically uniform distribution of point defects induced change in X-ray lattice parameter 222-4 induced shape change of body 221-2, 227, 228 induced stress and volume change of body 218 - 21 $\lambda^{(p)}$ tensor 221–2 Vegard's law 223 polynomial theorem for coherent ellipsoidal inclusion with polynomial transformation strain 126, 128, 152-3 potentials biharmonic 130 harmonic (of layer) 138 Newtonian 130 at external points around ellipsoid 139 at external points around sphere 142 at interior points in sphere 178 rational differential dislocation segment geometrical characterization of 257 stress field of 260 sextic formalism for two-dimensional problems 38 - 52coordinate systems used 38

sextic eigenvalue problem 39 eigenvalues 39–40 eigenvectors 40

Stroh vectors 40 completeness of 45-6 invariance of 46-9 normalization of 40 orthogonality of 43-5 sum rules for 49-50 source point, description of 35 spherical curvilinear coordinate system 411 Stokes' theorem 413-15 stress functions 60 see also Airy stress functions stress, types of applied 93 canceling 56 deviatoric 427 image 61-3 imposed 93 internal 93 St.-Venant's principle 31 "state of dislocation tensor", α_{ii} 345–6 relationship to curvature tensor 347-8 surface dislocation 352 tensor product of two vectors 416 transformation strain description of 56 procedure for creation of 56 see also elasticity, basic elements of linear theory of (when transformation strain present); elasticity theory for defects, basic methods of (for solving problems) vector field

irrotational 79 solenoidal 79 Vegard's law 223 Volterra equation 246, 262