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978-1-107-00854-0 - Linear Algebraic Groups and Finite Groups of Lie Type

Gunter Malle and Donna Testerman

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