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Introduction to Modeling Real-World Problems

1.1 Introduction: Goals of this Book

In many situations faced by the product designer or researcher, the initial step – how to formulate the problem – is unclear, and the presence of multiple thermal phenomena complicate this initial step further. The central challenge is often to make a simplified model of the system or process and to obtain some first-order approximations for the magnitude of the key variables. This textbook presents an introduction to methods of thermal modeling and associated approximation tools. A broad rang of techniques is discussed. Some will give only order of magnitude estimates, while others may provide very close approximations, often without laborious calculations. We will generally approach these tools and methods as means of evaluating new, initially complicated heat transfer problems, identifying the governing physical phenomena and establishing their magnitude. Our hope is to provide readers with a much greater facility to deal with these challenges.

In this chapter, we review the engineering approach for defining a problem and exploring possible solutions. Several broad tools are considered, ranging from examination of the very general context of the problem to specific steps used to begin an initial estimate of the performance. The application to complex real-world problems is contrasted with the more controlled approach used in simplified academic exercises.

The first step in proper modeling is to make a careful definition of the goals of the project. The important factors to be considered in identifying goals are discussed next. The balance of the first chapter is devoted to an introduction to modeling techniques that can be employed in the start-up phase of the new concept or design. These same techniques are illustrated in the succeeding chapters, where we will develop modeling approximations for a number of different heat transfer processes.

1.2 The Art of Engineering: What Textbooks Don't Cover

Traditional textbooks in heat transfer and other thermal sciences concentrate on the development of fundamental principles and their corollaries, usually through the analysis of well-defined situations with relatively simple geometries. In many cases, a closed-form solution results. Emphasis is placed on obtaining very accurate solutions to very well defined problems. Although this may be a satisfactory way to initiate students into a new field, it leaves many ill prepared to deal with challenges that arise in engineering practice.

In the design of new processes or innovative techniques to improve present systems, problems are seldom well defined. It is often unclear what analysis is required. In the initial design phase, thermal performance must be evaluated in concert with many other considerations such as size, cost, weight, and other indices related to (for example) electronics or kinematic performance. Usually, many different designs are being considered simultaneously. What are needed at this stage are modeling techniques and tools that allow the engineer to do a first-order estimate of thermal performance that can be used in the initial design stage.

Consider the opposite approach: the temptation is to immediately begin a detailed comprehensive thermal analysis of specific designs, usually with a major component of the analysis being carried out by computer. The analysis is comprehensive enough to include all possible parameters that may come into play. Alternatively, an engineer working on improvements to a complex existing system may be unable to define and carry out a complex analysis, and may instead react by carrying out a detailed cycle of experimental testing in which multiple parameters are varied across a wide range.

The problem with either the detailed analysis or the experimental approach is twofold: the engineer does not initially concentrate on the most important aspects of the problem – understanding what is really happening – and in the rush to develop complicated computer solutions or experiments no time is left to separate the forest from the trees. In many cases, this leads to very sophisticated analyses or detailed experimental measurements of the wrong problem.

EXAMPLE

The need for proper synthesis and modeling is best illustrated by a practical example. In the following, we discuss how simple models can help guide the engineer in defining the problem and exploring possible solutions.

Example 1.1 ***Office copying machines*** Photocopiers use static charge to attach fine black particles, the toner, to a drum and then to blank paper. In large copying machines, the toner is fused to the paper by heating it while simultaneously applying pressure. The paper, with the toner attached, is passed over a heated roll called a fuser roll (see Fig. 1.1). A second roll presses the paper tightly against the fuser roll. To avoid delays in warming up the system, the fuser roll is kept at an elevated temperature when the machine is idling between jobs. The heat lost from the fuser roll results in substantial standby energy consumption.

The goal is to design a copier that is very energy efficient while still offering quick startup. The desire for fast on-demand performance from an office copying

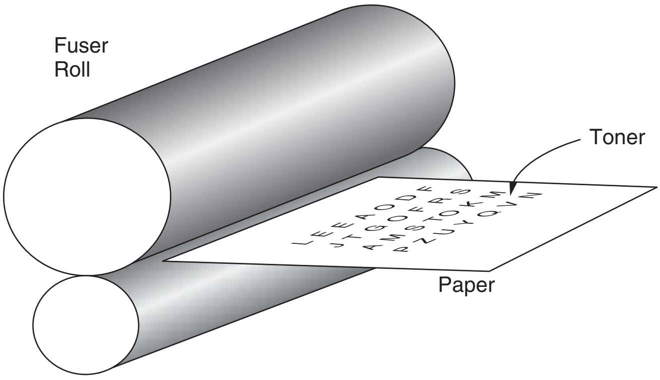

Figure 1.1. Fuser roll configuration for copying machines.

machine is in conflict with the desire to make the office equipment more energy efficient. In particular, current copiers implement a fast startup feature by keeping the fuser roll warm at all times, thereby using substantial energy in standby mode.

An innovative new design would maintain rapid response while doing away with most of the heat loss. There are several possible design strategies. One approach is better insulation of the fuser roll to minimize heat loss while it is idling at elevated temperature. Another is to find a means to heat the fuser roll rapidly when copies are needed. In this case, the fuser roll is kept at low temperatures during idling, reducing the heat lost. Still another possibility is the development of a new technique to fuse the print on the paper without the need for a fuser roll.

Before embarking on any of these design innovations, it would be well to verify that the heat lost from the fuser roll is the major source of energy consumption during the idling period. This involves approximating the heat lost from the surface of the fuser roll by convection and radiation, as well as estimating the conduction heat loss through the bearings and supports of the roll. A simple measurement of the energy consumption during idling may be called for to check those estimates.

The estimation of the energy loss from the fuser roll will also identify the most important sources of heat loss. This can point to the most effective ways to reduce that loss while the fuser is idling at high temperature. For example, if infrared radiation is the largest source of heat transfer from the fuser roll, the use of radiation shields around the fuser and the use of a low-emissivity coating on the fuser could be effective means to improve the copier's energy efficiency.

To explore the second strategy, rapid fuser heat-up, a completely different set of issues must be investigated. These issues include possible designs of a fuser with a low thermal capacitance (so-called thermal inertia) or a more rapid way of heating the surface of the fuser in contact with the paper.

A technology that omits the fuser would include consideration of new ways to heat the paper and print, possibly without direct contact, such as infrared sources.

These alternative design options for the energy-efficient copier can be explored effectively by the use of very simple thermal models. Such simple thermal models are appropriate as part of an overall design exploration that must also include possible new geometries and various materials of construction.

1.3 The Engineering Approach

The focus of this text is on thermal modeling. However, it is important to think of modeling in the context of the overall design project or desired problem solution. To begin a design process, several broad rules should be considered, as we outline next. That is not to say each project is best handled by following these steps by rote. The nature of the creative process requires considerable flexibility in the thought process as well.

The general steps to formulating a useful model follow this pattern:

- Identify what is the overall objective. What are the performance criteria in terms of energy efficiency, weight, and speed of operation, for example? What are the costs, time frame, and resources available to achieve these? If this is an improvement of an existing process, what are the key bottlenecks or shortcomings of that process?
- What is the initial set of likely new systems that can possibly meet the performance and other, desired criteria? For existing processes, what new concepts might help overcome the bottlenecks? For each of these prospective systems, an initial estimate of performance should be obtained. The focus here will be on the estimates of performance, but it should be understood that the other criteria must also be examined. In some cases, criteria unrelated to physical performance may be the overriding concern.
- For the system selected, an approximate definition should be stated. Again, what are the most important performance characteristics and system constraints? Attention should be focused on key elements of those performance criteria. A typical system configuration is assumed; alternatively, desired performance goals are set and the system configuration to meet those goals is found.
- Approximations are made about the system. Here the goal is to simplify the initial modeling and focus on the controlling parameters. The process might include developing assumptions that simplify the modeling. The assumptions will need to be justified, for example, by order of magnitude calculations.
- The performance of the system is estimated keeping in mind the level of detail necessary at this stage. The results are used to compare various design concepts and to eliminate those that are clearly inferior or cannot reach the desired goals. It is also important at this stage to ensure that the approximations, system definitions, and performance estimates look reasonable. That is, does the model give

believable results? If it does not, the approximations, definitions, and modeling steps need to be repeated. One simple check is to use the same modeling steps for a related, simplified system in which the performance or solution is known.

- The performance of the new system must be considered in light of existing devices and desired performance improvements. In some cases, it will be necessary to make a fresh start, identifying other candidate system designs.

Some engineering textbooks formulate problem solving into a rigid sequence of steps that resemble the latter steps in the preceding list. The problem is usually narrow and well defined and, in some examples, the context of the problem is not even stated. It is difficult to judge whether a more complete solution is required. The unspoken assumption is that a very thorough and complete solution of the narrowly defined task is the only "correct answer." Finally, although the steps outlined were given in the context of a design challenge, an analogous approach applies to the initial steps in the research of a new area.

1.4 Too Many Significant Figures

In many introductory scientific and engineering subjects, students are expected to come up with answers that in many cases represent an exact solution. If one carefully looks at a number of books in a field such as fluid mechanics or heat transfer, the same situations and boundary conditions are always considered in the derivations. These cases represent important limiting solutions and are valuable for study. The same examples appear repeatedly because they are the only ones for which an exact elegant closed-form solution is available. This can mislead readers into the expectation that all real problems will yield similarly elegant solutions and that such a level of precision can routinely be obtained. Unfortunately, when faced with a real-world problem, the situation is often so complicated that an exact solution is out of the question. On the other hand, a precise solution is usually not required. For the design example in the previous section, preliminary comparisons sometimes require only an order of magnitude approximation.

Are extended computer solutions with jazzy color graphics the order of the day? Enlightened managers and customers will shy away from such computer overkill, especially when they see the manpower requirements or the consulting bill for such an effort. In particular, when the system is too narrowly defined or does not consider all of the most relevant factors at the beginning of a design process, a solution with more than one significant figure is generally useless. The engineer needs to keep in mind what level of accuracy or approximation is needed for the question at hand: What is needed to make a design decision; what factors are most important; what are the real constraints; and what is controlling the physical process?

The administration in a Caribbean island was considering a solar desalination system. They wanted an accurate estimate of the flat plate thermal collector size that is needed to produce the desired output. Accuracy was needed because the cost of such a system was very high and overcapacity would be costly. Because the desired goal was increased freshwater supply, before doing an accurate estimate the set of possible solutions was broadened. It was found that the annual rainfall was high; a modest sized horizontal surface for rainfall collection gave the same results as the much larger solar collector (whose minimum size could be quickly estimated).

1.5 Property Values

In many situations, there is a large degree of uncertainty in thermophysical property values. Again, it doesn't make sense to carry out a numerical or analytical solution to three or four significant figures when the controlling property values may only be known within $\pm$ 25% This limitation is frequently overlooked when engineers or scientists produce "very precise solutions."

Handbooks and textbooks contain a variety of tables giving thermophysical properties. What is often omitted is the range of *uncertainty* of the reported data. For even such a mundane and commonplace material as steel, the thermal conductivity can vary by as much as a factor of four as the carbon and chromium content of this steel is varied between pure iron and stainless steel. Thermal radiation properties of a shiny metal surface can vary over time by a factor of five or ten as the surface is oxidized or becomes dirty! Not only is a high precision calculation uncalled-for in such a case, but it also may be totally misleading if the properties are poorly understood.

As an example, manufacturers of polyurethane closed-cell foam insulation are concerned about predicting the thermal conductivity as a function of time. Foam insulation is made with a high molecular weight, low-conductivity gas contained in closed cells. When newly made, these foams have a higher resistance to heat transfer than almost any other conventional insulation of comparable thickness. Over time, air diffuses into the foam interior, mixes with the lower conductivity blowing gas, raising the effective conductivity of the foam by 30% or more. To find the rate at which the air concentration increases within the foam, one needs only to solve a one-dimensional diffusion equation – Fick's equation for mass transfer of the intruding air and original gas. The solutions exactly parallel the well-known solutions for one-dimensional transient heat conduction in a homogeneous body. Unfortunately, the diffusion coefficients of the various gases through the foam matrix are poorly known. Some measurements of the diffusion coefficient disagree by more than an order of magnitude. The main resistance to diffusion is the series of solid cell walls. In some foams, the solid polyurethane is not uniformly distributed, the center of the cell walls are much thinner than the edges. What was initially conceived as a project that could be carried out in a matter of weeks, ended up as a multiyear project to measure the diffusion coefficients and the foam morphology accurately!

1.6 Introduction to Modeling Tools and Techniques

One of the most challenging questions posed by a new design, research project, or application of an existing system is "How do we get started"? At this stage the engineer is faced with the interplay of ideas, constraints, and performance goals. It is important to determine quickly if the approach, design, or physical model is on the right track. This usually requires a quick estimate at the beginning. In many cases, this may be sufficient to rank order competing proposals or to identify the correct direction for further inquiry. The estimate may relate to the thermal efficiency, the maximum capacity for heating and cooling, or the safe temperature levels of operation.

In the starting phases of the project it is necessary to define the goal. Is it a new or improved project? What are the technical performance goals, and how important are these relative to reliability or cost? The level of accuracy needed for the performance prediction should be established. If the new concept is intended to double a performance parameter then initial estimates of the performance that have, say $\pm$ 10% uncertainty, may be more than adequate for initial evaluations. If cost is the main consideration, and it is not known with high precision (which is the usual case), then high precision in technical performance predictions may not carry much weight in the overall evaluation of the concept.

In other cases in which a high degree of accuracy is required for predicting the physical behavior, an initial, more approximate approach may still be the proper first step.

The watchword of the opening study is to keep things as simple as possible, sometimes referred to, humorously, as the KISS rule: "Keep it simple, stupid!" This approach has several advantages:

- It forces the modeler to concentrate on understanding the controlling physics without the possible confusion of elaborate analytical or numerical results.
- It can be carried out quickly – a virtue when there is a short deadline.
- It provides a quick check on the feasibility of the proposed approach.
- It is far easier to explain to managers who may not be technically adept.
- It establishes an initial credibility for your efforts and sometimes helps to limit unnecessary effort or going down a blind alley.

The authors have evaluated many projects that have been carried out in elaborate detail when a far simpler approach would demonstrate that the concept just won't work or is far inferior to other options. The goal of this text is to introduce such an approach, with the use of modeling and simplified tools as the proper starting point.

1.7 Modeling Techniques

A number of simple techniques can be employed in the startup phase of a new concept or design. There is no set order or pattern in which they should be applied and some may not be useful for a particular case. They are introduced in the following paragraphs and discussed in detail throughout the text.

Order of Magnitude Estimates

Order of magnitude estimates are very useful to identify which physical mechanisms are controlling and, equally important, which mechanisms can be neglected as

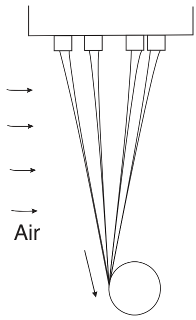

Figure 1.2. Glass fiber spinning.

second-order effects. Based on the approximate size, temperature, and other parameters, it is usually possible to make an order of magnitude estimate. How large must a heat exchanger be to provide a heat transfer rate of a certain magnitude? For a given power level and surroundings, how high a temperature will a body achieve?

Energy Balance

In thermal problems, an approximate energy balance will help identify important parameters. For example, if the heat generated within a body at steady state is 100 W and the heat transferred from it to the surrounding air by convection can be estimated to be about 90 W then conduction to adjoining solid bodies and radiation to the surroundings are second-order effects and may reasonably be omitted from initial consideration. If the convective heat transfer is estimated to be no more than 10 W, then it can be neglected and attention should be focused on radiation and/or conduction. For a proposed new design with a heat generation of 100 W, if the levels of conduction, convection, and radiation are all at most 10 W or less at the proposed design temperature, the design is most probably a dead end or in need of major revisions: there's no need to study it in greater detail without changing it first.

EXAMPLE

The use of modeling and approximations can sometimes be thought of as part of the "art" of engineering. To master this art requires practice with a variety of situations. Readers are advised to read the examples and to attempt their own modeling of the situations presented. In most cases, there is not a unique "correct" solution. The usefulness of the process depends on the ingenuity and creativity of the individual. In all cases, it is good to keep in mind that the progression of approaches should start with the simplest model first.

Example 1.2 ***Glass fiber spinning*** Continuous glass fibers are used for reinforcing polymers in structures such as boats and car bodies. They are also used in textiles. The glass fibers are formed from molten glass (see Fig. 1.2). The glass is heated to about 1000°C, at which temperature it is a liquid with a viscosity similar to that of heavy motor oil or syrup. It is delivered to a platinum vessel that has a thousand or more small nozzles or tips on the bottom. The glass flows out of the tips and forms continuous strands or fibers that are pulled vertically downward. The fibers are cooled by a horizontal flow of ambient air. They are gathered at the bottom and wound around a rotating wheel. The wheel applies tension to the glass and attenuates it down to its final diameter as the glass is cooled.

An important design parameter of the process is the amount of ambient temperature air that must be blown horizontally across the glass to achieve a temperature drop of the fibers of 500°C.

The first estimate of the airflow can be obtained from an energy balance. For a control volume around all of the fibers in steady state, neglecting radiation from the glass to the surroundings,

$$(\dot{m}c_p\Delta T)_{\text{glass}} = (\dot{m}c_p\Delta T)_{\text{air}} \tag{1.1}$$

where $\dot{m}$ is the mass flow rate, c_p is the specific heat capacity at constant pressure, and ΔT is the change in temperature. The entire energy change of the glass can be considered sensible heat; there is not an abrupt phase change as the glass cools. The mass flow of air depends on the air temperature rise as it passes over all of the fibers.

Furthermore, for all of the glass fibers to cool evenly, each fiber, nearest the air flow entrance and those furthest away, must experience a similar temperature level of the air so that for convective cooling

$$\dot{m}\,c_p\Delta T_{\text{glass}} = hA_{\text{fiber}}(T_{\text{glass}} - T_{\text{air}}) \tag{1.2}$$

where h is the convective heat transfer coefficient and A_{fiber} is the surface area of all the fibers. It can be shown that radiation is a negligible source of cooling once the glass fiber has been reduced in size.

For the cooling to be approximately the same for all fibers, the change in the air temperature must be much smaller than the glass temperature change,

$$\Delta T_{\text{air}} \approx \frac{1}{10}\Delta T_{\text{glass}} \tag{1.3}$$

Combining this with the energy balance, an initial estimate of the air flow can be made. The next step is to ensure a uniform distribution of air over all of the fibers. Near the bottom of the platinum vessel the air is heated by convection from the vessel surface and the platinum nozzles. Our estimate did not take this heating into account so it must be treated as a first, lower limit estimate, for the proper air flow rate.

Maximum/Minimum Bounds

Although systems with complicated geometries usually require intense efforts to obtain close predictions of their performance, it may be sufficient to bracket the correct answer within upper and lower bounds or limiting solutions. This can be

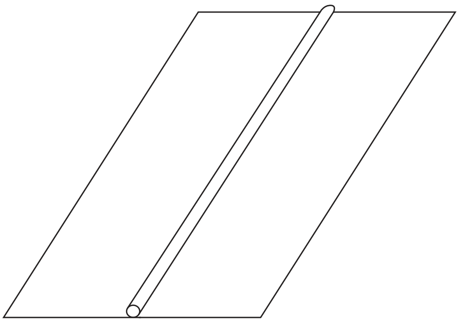

Figure 1.3. Wire-heated plate.

accomplished by making several simplifications to the geometry or assumptions about the process or system that will result in answers that can be easily shown to either exceed the correct results or provide a lower limit. The upper and lower limits may provide sufficient bounds for the purposes of an initial evaluation. In some cases, upper and lower bounds can be found that are very close to one another, resulting in a good estimate of the exact solution.

Example 1.3 ***Microprocessor chip*** Consider a microprocessor chip. Its steady-state energy consumption is 10 W and it is to be cooled by fins with a total area of 0.01 m^2. The designers are concerned about the maximum chip temperature relative to the surrounding air temperature. If the cooling relies on natural convection, then the minimum possible temperature difference between the surface of the chip and the surrounding ambient air occurs when the fin surface temperature everywhere is equal to the chip temperature,

$$\Delta T_{\text{min}} = \frac{Q}{hA_{\text{total}}} \tag{1.4}$$

where Q is the heat flow out of the chip. Taking an upper limit on h for natural convection as 25 W/m^{2}K, then the minimum ΔT between the air and chip with natural convection is 10/[(0.01) (25)] or 40°C.

An extension of this example can be used to establish upper bounds, as well. To provide a heated surface for temperature control of an experiment, a heater wire is attached to one side of a vertical thin flat plate (Fig. 1.3). The wire is 10 cm in length with a diameter of 2 mm. The overall heating of the wire at steady state is 2 W. The plate dimensions are 10 cm by 5 cm, and it is 5 mm thick. The wire and plate are surrounded by ambient air. An upper bound on the wire temperature will determine whether the wire will soften or melt the plate adjacent to it. The lower limit of wire temperature is obtained by assuming the plate is highly conductive and that the wire is in good contact with it, similar to the preceding microprocessor example.

An upper bound on the wire temperature is obtained by assuming that the plate is made of very low conductivity material (a plastic, for example) or by assuming that a large contact resistance exists between the wire and the plate. In this extreme, negligible heat is transferred between the wire and the plate; and at steady state, all