Continuum Mechanics and Thermodynamics

Continuum mechanics and thermodynamics are foundational theories of many fields of science and engineering. This book presents a fresh perspective on these important subjects, exploring their fundamentals and connecting them with micro- and nanoscopic theories.

Providing clear, in-depth coverage, the book gives a self-contained treatment of topics directly related to nonlinear materials modeling with an emphasis on the thermo-mechanical behavior of solid-state systems. It starts with vectors and tensors, finite deformation kinematics, the fundamental balance and conservation laws, and classical thermodynamics. It then discusses the principles of constitutive theory and examples of constitutive models, presents a foundational treatment of energy principles and stability theory, and concludes with example closed-form solutions and the essentials of finite elements.

Together with its companion book, *Modeling Materials* (Cambridge University Press, 2011), this work presents the fundamentals of multiscale materials modeling for graduate students and researchers in physics, materials science, chemistry, and engineering.

A solutions manual is available at www.cambridge.org/9781107008267, along with a link to the authors' website which provides a variety of supplementary material for both this book and *Modeling Materials*.

Ellad B. Tadmor is Professor of Aerospace Engineering and Mechanics, University of Minnesota. His research focuses on multiscale method development and the microscopic foundations of continuum mechanics.

Ronald E. Miller is Professor of Mechanical and Aerospace Engineering, Carleton University. He has worked in the area of multiscale materials modeling for over 15 years.

Ryan S. Elliott is Associate Professor of Aerospace Engineering and Mechanics, University of Minnesota. An expert in stability of continuum and atomistic systems, he has received many awards for his work.

Continuum Mechanics and Thermodynamics

From Fundamental Concepts to Governing Equations

ELLAD B. TADMOR
University of Minnesota, USA

RONALD E. MILLER
Carleton University, Canada

RYAN S. ELLIOTT
University of Minnesota, USA

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Preface

This book on *Continuum Mechanics and Thermodynamics* (CMT) (together with the companion book, by Tadmor and Miller, on *Modeling Materials* (MM) [TM11]) is a comprehensive framework for understanding modern attempts at modeling materials phenomena from first principles. This is a challenging problem because material behavior is dictated by many different processes, occurring on vastly different length and time scales, that interact in complex ways to give the overall material response. Further, these processes have traditionally been studied by different researchers, from different fields, using different theories and tools. For example, the bonding between individual atoms making up a material is studied by physicists using quantum mechanics, while the macroscopic deformation of materials falls within the domain of engineers who use continuum mechanics. In the end a multiscale modeling approach – capable of predicting the behavior of materials at the macroscopic scale but built on the quantum foundations of atomic bonding – requires a deep understanding of topics from a broad range of disciplines and the connections between them. These include quantum mechanics, statistical mechanics and materials science, as well as continuum mechanics and thermodynamics, which are the focus of this book.

Together, continuum mechanics and thermodynamics form the fundamental theory lying at the heart of many disciplines in science and engineering. This is a nonlinear theory dealing with the macroscopic response of material bodies to mechanical and thermal loading. There are many books on continuum mechanics, but we believe that several factors set our book apart. First, is our emphasis on fundamental concepts. Rather than just presenting equations, we attempt to explain where the equations come from and what are the underlying assumptions. This is important for those seeking to integrate continuum mechanics within a multiscale paradigm, but is also of great value for those who seek to master continuum mechanics on its own, and even for experts who wish to reflect further upon the basis of their field and its limitations. To this end, we have adopted a careful expository style, developing the subject in a step-by-step fashion, building up from fundamental ideas and concepts to more complex principles. We have taken pains to carefully and clearly discuss many of the subtle points of the subject which are often glossed over in other books.

A second difference setting our CMT apart from other books on the subject is the integration of thermodynamics into the discussion of continuum mechanics. Thermodynamics is a difficult subject which is normally taught using the language of heat engines and Carnot cycles. It is very difficult for most students to see how these concepts are related to continuum mechanics. Yet thermodynamics plays a vital role at the foundation of continuum mechanics. In fact, we think of continuum mechanics and thermodynamics as a single unified subject. It is simply impossible to discuss thermomechanical processes in materials without including thermodynamics. In addition, thermodynamics introduces key constraints on allowable forms of constitutive relations, the fundamental equations describing material response, that form the gateway to the underlying microscopic structure of the material.

The third difference is that we have written CMT with an eye to making it accessible to a broad readership. Without oversimplifying any of the concepts, we endeavor to explain everything in clear terms with as little jargon as possible. We do not assume prior knowledge of the subject matter. Thus, a reader from any field with an undergraduate education in engineering or science should be able to follow the presentation. We feel that this is particularly important as it makes this vital subject accessible to researchers and students from physics, chemistry and materials science who traditionally have less exposure to continuum mechanics.

The philosophy underlying CMT and its form provide it with a dual role. On its own, it is suitable as a first introduction to continuum mechanics and thermodynamics for graduate students or researchers in science and engineering. Together with MM, it provides a comprehensive and integrated framework for modern predictive materials modeling. With this latter goal in mind, CMT is written using a similar style, notation and terminology to that of MM, making it easy to use the two books together.

Acknowledgments

As we explained in the preface, this book is really one part of a two-volume project covering many topics in materials modeling beyond continuum mechanics and thermodynamics (CMT). In the following few pages, we choose to express our thanks to everyone involved in the *entire* project, whether their contribution directly affected the words on these pages or only the words in the companion volume (*Modeling Materials* or MM for short). We mention this by way of explanation, in case a careful reader is wondering why we thank people for helping us with topics that clearly do not appear in the table of contents. The people thanked below most certainly helped shape our understanding of materials modeling in general, even if not with respect to CMT specifically.

Our greatest debt goes to our wives, Jennifer, Granda and Sheila, and to our children: Maya, Lea, Persephone and Max. They have suffered more than anyone during the long course of this project, as their preoccupied husbands and fathers stole too much time from so many other things. They need to be thanked for such a long list of reasons that we would likely have to split these two books into three if we were thorough with the details. Thanks, all of you, for your patience and support. We must also thank our own parents Zehev and Ciporah, Don and Linda, and Robert and Mary for giving us the impression – perhaps mistaken – that everybody will appreciate what we have to say as much as they do.

The writing of a book is always a collaborative effort with so many people whose names do not appear on the cover. These include students in courses, colleagues in the corridors and offices of our universities and unlucky friends cornered at conferences. The list of people that offered a little piece of advice here, a correction there or a word of encouragement somewhere else is indeed too long to include, but there are a few people in particular that deserve special mention.

Some colleagues generously did calculations for us, verified results or provided other contributions from their own work. We thank Quiying Chen at the NRC Institute for Aerospace Research in Ottawa for his time in calculating UBER curves with density functional theory. Tsveta Sendova, a postdoctoral fellow at the University of Minnesota (UMN), coded and ran the simulations for the two-dimensional NEB example we present. Another postdoctoral fellow at UMN, Woo Kyun Kim, performed the indentation and thermal expansion simulations used to illustrate the hot-QC method. We thank Yuri Mishin (George Mason University) for providing figures, and Christoph Ortner (Oxford University) for providing many insights into the problem of full versus sequential minimization of multivariate functions, including the example we provide in the MM book. The hot-QC project has greatly benefited from the work of Laurent Dupuy (SEA Saclay) and Frederic Legoll (École Nationale des Ponts et Chaussées). Their help in preparing a journal paper on the subject has also proven extremely useful in preparing the chapter on dynamic multiscale methods. Furio Ercolessi must be thanked in general for his fantastic web-based notes on so many important subjects discussed herein, and specifically for providing us with his molecular dynamics code as a teaching tool to provide with MM.

Other colleagues patiently taught us the many subjects in these books about which we are decidedly *not* experts. Dong Qian at the University of Cincinnati and Michael Parks at Sandia National Laboratories very patiently and repeatedly explained the nuances of various multiscale methods to us. Similarly, we would like to thank Catalin Picu at the Rensselaer Polytechnic Institute for explaining CACM, and Leo Shilkrot for his frank conversations about CADD and the BSM. Noam Bernstein at the Navy Research Laboratories (NRL) was invaluable in explaining DFT in a way that an engineer could understand, and Peter Watson at Carleton University was instrumental in our eventual understanding of quantum mechanics. Roger Fosdick (UMN) discussed, at length, many topics related to continuum mechanics including tensor notation, material frame-indifference, Reynolds transport theorem and the principle of action and reaction. He also took the time to read and comment on our take on material frame-indifference.

We are especially indebted to those colleagues that were willing to take the time to carefully read and comment on drafts of various sections of the books – a thankless and delicate task. James Sethna (Cornell University) and Dionisios Margetis (University of Maryland) read and commented on the statistical mechanics chapter. Noam Bernstein (NRL) must be thanked more than once, for reading and commenting on both the quantum mechanics chapter and the sections on cluster expansions. Nikhil Admal, a graduate student working with Ellad at UMN, contributed significantly to our understanding of stress and read and commented on various continuum mechanics topics, Marcel Arndt helped by translating an important paper on stress by Walter Noll from German to English and worked with Ellad on developing algorithms for lattice calculations, while Gang Lu at the California State University (Northridge) set us straight on several points about density functional theory. Other patient readers to whom we say "thank you" include Mitch Luskin from UMN (numerical analysis of multiscale methods and quantum mechanics), Bill Curtin from Brown University (static multiscale methods), Dick James from UMN (restricted ensembles and the definition of stress) and Leonid Berlyand from Pennsylvania State University (thermodynamics).

There are a great many colleagues who were willing to talk to us at length about various subjects in these books. We hope that we did not overstay our welcome in their offices too often, and that they do not sigh too deeply anymore when they see a message from us in their inbox. Most importantly, we thank them very much for their time. In addition to those already mentioned above, we thank David Rodney (Institut National Polytechnique de Grenoble), Perry Leo and Tom Shield (UMN), Miles Rubin and Eli Altus (Technion), Joel Lebowitz, Sheldon Goldstein and Michael Kiessling (Rutgers)[1] and Andy Ruina (Cornell). We would also be remiss if we did not take the time to thank Art Voter (Los Alamos National

[1] Ellad would particularly like to thank the Rutgers trio for letting him join them on one of their lunches to discuss the foundations of statistical mechanics – a topic which is apparently standard lunch fare for them along with the foundations of quantum mechanics.

Laboratory), John Moriarty (Lawrence Livermore National Laboratory) and Mike Baskes (Sandia National Laboratories) for many insightful discussions and suggestions of valuable references.

There are some things in these books that are so far outside our area of expertise that we have even had to look beyond the offices of professors and researchers. Elissa Gutterman, an expert in linguistics, provided phonetic pronunciation of French and German names. As none of us are experimentalists, our brief foray into pocket watch "testing" would not have been very successful without the help of Steve Truttman and Stan Conley in the structures laboratories at Carleton University. The story of our cover images involves so many people, it deserves its own paragraph.

As the reader will see in the introduction to both books, we are fond of the symbolic connection between pocket watches and the topics we discuss herein. There are many beautiful images of pocket watches out there, but obtaining one of sufficient resolution, and getting permission to *use* it, is surprisingly difficult. As such, we owe a great debt to Mr. Hans Holzach, a watchmaker and amateur photographer at Beyer Chronometrie AG in Zurich. Not only did he generously agree to let us use his images, he took over the entire enterprise of retaking the photos when we found out that his images did not have sufficient resolution! This required Hans to coordinate with many people that we also thank for helping make the strikingly beautiful cover images possible. These include the photographer, Dany Schulthess (www.fotos.ch), Mr. René Beyer, the owner of Beyer Chronometrie AG in Zurich, who compensated the photographer and permitted photos to be taken at his shop, and also to Dr. Randall E. Morris, the owner of the pocket watch, who escorted it from California to Switzerland (!) in time for the photo shoot. The fact that total strangers would go to such lengths in response to an unsolicited e-mail contact is a testament to their kind spirits and, no doubt, to their proud love of the beauty of pocket watches.

We cannot forget our students. Many continue to teach us things every day just by bringing us their questions and ideas. Others were directly used as guinea pigs with early drafts of parts of these books.[2] Ellad would like to thank his graduate students and postdoctoral fellows over the last five years who have been fighting with this project for attention, specifically Nikhil Admal, Yera Hakobian, Hezi Hizkiahu, Dan Karls, Woo Kyun Kim, Leonid Kucherov, Amit Singh, Tsvetanka Sendova, Valeriu Smiricinschi, Slava Sorkin and Steve Whalen. Ron would likewise like to thank Ishraq Shabib, Behrouz Shiari and Denis Saraev, whose work helped shape his ideas about atomistic modeling. Ryan would like to thank Kaushik Dayal, Dan Gerbig, Dipta Ghosh, Venkata Suresh Guthikonda, Vincent Jusuf, Dan Karls, Tsvetanka Sendova, Valeriu Smirichinski and Viacheslav (Slava) Sorkin. Harley Johnson and his 2008–2009 and 2010–2011 graduate classes at the University of Illinois (Urbana-Champaign) who used the books extensively provided great feedback to improve the manuscripts, as did Bill Curtin's class at Brown in 2009–2010. The 2009 and 2010 classes of Ron's "Microstructure and Properties of Engineering Materials" class caught many initial errors in the chapters on crystal structures and molecular statics and

[2] Test subjects were always treated humanely and no students were irreparably harmed during the preparation of these books.

dynamics. Some students of Ellad's Continuum Mechanics course are especially noted for their significant contributions: Yilmaz Bayazit (2008), Pietro Ferrero (2009), Zhuang Houlong (2008), Jenny Hwang (2009), Karl Johnson (2008), Dan Karls (2008), Minsu Kim (2009), Nathan Nasgovitz (2008), Yintao Song (2008) and Chonglin Zhang (2008).

Of course, we should also thank our own teachers. Course notes from Michael Ortiz, Janet Blume, Jerry Weiner, Nicolas Triantafyllidis and Tom Shield were invaluable to us in preparing our own notes and this book. Thanks also to Ellad and Ron's former advisors at Brown University, Michael Ortiz and Rob Phillips (both currently at Caltech) and Ryan's former advisors Nicolas Triantafyllidis and John A. Shaw at the University of Michigan (Nick is currently at the École Polytechnique, France), whose irresistible enthusiasm, curiosity and encouragement pulled us down this most rewarding of scientific paths.

Ryan would like to thank the University of Minnesota and the McKnight Foundation whose *McKnight Land-Grant Professorship* helped support his effort in writing this book. Further, he would like to sincerely thank Patrick Le Tallec, Nicolas Triantafyllidis, Renata Zwiers, Kostas Danas, Charis Iordanou and everyone at the *Laboratoire de Mécanique des Solides* (LMS), the École Polytechnique, France for their generous support, hosting and friendship during Ryan and Sheila's "Paris adventure" of 2010. Finally, Ryan would like to acknowledge the support of the National Science Foundation.

We note that many figures in these books were prepared with the drawing package Asymptote (see http://asymptote.sourceforge.net/), an open-source effort that we think deserves to be promoted here. Finally, we thank our editor Simon Capelin and the entire team at Cambridge, for their advice, assistance and truly astounding patience.

Notation

This book is devoted to the subject of continuum mechanics and thermodynamics. However, together with the companion book by Tadmor and Miller, *Modeling Materials* (MM) [TM11], it is part of a greater effort to create a unified theoretical foundation for multiscale modeling of material behavior. Such a theory includes contributions from a large number of fields including those covered in this book, but also quantum mechanics, statistical mechanics and materials science. We have attempted as much as possible to use the most common and familiar notation from within each field as long as this does not lead to confusion. To keep the amount of notation to a minimum, we generally prefer to append qualifiers to symbols rather than introducing new symbols. For example, $\boldsymbol{f}$ is force, which if relevant can be divided into internal, $\boldsymbol{f}^{\text{int}}$, and external, $\boldsymbol{f}^{\text{ext}}$, parts.

We use the following general conventions:

- Descriptive qualifiers generally appear as superscripts and are typeset using a Roman (as opposed to Greek) nonitalic font.
- The weight and style of the font used to render a variable indicates its type. Scalar variables are denoted using an italic font. For example, T is temperature. Array variables are denoted using a sans serif font, such as A for the matrix A. Vectors and tensors (in the mathematical sense of the word) are rendered in a boldface font. For example, $\boldsymbol{\sigma}$ is the stress tensor.
- Variables often have subscript and superscript indices. Indices referring to the components of a matrix, vector or tensor appear as subscripts in italic Roman font. For example, v_i is the ith component of the velocity vector. Superscripts will be used as counters of variables. For example, $\boldsymbol{F}^e$ is the deformation gradient in element e. Iteration counters appear in parentheses, for example $\boldsymbol{f}^{(i)}$ is the force in iteration i.
- The Einstein summation convention will be followed on repeated indices (e.g. $v_i v_i = v_1^2 + v_2^2 + v_3^2$), unless otherwise clear from the context. (See Section 2.2.2 for more details.)
- A subscript is used to refer to multiple equations on a single line, for example, "Eqn. $(3.32)_2$" refers to the second equation in Eqn. (3.32) ("$a_i(\boldsymbol{x}, t) \equiv \dots$").
- Important equations are emphasized by placing them in a shaded box.

Below, we describe the main notation and symbols used in the book, and indicate the page on which each is first defined.

Mathematical notation

Notation	Description	Page
$\equiv$	equal to by definition	22
$:=$	variable on the left is assigned the value on the right	283
$\forall$	for all	22
$\in$	contained in	22
$\subset$	a subset of	107
iff	if and only if	22
$O(n)$	orthogonal group of degree n	32
$SL(n)$	proper unimodular (special linear) group of degree n	217
$SO(n)$	proper orthogonal (special orthogonal) group of degree n	32
$\mathbb{R}$	set of all real numbers	22
$\mathbb{R}^n$	real coordinate space (n-tuples of real numbers)	25
$\lvert \bullet \rvert$	absolute value of a real number	25
$\lVert \bullet \rVert$	norm of a vector	25
$\langle \bullet, \bullet \rangle$	inner product of two vectors	25
$\langle \mathcal{D}_{\boldsymbol{x}} \bullet; \boldsymbol{u} \rangle$	nonnormalized directional derivative with respect to $\boldsymbol{x}$ in the direction $\boldsymbol{u}$	57
$f[\bullet]$	square brackets indicate f is a linear function of its arguments	24
$\boldsymbol{A}^T$	transpose of a second-order tensor or matrix: $[\boldsymbol{A}^T]_{ij} = A_{ji}$	19
$\boldsymbol{A}^{-T}$	transpose of the inverse of $\boldsymbol{A}$: $\boldsymbol{A}^{-T} \equiv (\boldsymbol{A}^{-1})^T$	43
$\boldsymbol{a} \cdot \boldsymbol{b}$	dot product (vectors): $\boldsymbol{a} \cdot \boldsymbol{b} = a_i b_i$	25
$\boldsymbol{a} \times \boldsymbol{b}$	cross product (vectors): $[\boldsymbol{a} \times \boldsymbol{b}]_k = \epsilon_{ijk} a_i b_j$	29
$\boldsymbol{a} \otimes \boldsymbol{b}$	tensor product (vectors): $[\boldsymbol{a} \otimes \boldsymbol{b}]_{ij} = a_i b_j$	39
$\boldsymbol{A} : \boldsymbol{B}$	contraction (second-order tensors): $\boldsymbol{A} : \boldsymbol{B} = A_{ij} B_{ij}$	44
$\boldsymbol{A} \cdot\cdot \boldsymbol{B}$	transposed contraction (second-order tensors): $\boldsymbol{A} \cdot\cdot \boldsymbol{B} = A_{ij} B_{ji}$	44
$A_{(ij)}$	symmetric part of a second-order tensor: $A_{(ij)} = \frac{1}{2}(A_{ij} + A_{ji})$	48
$A_{[ij]}$	antisymmetric part: $A_{[ij]} = \frac{1}{2}(A_{ij} - A_{ji})$	48
$\lambda_\alpha^{\boldsymbol{A}}, \boldsymbol{\Lambda}_\alpha^{\boldsymbol{A}}$	αth eigenvalue and eigenvector of the second-order tensor $\boldsymbol{A}$	49
$I_k(\boldsymbol{A})$	kth principal invariant of the second-order tensor $\boldsymbol{A}$	49
$đ$	inexact differential	159
$\det \boldsymbol{A}$	determinant of a matrix or a second-order tensor	21
$\operatorname{tr} \boldsymbol{A}$	trace of a matrix or a second-order tensor: $\operatorname{tr} \boldsymbol{A} = A_{ii}$	19
$\nabla \bullet$, grad $\bullet$	gradient of a tensor (deformed configuration)	57
$\nabla_0 \bullet$, Grad $\bullet$	gradient of a tensor (reference configuration)	77
curl $\bullet$	curl of a tensor (deformed configuration)	58
Curl $\bullet$	curl of a tensor (reference configuration)	77
div $\bullet$	divergence of a tensor (deformed configuration)	59
Div $\bullet$	divergence of a tensor (reference configuration)	77
$\nabla^2 \bullet$	Laplacian of a tensor (deformed configuration)	60
$\vec{\alpha}_e$	local node number on element e for global node number α	294

General symbols – Greek

Symbol	Description	Page
α	stretch parameter	78
$\mathbf{\Gamma}$, Γ_i	set of extensive state variables	135
$\mathbf{\Gamma}^{\mathrm{i}}$, Γ^{i}_i	set of intensive state variables obtained from $\mathbf{\Gamma}$	173
$\boldsymbol{\gamma}$, γ_i	set of intensive state variables work conjugate with $\mathbf{\Gamma}$	157
δ_{ij}	Kronecker delta	19
ϵ_{ijk}	permutation symbol	20
$\boldsymbol{\epsilon}$, ϵ_{ij}	small strain tensor	93
θ	polar coordinate in polar cylindrical system	61
θ	zenith angle in spherical system	62
θ^i	curvilinear coordinates in a general coordinate system	60
κ	bulk viscosity (fluid)	220
λ	Lamé constant	235
μ	shear viscosity (fluid)	220
μ	shear modulus (solid)	235
ν	Poisson's ratio	235
$\boldsymbol{\xi}$, ξ_I	parent space for a finite element	294
Π	total potential energy of a system and the applied loads	247
ρ	mass density (deformed configuration)	106
ρ_0	mass density (reference configuration)	107
$\boldsymbol{\sigma}$, σ_{ij}	Cauchy stress tensor	116
$\boldsymbol{\tau}$, τ_{ij}	Kirchhoff stress tensor	123
ϕ	azimuthal angle in spherical system	63
$\boldsymbol{\varphi}$, φ_i	deformation mapping	72
ψ	specific Helmholtz free energy	193
$\boldsymbol{\psi}$, ψ_i	spin axial vector	98

General symbols – Roman

Symbol	Description	Page
$\breve{\boldsymbol{a}}$, $\breve{a}_i$	acceleration vector (material description)	94
$\boldsymbol{a}$, a_i	acceleration vector (spatial description)	94
B	bulk modulus	241
$\boldsymbol{B}$, B_{ij}	left Cauchy–Green deformation tensor	85
B	matrix of finite element shape function derivatives	302
$\breve{\boldsymbol{b}}$, $\breve{b}_i$	body force (material description)	122
$\boldsymbol{b}$, b_i	body force (spatial description)	112
C_v	molar heat capacity at constant volume	144
$\boldsymbol{C}$, C_{IJ}	right Cauchy–Green deformation tensor	79

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$\boldsymbol{C}$, C_{IJKL}	referential elasticity tensor	226
c_v	specific heat capacity at constant volume	320
$\boldsymbol{c}$, c_{ijkl}	spatial (or small strain) elasticity tensor	228
c, c_{mn}	elasticity matrix (in Voigt notation)	230
$\boldsymbol{D}$, D_{iJkL}	mixed elasticity tensor	227
D	matrix representation of the mixed elasticity tensor	303
$\boldsymbol{d}$, d_{ij}	rate of deformation tensor	96
$\mathcal{E}$	total energy of a thermodynamic system	141
E	Young's modulus	235
$\boldsymbol{E}$, E_{IJ}	Lagrangian strain tensor	87
E	finite element strain operator matrix	302
$\boldsymbol{e}_i$	orthonormal basis vectors	23
$\boldsymbol{e}$, e_{ij}	Euler–Almansi strain tensor	90
$\mathcal{F}$	frame of reference	196
$\boldsymbol{F}^{\mathrm{ext}}$, F_i^{ext}	total external force acting on a system	10
$\boldsymbol{F}$, F_{iJ}	deformation gradient	78
F	matrix representation of the deformation gradient	301
f	column matrix of finite element nodal forces	281
G	material symmetry group	216
g	specific Gibbs free energy	195
$\boldsymbol{g}^i$, $\boldsymbol{g}_i$	contravariant and covariant basis vectors, respectively	28
$\boldsymbol{H}_0$, H_{0i}	angular momentum about the origin	120
h	outward heat flux across a body surface	173
h	specific enthalpy	194
$\boldsymbol{I}$	identity tensor	41
I	identity matrix	20
J	Jacobian of the deformation gradient	79
$\hat{J}$	Jacobian of the finite element parent space mapping	295
J	affine mapping from the parent element space to physical space	295
$\mathcal{K}$	macroscopic (continuum) kinetic energy	140
K	finite element stiffness matrix	287
k	thermal conductivity	210
$\boldsymbol{L}$, L_i	linear momentum	110
$\boldsymbol{l}$, l_{ij}	spatial gradient of the velocity field	95
$\boldsymbol{M}_0^{\mathrm{ext}}$, M_{0i}^{ext}	total external moment about the origin acting on a system	120
N	number of particles or atoms	110
n_{d}	dimensionality of space	16
n	number of moles of a gas	144
$\mathcal{P}^{\mathrm{def}}$	deformation power	172
$\mathcal{P}^{\mathrm{ext}}$	external power	170
$\boldsymbol{P}$, P_{iJ}	first Piola–Kirchhoff stress tensor	122
P	matrix representation of the first Piola–Kirchhoff stress	301
p	pressure (or hydrostatic stress)	119

$\Delta\mathcal{Q}$	heat transferred to a system during a process	140
$\mathcal{Q}_t$	orthogonal transformation between frames of reference	197
$\mathbf{Q}$,$\mathsf{Q}_{\alpha i}$	orthogonal transformation matrix	31
$\boldsymbol{q}$, q_i	spatial heat flux vector	174
$\boldsymbol{q}_0$, q_{0I}	reference heat flux vector	175
$\mathcal{R}$	rate of heat transfer	170
$\boldsymbol{R}$, R_{iJ}	finite rotation (polar decomposition)	83
r	radial coordinate in polar cylindrical (and spherical) system	61
r	spatial strength of a distributed heat source	173
r_0	reference strength of a distributed heat source	175
$\mathcal{S}$	entropy	150
$\dot{\mathcal{S}}^{\text{ext}}$	external entropy input rate	176
$\dot{\mathcal{S}}^{\text{int}}$	internal entropy production rate	176
S^α	shape function for finite element node α (physical space)	280
$\boldsymbol{S}$, S_{IJ}	second Piola–Kirchhoff stress tensor	125
$\mathbf{S}$	matrix of finite element shape functions	279
s	specific entropy	175
$\dot{s}^{\text{ext}}$	specific external entropy input rate	176
$\dot{s}^{\text{int}}$	specific internal entropy production rate	176
$\boldsymbol{s}$, s_{ijkl}	spatial (or small strain) compliance tensor	230
$\mathbf{s}$, s_{mn}	compliance matrix (in Voigt notation)	231
$\mathfrak{s}^\alpha$	shape function for finite element node α (parent space)	294
T	temperature	137
$\boldsymbol{T}$, T_i	nominal traction (stress vector)	123
$\boldsymbol{t}$, t_i	true traction (stress vector)	113
$\bar{\boldsymbol{t}}$, $\bar{t}_i$	true external traction (stress vector)	112
$\mathcal{U}$	internal energy	140
$\boldsymbol{U}$, U_{IJ}	right stretch tensor	83
u	spatial specific internal energy	170
u_0	reference specific internal energy	175
$\boldsymbol{u}$, u_i	displacement vector	91
$\widetilde{\boldsymbol{u}}$, $\widetilde{u}_i$	finite element approximation to the displacement field	279
$\mathbf{u}$	column matrix of finite element nodal displacements	278
V_0	volume (reference configuration)	79
V	volume (deformed configuration)	79
$\boldsymbol{V}$, V_{ij}	left stretch tensor	83
v	specific volume	132
$\breve{\boldsymbol{v}}$, $\breve{v}_i$	velocity vector (material description)	94
$\boldsymbol{v}$, v_i	velocity vector (spatial description)	94
$\Delta\mathcal{W}$	work performed on a system during a process	140
W	strain energy density function	194
$\boldsymbol{w}$, w_{ij}	spin tensor	97

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$\boldsymbol{X}$, X_I	position of a continuum particle (reference configuration)	72
$\boldsymbol{x}$, x_i	position of a continuum particle (deformed configuration)	72
X	column matrix of finite element nodal coordinates	278
z	axial coordinate in polar cylindrical system	61