A Student's Guide to Entropy

Striving to explore the subject in as simple a manner as possible, this book helps readers understand the elusive concept of entropy.

Innovative aspects of the book include the construction of statistical entropy from desired properties, the derivation of the entropy of classical systems from purely classical assumptions, and a statistical thermodynamics approach to the ideal Fermi and ideal Bose gases. Derivations are worked through step-by-step and important applications are highlighted in over 20 worked examples. Around 50 end-of-chapter exercises test readers' understanding.

The book also features a glossary giving definitions for all essential terms, a time line showing important developments, and a list of books for further study. It is an ideal supplement to undergraduate courses in physics, engineering, chemistry, and mathematics.

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Preface

The mathematician John von Neumann once urged the information theorist Claude Shannon to assign the name *entropy* to the measure of uncertainty Shannon had been investigating. After all, a structurally identical measure with the name *entropy* had long been an element of statistical mechanics. Furthermore, "No one really knows what entropy really is, so in a debate you will always have the advantage." Most of us love clever one-liners and allow each other to bend the truth in making them. But von Neumann was wrong about entropy. Many people have understood the concept of entropy since it was first discovered 150 years ago.

Actually, scientists have no choice but to understand entropy because the concept describes an important aspect of reality. We know how to calculate and how to measure the entropy of a physical system. We know how to use entropy to solve problems and to place limits on processes. We understand the role of entropy in thermodynamics and in statistical mechanics. We also understand the parallelism between the entropy of physics and chemistry and the entropy of information theory.

But von Neumann's witticism contains a kernel of truth: entropy is difficult, if not impossible, to visualize. Consider that we are able to invest the concept of the *energy* of a rod of iron with meaning by imagining the rod broken into its smallest parts, atoms of iron, and comparing the energy of an iron atom to that of a macroscopic, massive object attached to a network of springs that model the interactions of the atom with its nearest neighbors. The object's energy is then the sum of its kinetic and potential energies – types of energy that can be studied in elementary physics laboratories. Finally, the energy of the entire system is the sum of the energy of its parts.

These imaginative transitions – first to analyze a whole into its parts, second to compare each part with a familiar object, third to recognize the quantity sought in the familiar object, and finally to recompose the whole out of its parts – fail to shed light on entropy. The difficulty lies in recognizing the entropy in the smallest part. A single, localized molecule or atom has no entropy. Nevertheless, a rod of iron has entropy. Entropy is not a localized phenomenon at which we can point, even in our imaginations, and say: "Look! There is entropy." And, if we insist on trying to understand a subject in ways inconsistent with its nature, we will be disappointed.

This student guide introduces us to the ways in which entropy can be understood. It emphasizes conceptual foundations and exemplary illustrations and distinguishes among different kinds of entropy: thermodynamic entropy, the entropy of classical and quantized statistical systems, and the entropy of information. These entropies differ in the classes of systems to which they apply. But all name the same concept in the sense that they reduce to each other when they can be applied to the same object.

Several features make this text appropriate as a student guide and appropriate for self-study. Never does the text depend upon "It can be shown." Derivations are included and each step of each derivation is made explicit. Mathematical techniques are used and reused and each piece of physics is revisited in different contexts. Worked examples illustrate important applications. Italicized words usually mean, "Note well. Here is a definition." These definitions, often in expanded form, are collected in a Glossary. The answers to all end-of-chapter problems are appended and an Annotated Further Reading list identifies sources and suggests books for further study. Rudolph Clausius, Ludwig Boltzmann, Max Planck, Claude Shannon, and others developed the concept of entropy over a period of 100 years. I have not been reluctant to assign credit where credit is due and to associate people and developments with dates. A Time Line organizes this information. A Formulary contains useful identities and expansions.

The middle chapters of this guide, Chapters 2 to 7, compose an entropy-centered introduction to equilibrium statistical mechanics. Not only is entropy a smooth pathway to major results, including blackbody radiation and the ideal quantum gases, but concentrating on entropy keeps one close to the physics.

The experienced user of thermal and statistical physics texts will notice several original features. One of these is the way statistical entropy is constructed, step-by-step, out of desired properties. This first phase of this construction is accomplished in Section 2.4 and the last in Section 4.5. Another is that the entropy of a classical system is derived from purely classical presuppositions in Section 2.4. Most texts do not bother with constructing a coherent classical statistical mechanics, but instead make do with the semi-classical limit of the corresponding quantum description. A third is that the ideal Fermi gas (Section 6.3) and ideal Bose gas (Sections 7.3 and 7.4) are developed from the properties of their entropy functions via the average energy approximation (Section 6.4) rather than from partition functions and density of states functions. The logic of these constructions and derivations is uncluttered and efficient.

What has been most important to me in writing this guide has been to closely correlate verbal and mathematical formulations and to translate simple concepts into simple mathematics. If the whole is complex, I have always meant the parts to be simple. My goal has been to write simply of the simple and clearly of the complex.

If I have, in some measure, been successful, no small credit is due those who helped me. Ralph Baierlein, in particular, deserves my thanks for thoughtfully reading and rereading the entire text and giving unselfishly of his expert advice. Ralph saved me from several consequential missteps and improved the readability of the text. Ralph also summarized content from German language sources. Anthony Gythiel provided translations from the German. Rick Shanahan also read and commented thoughtfully on the entire text. Dale Allison, Clayton Gearhart, Galen Gisler, Bob Harrington, Carl Helrich, and Bill Peter read and commented on various parts of the text. Hans von Baeyer, Andrew Rex, and Wolfgang Reiter graciously answered my questions. My students in a thermal physics course at Wichita State University constantly reminded me of my purpose. I am indebted to these students, colleagues, and friends.