DYNAMICS OF QUANTISED VORTICES IN SUPERFLUIDS

A comprehensive overview of the basic principles of vortex dynamics in superfluids, this book addresses the problems of vortex dynamics in all three superfluids available in laboratories – ^{4}He, ^{3}He and Bose–Einstein condensate of cold atoms – alongside discussions of the vortex elasticity, forces on vortices and vortex mass. Beginning with a summary of classical hydrodynamics, the book guides the reader through examinations of vortex dynamics from large scales to the microscopic scale. Topics such as vortex arrays in rotating superfluids, bound states in vortex cores and interaction of vortices with quasiparticles are discussed. The final chapter of the book considers implications of vortex dynamics for superfluid turbulence using simple scaling and symmetry arguments. Written from a unified point of view that avoids a complicated mathematical approach, this text is ideal for students and researchers working with vortex dynamics in superfluids, superconductors, magnetically ordered materials, neutron stars and cosmological models.

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Frontmatter
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Preface

The motion of vortices has been an area of study for more than a century. During the classical period of vortex dynamics, from the late 1800s, many interesting properties of vortices were discovered, beginning with the notable Kelvin waves propagating along an isolated vortex line (Thompson, 1880). The main object of theoretical studies at that time was a dissipationless perfect fluid (Lamb, 1997). It was difficult for the theory to find a common ground with experiment since any classical fluid exhibits viscous effects. The situation changed after the works of Onsager (1949) and Feynman (1955) who revealed that rotating superfluids are threaded by an array of vortex lines with quantised circulation. With this discovery, the quantum period of vortex dynamics began. Rotating superfluid ^{4}He provided the testing ground for the theories of vortex motion developed for the perfect fluid. At the same time, some effects needed an extension of the theory to include two-fluid effects, and the quantum period of vortex studies was marked by progress in the understanding of vortex dynamics in the framework of the two-fluid theory. The first step in this direction was taken by Hall and Vinen (1956a), who introduced the concept of mutual friction between vortices and the normal part of the superfluid and derived the law of vortex motion in two-fluid hydrodynamics. Hall (1958) and Andronikashvili et al. (1961) were the first to study experimentally the elastic properties of vortex lines using torsional oscillators. This made it possible to observe Kelvin waves with a spectrum modified by the interaction between vortices. Elastic deformations of vortex lines were caused by *pinning* of vortices at solid surfaces confining the superfluid. Vortex pinning was another important concept, which emerged during the study of dynamics of quantised vortices.

The third important theoretical framework, invented to describe vortex motion in rotating superfluids, was so-called macroscopic hydrodynamics. This relied on a coarse-graining procedure of averaging hydrodynamical equations over scales much larger than the intervortex spacing. Such hydrodynamics was used in the pioneering work on dynamics of superfluid vortices by Hall and Vinen (1956a) and further developed by Hall (1960) and Bekarevich and Khalatnikov (1961). It was a continuum theory similar to the elasticity theory. However, it only included bending deformations of vortex lines and ignored the crystalline order of the vortex array. This theory, called the Hall–Vinen–Bekarevich–Khalatnikov (HVBK) theory (Khalatnikov, 2000), was successful in explaining a variety of experiments on rotating superfluids.

In the late 1960s attention was attracted by phenomena connected with crystalline order in a vortex array predicted by Tkachenko (1965). He also predicted (Tkachenko, 1966) that the vortex lattice sustains collective elastic waves, later called 'Tkachenko waves', in which vortex lines undergo displacements which are homogeneous along the vortex lines and transverse to the wave vector. This type of wave is a transverse-sound mode of the vortex lattice and is derived from the elasticity theory of the two-dimensional vortex lattice when the wavelength is much larger than the distance between vortices (Tkachenko, 1969). Tkachenko modes could not be described within HVBK hydrodynamics, but later the continuum theory was developed which incorporated the effects of vortex lattice rigidity (Sonin, 1976; Williams and Fetter, 1977; Baym and Chandler, 1983). Only 14 years after the paper by Tkachenko (1965), the existence of a regular vortex lattice in rotating superfluid ^{4}He was demonstrated experimentally, although for a rather small number of vortices (Gordon et al., 1978; Yarmchuk et al., 1979). Indirect evidence was obtained by Tsakadze (1978), who deduced a value of the vortex-lattice shear rigidity by observing a slow mode of vortex oscillations (the Tkachenko mode modified by vortex bending) in a free-spinning container with superfluid ^{4}He.

New problems challenged vortex dynamics after the discovery of superfluid phases of ^{3}He. The *A* phase turned out to be especially unusual. This is an example of a chiral *p*-wave superfluid with an intrinsic angular momentum. It possessed a remarkable property unknown before: the rotating *A* phase, while remaining a superfluid, sustains a continuous vorticity. The latter is not homogeneous in space as in a rotating classical fluid, but forms a two-dimensional periodic texture sometimes with more intricate symmetry compared with the simple hexagonal symmetry of the triangular vortex array in superfluid ^{4}He (Volovik and Kopnin, 1977). One can call this *quantum* continuous vorticity. Quantum continuous vorticity in the rotating *A* phase was detected by Hakonen et al. (1982) using the NMR technique. A dynamical theory of quantum continuous vorticity was developed by Kopnin (1978a) who derived the law of motion for a continuous axisymmetric vortex in the *A* phase. However, vortex dynamics in a chiral superfluid is rather far from being understood completely because of problems with the hydrodynamical theory at $T = 0$.

The family of two helium superfluids available in laboratories (putting aside superconducting electron fluids in metals) was extended significantly after the discovery of Bose–Einstein Condensation (BEC) of cold-atom gases. This brought new possibilities and new challenges. In contrast to strong interaction in ^{4}He and ^{3}He superfluids, in cold-atom gases the interaction is weak, and weak-coupling theories (the Gross–Pitaevskii theory, for example) became both qualitatively and quantitatively reliable for the prediction and interpretation of experiments. An important feature of cold-atom superfluids is a much higher compressibility than in ^{4}He and ^{3}He superfluids and non-uniform density. This required revision of the theory of vortex motion, especially for slow Tkachenko modes. Very effective optical methods which were not accessible in old superfluids allowed clear visualisations of equilibrium and oscillating vortex arrays (Abo-Shaeer et al., 2001; Coddington et al., 2003). Cold-atom superfluids are confined by potential traps formed by laser beams. They have no contacts with rough solid surfaces like the old superfluids. This excludes pinning, which was the main hurdle for the observation of pure Tkachenko waves. While evidence of the Tkachenko mode in the old superfluids was rather circumstantial and did not allow a decisive quantitative comparison with the theory, experiments with cold-atom superfluids provided the first unambiguous observation of Tkachenko waves (Coddington et al., 2003).

Superfluids available in laboratories do not exhaust all applications of superfluid vortex dynamics. Long ago it was supposed that the interior matter of neutron stars is in the superfluid state and is threaded by quantised vortices because of rotation (Ginzburg and Kirzhnitz, 1964). A rich variety of phenomena in pulsars can be interpreted using the concept of quantum vortices. In the past, astrophysical applications greatly stimulated vortex dynamics studies. For example, experimental studies of Tkachenko modes by Tsakadze and Tsakadze (1973) were encouraged by the theory of Ruderman (1970) associating periodic variations observed in the pulse period of pulsars with Tkachenko waves.

The study of vortex dynamics in rotating superfluids is advantageous because rotation creates vortices with well controlled form and density. But vortices can appear chaotically after the transition to the turbulent regime at high fluid velocities. The phenomenon of turbulence was studied intensively in classical fluids for centuries. In superfluids, turbulence can also emerge, but it is strongly affected by the fact that only vortices with quantised circulation are possible, hence superfluid turbulence is also called quantum turbulence. Quantum turbulence has been studied for more than a half-century, starting with the pioneering theoretical and experimental investigations by Vinen (1957c, 1961b). The interest in this field has not subsided. In principle, one might expect that the theory of quantum turbulence could be derived from the dynamics of quantised vortices, and that the non-linear Navier–Stokes equation should describe all the features of classical turbulence. However, such development of the theory from first principles is not feasible practically and is hardly useful. Inevitably the theory of turbulence requires introduction of essential assumptions and concepts. This transforms it into a special area of research, which does not reduce to hydrodynamics of laminar flows or to pure vortex dynamics, in full analogy with the fact that solid state physics does not reduce to atomic physics. But the theory of turbulence, nevertheless, is an important application of vortex dynamics.

Earlier theoretical and experimental investigations of vortex dynamics were treated in a number of comprehensive reviews and books (Hall, 1960; Andronikashvili et al., 1961; Andronikashvili and Mamaladze, 1966, 1967; Putterman, 1974; Donnelly, 1991; Khalatnikov, 2000) addressing mostly vortices in superfluid ^{4}He. Phenomena associated with crystalline order in the vortex lattice were considered only fragmentarily. There were books on superfluid ^{3}He with some chapters addressing vortex dynamics (Vollhardt and Wölfle, 1990; Volovik, 2003b) and there was an extensive review by Salomaa and Volovik (1985) who focused on the topological analysis of various vortex textures. A review focusing more on the effects of crystalline ordering of vortices on vortex dynamics in ^{4}He and ^{3}He was also published (Sonin, 1987). Investigations of vortex dynamics in cold-atom BEC are still at an early stage, but one can already find reviews or chapters of books addressing vortex dynamics in BEC of cold atoms (Pitaevskii and Stringari, 2003; Fetter, 2009; Ueda, 2010).

In the present book I have made an attempt to bring together vortex dynamics and its applications to all known neutral superfluids available now in laboratories (^{4}He, ^{3}He, and cold-atom BEC), addressing them from a unified position, which allows us to see the generality and differences of vortex dynamics in various media. A natural question is why vortices in charged superfluids (superconductors) were excluded from the list. An honest answer is that it would make the project too ambitious and the chance of completing it in a reasonable time bleak. The same applies to vortices in exciton and polariton BEC. Consideration of vortices in superconductors and excitonic superfluids would require addressing two important effects which are beyond the scope of the book: Meissner screening in superconductors and non-conservation of the particle number in excitonic superfluids. On the other hand, vortex dynamics in neutral and charged superfluids are definitely closely interrelated and have general problems and approaches. Therefore I hope that the content of the book will also be useful for people interested in vortex dynamics in superconductors. One can find an extensive theoretical investigation of vortex dynamics in superconductors in the book by Kopnin (2001), and some problems and approaches in Kopnin's book are also included in the present book. The present book includes the material of the old review (Sonin, 1987) except for some outdated parts. A huge work on vortex dynamics done after publication of the review required a new analysis, which this book is supposed to present.

This book mostly addresses the theory, it is written by a theorist, and from the position of a theorist. This does not mean that no attention is paid to the connection with experimental investigations, which have been done or can be done. However, only the main experimental results, and not techniques, are discussed. The experiments deal mostly with effects produced by a large number of vortices. So the book widely exploits macroscopic hydrodynamics referring to infinite vortex arrays. Even though a lot of attention is devoted to boundary problems for finite vortex arrays (because of their great importance for contact of the theory with experiments on vortex oscillations) they are assumed to be large enough to be treated within macroscopic hydrodynamics. In order to make the book self-contained we must inevitably address problems of classical and superfluid hydrodynamics as a whole, but only to an extent which is necessary for understanding vortex dynamics. In this book the principle 'from particular to general' is preferred to the principle 'from general to particular'. Though the latter makes the text more compact and helps to avoid repetition, the former is more convenient for readers who have no intention to enter deeply into the theory and want to stop at some level.

Chapter 1 addresses problems of classical hydrodynamics relevant for the main content of the book. The chapter deals with the hydrodynamical approach in general and with hydrodynamical modes in perfect and viscous fluids, either resting or rotating. The chapter analyses the motion of a cylinder with and without circulation of velocity around it through a perfect and a viscous fluid in the light of the key importance of this classical problem for the motion of quantised vortices. Only the last section of the chapter, Section 1.15, turns from classical theory to quantum theory, addressing the Gross–Pitaevskii theory of a weakly interacting Bose gas. This section demonstrates how under some conditions the quantum mechanical Gross–Pitaevskii theory justifies the application of classical hydrodynamics to quantum problems.

In Chapter 2, the dynamics of a single quantised vortex is considered at zero temperature when the normal component is totally absent. Apart from quantisation of circulation around the vortex, this problem can also be analysed within classical hydrodynamics of a perfect fluid. Basic principles of vortex dynamics are introduced for rectilinear vortices. A key feature of vortex dynamics is that an external force on a vortex is balanced by the Magnus force proportional to the vortex velocity, while the inertia force proportional to acceleration usually (but not always) can be neglected. This is supported by calculation of the vortex mass. The dynamical theory for curved vortex lines starts from the Biot–Savart law for a vortex line moving in a perfect fluid. Then the local induction approximation is introduced, which reduces the integral Biot–Savart equation of vortex motion to a differential equation. The latter is mostly used later in the chapter, which addresses various examples of vortex dynamics applications: Kelvin waves, vortex rings, helical vortices and helical vortex rings.

Chapter 3 formulates macroscopic hydrodynamics of rotating superfluids derived from coarse-graining over scales exceeding the intervortex distance. The analysis addresses the HVBK theory, which neglects shear rigidity of the vortex array, and its extension which takes into account shear rigidity. Elastic moduli of the vortex lattice are introduced, and the Tkachenko theory, which provides an exact value of the shear modulus, is surveyed. Then macroscopic hydrodynamics is used for investigation of collective modes in a rotating superfluid. These are Kelvin modes modified by long-range intervortex interaction, which introduces a gap into the originally gapless spectrum. Another important mode is the slow mode, which is a hybrid of an inertial wave in a classical rotating fluid and of an elastic shear Tkachenko mode in a lattice of quantised vortex lines. The analysis extends to a compressible fluid describing the Tkachenko mode in a rotating BEC of cold atoms. Most of the chapter assumes the existence of an array of singular vortex lines with core radius very small compared with the intervortex distance. This is called the Vortex Line Lattice (VLL) state. The last section, of the chapter, Section 3.12, considers the opposite case of a rapidly rotating Bose–Einstein condensate, when the vortex array is very dense and vortex cores strongly overlap. This is called the Lowest Landau Level (LLL) state. The state is an analogue of the mixed state of type II superconductors in strong magnetic field close to the second critical field.

Chapter 4 addresses two-dimensional boundary problems of macroscopic hydrodynamics. This requires the formulation of boundary conditions at lateral walls or borders of vertically uniform vortex bundles. After this, edge waves are considered. Edge modes on the boundary of the vortex bundle are chiral unidirectional modes similar to edge modes in the quantum Hall effect. But the focus of the chapter is pure Tkachenko modes in an incompressible and in a compressible fluid. The latter case is relevant for rotating BEC of cold atoms, where visual observation of Tkachenko modes becomes possible. The theory is compared with these observations.

In Chapter 5, three-dimensional boundary problems of macroscopic hydrodynamics are considered. This requires boundary conditions at the tops and bottoms of containers, which take into account pinning of vortices at rough solid surfaces. The concept of collective surface pinning is discussed. The theory is applied for the analysis of pile-of-disks torsional oscillations, which have been intensively investigated experimentally during many decades of study of superfluids. The analysis of the slow mode in a finite container provides an explanation of why it was so difficult to observe the Tkachenko mode in helium superfluids: even rather weak pinning transforms it to a classical inertial wave with the spectrum independent of the shear rigidity of the lattice of quantised vortices.

Chapter 6 turns from one-fluid vortex dynamics to vortex dynamics in two-fluid macroscopic hydrodynamics. The equations of macroscopic hydrodynamics are reformulated, including the effect of mutual friction and other dissipative processes. These equations allow us to investigate the temperature dependences and damping of collective modes discussed in previous chapter in a perfect fluid at $T = 0$. The chapter also considers the clamped regime, when the normal component is rigidly connected with a container and rotates as a solid body together with it. The regime is realised in rotating superfluid ^{3}He because of its very large normal viscosity. The last section of the chapter, Section 6.9, surveys the phenomenological Ginzburg–Pitaevskii theory and its extensions, which must be used to study vortex motion close to the critical temperature.

Chapter 7 reconsiders boundary problems investigated in previous chapters in the framework of two-fluid hydrodynamics. The last sections of the chapter, Sections 7.6 and 7.7, address a single vortex line and a vortex tangle terminated at the lateral walls of a container. This geometry became an object of experimental investigations of superfluid turbulence emerging at the propagation of a vortex front separating the vortex bundle from a vortex-free area (Eltsov et al., 2014).

Chapter 8 descends from macroscopic hydrodynamics down to scales much less than the intervortex distance. This is necessary for derivation of mutual friction parameters, which entered macroscopic hydrodynamics as phenomenological parameters. This issue was a matter of decades long dispute and controversy. Mutual friction originates from scattering by a vortex of quasiparticles (phonons and rotons) forming the normal component. The presence of a long-range velocity field $\sim 1/r$ around the vortex complicates the scattering theory of quasiparticles. This field leads to an effect similar to the Aharonov–Bohm effect for electrons interacting with an electromagnetic vector potential $\sim 1/r$. The Aharonov–Bohm effect for scattering of quasiparticles leads to the emergence to a transverse force on a vortex (the Lifshitz–Pitaevskii force from rotons and the Iordanskii force from phonons), which must be added to the Magnus force produced by the superfluid component. In the past, the Iordanskii force was rejected on the basis of topological arguments using the Berry phase of a moving vortex. These topological arguments ignored the contribution of the normal component to the Berry phase. The chapter also considers mutual friction near the critical temperature, where the analysis based on the scattering theory of non-interacting quasiparticles becomes invalid. Instead of this, mutual friction parameters are derived from the phenomenological time-dependent Ginzburg–Pitaevskii theory.

Peculiar features of vortex dynamics in Fermi superfluids are considered in Chapter 9, which starts with a short overview of some aspects of the Bardeen–Cooper–Schrieffer (BCS) theory which are important for vortex dynamics. There are peculiarities of the Aharonov–Bohm effect for BCS quasiparticles, which are important for determination of the contribution of high energy quasiparticles to the transverse force on the vortex. However, the main difference between vortex dynamics in the Bose and Fermi superfluids arises from the existence of core bound states in Fermi superfluids. Interaction of quasiparticles bound in the vortex core with free quasiparticles or with impurities leads to the transverse Kopnin–Kravtsov force. The force has opposite direction with respect to the Magnus force from the superfluid component and sometimes can fully cancel the latter. Bound states also make an essential contribution to the vortex mass. In the past the effects of bound states on mutual friction and vortex mass were interpreted as resulting from spectral flow of bound states across the gap in the BCS quasiparticle spectrum (Volovik, 2003b). The analysis presented in Chapter 9 shows that vortex motion does not produce spectral flow, and the spectral flow interpretation must be abandoned.

Chapter 10 focuses on hydrodynamics of chiral superfluids like the *A* phase of ^{3}He. Chirality is related to a spontaneous orbital moment of Cooper pairs (orbital ferromagnetism in the terminology of P. W. Anderson). The order parameter of chiral superfluids allows superfluid velocity fields with continuous vorticity. This has a dramatic effect on hydrodynamics. There is a fundamental still unresolved problem in the formulation of hydrodynamics of chiral superfluids at zero temperature. This is related to an intrinsic angular momentum connected with the orbital moment of Cooper pairs. The magnitude of the intrinsic angular momentum is debated and, as explained in Chapter 10, is ambiguous because there are different definitions of the intrinsic angular momentum. However, leaving the magnitude as an arbitrary phenomenological parameter, one can derive mutual friction parameters from two-fluid hydrodynamics at finite temperature.

Chapter 11 considers possible mechanisms of vortex nucleation. Vortex nucleation is impeded by a potential barrier for vortex creation. The Iordanskii–Langer–Fisher theory assumes that the barrier is overcome due to thermal fluctuations, and the nucleation rate is given by the Arrhenius law with the barrier height in the exponent. Approaching zero temperature, thermal nucleation becomes ineffective, and it was expected that vortices could be nucleated via quantum tunnelling through the potential barrier. A vortex is a macroscopic excitation of the fluid, and quantum tunnelling of vortices is an example of *macroscopic quantum tunnelling*. Thermal and quantum nucleation is discussed for various geometries of superflows. The chapter deals only with simple cases of vortex nucleation and does not address more realistic situations, which the theory of critical velocities in superfluids encounters.

Chapter 12 analyses vortex dynamics in superfluid films. It starts with a survey of the Berezinskii–Kosterlitz–Thouless theory and then turns to superfluid films on porous substrates, when one can observe an interesting interplay of two-dimensional and three-dimensional physics. The theory provides an explanation of a dissipation peak revealed in experiments on torsional oscillations of rotating porous substrates with superfluid ^{4}He film which is proportional to the angular velocity. This was absent in films on rotating plane substrates.

Chapter 13 deals with vortex dynamics in lattice superfluids. The best known example of a lattice superfluid is the Josephson junction array, where suppression of the Magnus force responsible for the Hall effect was revealed theoretically and experimentally. Nowadays, interest in the vortex dynamics in lattice superfluid has been boosted by investigations of BEC of cold atoms in optical lattices. The chapter analyses the phenomenological model decribing lattice superfluids in the continuous approximation. The model follows from the Bloch band theory for particles in periodic potentials. Within this model, forces on the vortex are derived from the balances of quasimomentum and true momentum. The analysis addresses the case of BEC close to the superfluid–insulator transition where the Magnus force changes its sign.

The last chapter of the book is devoted to superfluid turbulence, which emerges as a random tangle of quantised vortex lines. The discussion addresses only a restricted list of topics, which are chosen as examples of the role of vortex dynamics and illustrations of applications of scaling arguments and laws of symmetry for studying turbulence. The theory and experiments on superfluid turbulence in a broader sense are beyond the scope of the present book, and in order to study this field, the reader should use other reviews and books.

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