PATH INTEGRALS AND HAMILTONIANS

Providing a pedagogical introduction to the essential principles of path integrals and Hamiltonians, this book describes cutting-edge quantum mathematical techniques applicable to a vast range of fields, from quantum mechanics, solid state physics, statistical mechanics, quantum field theory, and superstring theory to financial modeling, polymers, biology, chemistry, and quantum finance.

Eschewing use of the Schrödinger equation, the powerful and flexible combination of Hamiltonian operators and path integrals is used to study a range of different quantum and classical random systems, succinctly demonstrating the interplay between a system's path integral, state space, and Hamiltonian. With a practical emphasis on the methodological and mathematical aspects of each derivation, this is a perfect introduction to these versatile mathematical methods, suitable for researchers and graduate students in physics, mathematical finance, and engineering.

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Principles and Methods

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This book is dedicated to the memory of
Kenneth Geddes Wilson (1936-2013).
Intellectual giant, visionary scientist,
exceptional educator, altruistic spirit.

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Preface

Quantum mechanics is undoubtedly one of the most accurate and important scientific theories in the history of science. The theoretical foundations of quantum mechanics have been discussed in depth in Baaquie (2013e), where the main focus is on the interpretation of the mathematical symbols of quantum mechanics and on its enigmatic superstructure. In contrast, the main focus of this book is on the mathematics of path integral quantum mechanics.

The traditional approach to quantum mechanics has been to study the Schrödinger equation, one of the cornerstones of quantum mechanics, and which is a special case of partial differential equations. Needless to say, the study of the Schrödinger equation continues to be a central task of quantum mechanics, yielding a steady stream of new and valuable results.

Interestingly enough, there are two other formulations of quantum mechanics, namely the operator approach of Heisenberg and the path integral approach of Dirac–Feynman, that provide a mathematical framework which is independent of the Schrödinger equation. In this book, the Schrödinger equation is never directly solved; instead the Hamiltonian operator is analyzed and path integrals for different quantum and classical random systems are studied to gain an understanding of quantum mathematics.

I became aware of path integrals when I was a graduate student, and what intrigued me most was the novelty, flexibility and versatility of their theoretical and mathematical framework. I have spent most of my research years in exploring and employing this framework.

Path integration is a natural generalization of integral calculus and is essentially the integral calculus of infinitely many variables, also called functional integration. There is, however, a fundamental feature of path integration that sets it apart from functional integration, namely the role played by the Hamiltonian in the formalism. All the path integrals discussed in this book have an underlying linear structure that is encoded in the Hamiltonian operator and its linear vector state space. It is this combination of the path integral and its underlying Hamiltonian that provides a powerful and flexible mathematical machinery that can address a vast variety and range of diverse problems. Path integration can also address systems that do not have a Hamiltonian and these systems are not studied. Instead, topics have been chosen that can demonstrate the interplay of the system's path integral, state space, and Hamiltonian.

The Hamiltonian operator and the mathematical formalism of path integration make them eminently suitable for describing quantum indeterminacy as well as classical randomness. In two chapters of the book, namely Chapter 7 on stochastic processes and Chapter 17 on compact degrees of freedom, path integrals are applied to classical stochastic and random systems. The rest of the chapters analyze systems that have quantum indeterminacy.

The range and depth of subjects that come under the sway of path integrals are unified by a common thread, which is the mathematics of path integrals. Problems seemingly unrelated to indeterminacy such as the classification of knots and links or the mathematical properties of manifolds have been solved using path integration. The applications of path integrals are almost as vast as calculus, ranging from finance, polymers, biology, and chemistry to quantum mechanics, solid state physics, statistical mechanics, quantum field theory, superstring theory, and all the way to pure mathematics. The concepts and theoretical underpinnings of quantum mechanics lead to a whole set of new mathematical ideas and have given rise to the subject of *quantum mathematics*.

The ground-breaking and pioneering book by Feynman and Hibbs (1965) laid the foundation for the study of path integrals in quantum mechanics and is always worth reading. More recent books such as those by Kleinert (1990) and Zinn-Justin (2005) discuss many important aspects of path integration and cover a wide range of applications. Given the complex theoretical and mathematical nature of the subject, no single book can conceivably cover the gamut of worthwhile topics that appear in the study of path integration and there is always a need for books that break new ground. The topics chosen in this book have a minimal overlap with other books on path integrals.

A major field of theoretical physics that is based on path integrals is quantum field theory, which includes the Standard Model of particles and forces. The study of quantum field theory leads to the concept of nonlinear gauge fields and to the concept of renormalization, both of which are beyond the scope this book.

The purpose of the book is to provide a pedagogical introduction to the essential principles of path integrals and of Hamiltonians; for this reason many examples have been worked out in full detail so as to elucidate some of the varied methods and techniques that have proven useful in actually performing path integrations. The emphasis in all the derivations is on the methodological and mathematical aspect of the problem – with matters of interpretation being discussed only in passing. Starting from the simplest examples, the various chapters lay the ground work for analyzing more advanced topics. The book provides an introduction to the foundations of path integral quantum mechanics and is a primer to the techniques and methods employed in the study of quantum finance, as formulated by Baaquie (2004) and Baaquie (2010).

Acknowledgements

I would like to acknowledge and express my thanks to many outstanding teachers, scholars, and researchers whose work motivated me to study path integral quantum mechanics and to grapple with its mathematical formalism.

I had the singular privilege of doing my Ph.D. thesis under the guidance of Nobel Laureate Kenneth G. Wilson; his visionary conception of quantum mechanics and of quantum field theory – rooted in the path integral – greatly enlightened and inspired me, and continues to do so today. As an undergraduate I had the honor of meeting and conversing a number of times with Richard P. Feynman, the legendary discoverer of the path integral, and this left a permanent impression on me.

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