

Cambridge University Press

978-1-107-00596-9 - The Theory of Fusion Systems: An Algebraic Approach

David A. Craven

Index

[More information](#)

Index

- Alperin's fusion theorem, 9, 15
 - for finite groups, **13**, 14
 - for fusion systems, **121**, 123
- augmentation
 - of a cosimplicial space, **80**
 - in a k -algebra, **51**
- augmentation ideal, **51**
- automizer, **3**
- automorphism of a fusion system, **147**

- bad space, 81
- balance, 309
- belong (module to a block), **31**
- biset, **252**
 - bifree, 257
 - characteristic, 254
 - stable, 256
- block, **27**
 - bijection between $\mathcal{O}G$ and kG , 33
 - Clifford theory for, 336
 - cover another, 336
 - defect group of, 41
 - fusion system is saturated, 49
 - and idempotents, 28, 29
 - module belongs to it, 31
 - nilpotent, 53
 - with trivial defect group, 50
- block fusion system, **47**
 - is saturated, 49
 - of sporadic groups, 333
- block idempotent, **29**
- Brauer morphism, **34**
- Brauer pair, **38**
 - b -Brauer pair, **41**
 - generalized, 337
 - inclusion of, 39
 - maximal b -Brauer pair, **41**
 - uniqueness of inclusion, 40
- Brauer's $k(B)$ conjecture, **50**
- Brauer's first main theorem, 45
- Burnside's normal p -complement theorem, **9**

- central product, **297**
 - and finite groups, 315
 - and normal subsystems, 298
- centralizer subgroup of a subsystem, **306**
- centralizer subsystem, **108**
 - for finite groups, 108
 - is saturated, 109
- centre, **161**
 - fusion systems without centre on
 - 2-groups of 2-rank 2, 178
 - and normal subsystems, 295
- centre functor, **346**
- centric, **117**, 129
 - for finite groups, **83**, 117
- centric linking system, **343**
 - and classifying spaces, 84
 - for finite groups, **84**, 344
 - and higher limits, 346
 - uniqueness for finite groups, 86
- characteristic biset, **254**
- characteristic element of the double Burnside ring, **256**
- characteristic subsystem, **294**
- classifying space, **66**
 - and centric linking systems, 84
 - is fibrant, 70
 - homotopy groups of, 72
 - of a p -local finite group, **349**

- Clifford theory of blocks, 336
 codegeneracy map, **75**
 coface map, **75**
 R -complete space, **81**
 R -completion, **80**
 component
 for finite groups, **183**
 for fusion systems, **299**
 H -conjugate, **3**
 conjugate in a fusion system, 21, **94**
 conjugation family, **14**
 fusion systems, **225**
 p -constrained finite group, **86**
 constrained fusion system, **87**
 bijection for normal subsystems, 291
 has a model, 87
 control constrained transfer, **264**
 control fusion, **215**
 control G -fusion in H , **5**
 control weak fusion in H with respect
 to G , **5**
 in groups with abelian Sylows, 6
 control transfer, **264**
 core of a finite group, 309
 cosimplicial object, **75**
 cosimplicial simplex, **76**
 cosimplicial space, **75**
 R^*X , 80
 cover for blocks, **336**
- defect group, **41**
 defect of a subnormal subsystem,
 286
 defined (a subsystem on a subgroup),
 95
 degeneracy map, **58**
 degenerate simplex, **58**
 detecting subgroup, **275**
 direct product, **211**
 of normal subsystems, 296
 disjoint union of two bisets, 253
 double Burnside ring, **255**
 characteristic element, 256
 double normalizer, **116**
- essential, **119**, 129
 exotic fusion system, **24**
 over p_+^{1+2} , 324
 exponent of a finite group, **4**
 extensions of morphisms, 98, 103
 extremal, **15**
- F -module, **353**
 face map, **58**
 factor system, **138**
 failure of factorization module, **353**
 family, **13**
 conjugation, 14
 weak conjugation, 14
 fibrant, **68**
 simplicial groups are, 75
 fibrant replacement, **73**
 filling, **69**
 first isomorphism theorem, 143
 Fitting subgroup, **183**
 focal subgroup
 for finite groups, **16**
 for fusion systems, 187, **238**
 focal subgroup theorem, **16**
 \mathcal{F} -Frattini subsystem, **154**
 and weakly normal subsystems, 156
 H -free for fusion systems, **227**
 Frobenius reciprocity, 257
 Frobenius's normal p -complement
 theorem, **9**, 18, 20, 133
 fully automized, **97**
 fully centralized, 21, **101**
 and receptive, 103–105
 for finite groups, 22
 fully K -normalized, **110**
 fully normalized, 21, **101**
 for finite groups, 22
 proper pairs, **194**
 and receptive/fully automized, 103,
 105
 function space, **63**
 adjunction with product, 63, 77
 cosimplicial spaces, **77**
 fused
 elements of a group, **4**
 fusion system, 20, **94**
 of a block, **47**
 centralizer, 108
 constrained have models, 87
 on dihedral 2-groups, 125
 exotic, 24
 generated by morphisms, **95**
 of $GL_3(2)$, 7
 for groups, **7**
 of a group is saturated, 100
 group fusion systems are saturated,
 22
 H -free, 227

Cambridge University Press

978-1-107-00596-9 - The Theory of Fusion Systems: An Algebraic Approach

David A. Craven

Index

[More information](#)

368

\mathcal{H} -generated, 193
 normalizer, 108
 on p_+^{1+2} , 326
 of a p -soluble group, 185
 on quaternion 2-groups, 125
 saturated, 22, 100
 on semidihedral 2-groups, 125
 simple, 160
 soluble, 181
 on Sylow 2-subgroup of Co_3 , 328
 universal, 95
 on wreathed 2-groups, 180

generalized Brauer pair, **337**
 generalized Fitting subgroup, **183**
 generalized Fitting subsystem, **306**
 generalized fusion system (of a block),
337
 \mathcal{H} -generated, **193**
 generated (as a fusion system), **95**
 generated by an invariant map, **202**
 geometric realization, **64**
 and singular simplices functor, 66
 Glauberman functor, **222**
 K^∞ and K_∞ are, 224
 for the prime 2, 222
 ZJ is, 222
 Glauberman's Z^* -theorem, 167
 Glauberman's ZJ -theorem, 12, 222
 Glauberman–Thompson normal
 p -complement theorem
 for finite groups, **11**
 for fusion systems, **235**
 Goldschmidt
 simple groups with strongly closed
 abelian subgroups, 185
 Goldschmidt group, **185**
 good space, 81
 group fusion system
 centric, 117
 fully centralized, 22
 fully normalized, 22
 is saturated, 22
 Grün
 first theorem, 9, **24**
 second theorem, **132**

homocyclic, **175**
 homology groups of a simplicial set,
79
 homotopy (simplicial sets), **70**

Index

homotopy category, 74
 homotopy equivalence for simplicial
 sets, **74**
 homotopy groups for simplicial sets,
71
 horn, **69**
 filling of, 69
 inner, outer, 89
 hypercentral subgroup theorem, 165
 hypercentre, **165**
 hyperfocal subgroup
 for finite groups, **19**
 for fusion systems, **236**
 hyperfocal subgroup theorem, **19**
 hyperfocal subsystem, **243**

idempotent, **27**
 and blocks, 28, 29
 of a block, 29
 lifting of, 33
 inclusion of Brauer pairs, **39**
 uniqueness, 40
 indecomposable ideal of k -algebra, **27**
 induces a morphism of fusion systems,
135
 inductively saturated, **191**
 inertia group of a block, **336**
 injective morphism of fusion systems,
 136
 inner horn, **89**
 intersection
 of normal subsystems, 281
 of weakly normal subsystems, 281
 intersection subsystem, **95**
 invariant map, **202**
 invariant subsystem, **151**
 involution, **5**
 isomorphic
 bisets, 252
 in a fusion system, 20, **94**
 isomorphism theorems
 first, 143
 second, 143
 third, 144

Jacobson radical, **29**

Kan complex, *see* fibrant
 Kelley product, **61**
 kernel of a morphism of fusion systems,
 136

- layer, **304**
 - is a central product, **304**
 - for finite groups, **183**
 - and normalizers, **309**
- p -local subgroup, **4**
- map
 - codegeneracy, **75**
 - coface, **75**
- Martino–Priddy conjecture, **55, 317**
- Maschke’s theorem, **30**
- metacyclic, **217**
- model, **87**
- model category, **74**
- p -modular system, **32**
- morphism (extensions of), **98, 103**
- morphism of fusion systems, **135**
 - $\bar{\mathcal{F}}_Q$, **137**
 - injective, **136**
 - kernel, **136**
 - kernels are strongly closed, **136**
 - surjective, **136**
 - when $\mathcal{F} = \mathcal{N}_{\mathcal{F}}(Q)$, **138**
- nerve, **60**
 - products of, **61**
- p -nilpotent, **9**
- nilpotent block, **53**
- non-degenerate simplex, **58**
- normal p -complement, **9**
- normal map, **275**
- normal subgroup, **87, 148**
 - equivalent conditions, **129, 130, 150, 153, 153**
- normal subsystem, **272**
 - and central products, **298**
 - and constrained fusion systems, **291**
 - and direct products, **296**
 - intersection of, **281**
 - and normal subgroups of fusion systems, **273**
 - and normal subgroups of groups, **273, 291**
 - and normalizers, **274**
 - and quotients, **274**
 - quotient by central subgroups, **295**
 - transitivity, **293**
 - and weakly normal subsystems, **279**
- K -normalizer subgroup, **110**
- normalizer subsystem, **108**
 - for finite groups, **108**
 - is saturated, **109**
- K -normalizer subsystem, **111**
 - is saturated, **111**
- offender, **353**
- Oliver conjecture, **351**
- Oliver subgroup, **351**
- orbit category, **345**
- orbit of a biset, **253**
- orthogonal idempotent, **28**
- outer horn, **89**
- p -complement theorems for fusion systems, **235**
- p -local finite group, **349**
- p -modular system, **32**
- p -nilpotent, **9**
- p -perfect finite group, **168**
- p -power index, **239**
- path connected simplicial set, **89**
- perfect fusion system, **168**
- positive characteristic p -functor, **222**
 - K^∞, K_∞ , **223**
- primitive idempotent, **28**
- principal block, **33**
 - fusion system of, **53**
- product
 - adjunction with function space, **63, 77**
 - of simplicial sets, **61**
- proper \mathcal{Q} -pair, **194**
- \mathcal{Q} -series, **350**
- $Qd(p)$ -free fusion system, **227**
 - is soluble, **227**
- quasisimple fusion system, **299**
- Quillen equivalence, **74**
- quotient
 - $\bar{\mathcal{F}}_Q$, **137**
 - \mathcal{F}/Q , **138**
 - \mathcal{F}/Q is saturated, **139**
 - $\mathcal{F}/Q = \bar{\mathcal{F}}_Q$, **141**
 - for finite groups, **145**
- R -bad, **81**
- R -completion, **80**
- R -good, **81**
- radical, **118, 129**
- receptive, **99**
 - for finite groups, **99**
 - and fully centralized, **103–105**
- R -receptive, **189**

Cambridge University Press

978-1-107-00596-9 - The Theory of Fusion Systems: An Algebraic Approach

David A. Craven

Index

[More information](#)

370

π -reduced, 88
 reduction modulo p , **32**
 relative trace, 35
 residual subsystem, **247**
 automorphisms in subsystem, 252
 resistant, **216**
 abelian groups are, 25
 $\text{PSU}_3(2^n)$, 221
 Suzuki 2-groups are, 220
 Suzuki simple groups, 220
 restriction of a biset, **254**
 Rosenberg's lemma, 30

 satisfies Frobenius reciprocity, 257
 saturated, 22, **100**
 for finite groups, 100
 other definitions, 106, 107
 saturated \mathcal{F} -conjugacy class, **103**
 Schur multiplier, **170**
 second isomorphism theorem, 143
 simple fusion system, **160**
 and normal subsystems, 279
 on 2-groups of 2-rank 2, 180
 simple groups with strongly closed
 abelian subgroups, 185
 simplex, **58**
 degenerate, 58
 non-degenerate, 58
 simplicial groups are fibrant, 75
 simplicial map, **59**
 simplicial object, **75**
 simplicial R -module
 $R \otimes X$, 78
 RX , 78
 simplicial set, **58**
 category, **59**
 path connected, 89
 from poset, 60
 from simplicial complex, 58
 singular simplex, **66**
 singular simplices functor, **66**
 and geometric realization, 66
 Solomon fusion system, 171
 and blocks of finite groups, 343
 soluble fusion system, **181**
 is constrained, 183
 and p -soluble groups, 185
 sparse fusion system, **232**
 stabilizer of a biset, **253**
 stable
 biset, 256

Index

 under a group action, **34**
 strongly closed, **127**
 abelian subgroup, 153
 for finite groups, 127
 and intersections, 128
 and products, 129, 147
 under quotients, 145
 strongly p -embedded, 14, **119**
 classification for $p = 2$, 120
 subgroup
 essential, 119
 fully automized, 97
 fully centralized, 101
 fully normalized, 101
 normal in a fusion system, 148
 radical, 118
 receptive, 99
 strongly closed, 127
 strongly p -embedded, 119
 weakly closed, 127
 subnormal, **286**
 subsystem, **95**
 be defined on, 95
 generated by an invariant map, 202
 normal, 272
 $N_P(Q)C_{\mathcal{F}}(Q)$, **115**
 $QC_{\mathcal{F}}(Q)$, **115**
 surjective morphism of fusion systems,
 136
 surjectivity property, **190, 202**
 Suzuki 2-group, 175, **220**

 tame intersection, **13**
 third isomorphism theorem, 144
 Thompson subgroup, **10**
 Thompson's $A \times B$ lemma, 310
 Thompson's normal p -complement
 theorem
 for finite groups, **11**
 for fusion systems, **235**
 three subgroup lemma, 302, 310
 TI subgroup, **24**
 total space, **77**
 transfer
 control of, 264
 control of constrained, 264
 for fusion systems, **261**
 transitive biset, **253**
 transitive factor of a biset, **253**
 transporter, **84**
 transporter system, **84**

Cambridge University Press

978-1-107-00596-9 - The Theory of Fusion Systems: An Algebraic Approach

David A. Craven

Index

[More information](#)*Index*

371

- trivial fusion system, **232**
- trivial intersection, **24**

- universal fusion system, **95**
- universally weakly closed, **357**

- weak conjugation family, **14**
- weak equivalence
 - for fibrant spaces, 74
 - simplicial sets, **70**
- weak homotopy equivalence, **56**
- weak intersection, **281**
 - of subnormal subsystems, **287**
- weakly characteristic subsystem, **159**
- weakly closed, **127**
 - for finite groups, 127
 - and products, 128
 - under quotients, 145
- weakly normal map, **205**
- weakly normal subsystem, **151**
 - equivalent conditions, 156
 - generated by a weakly normal map, 207
 - intersection of, 281
 - and normal subgroups of fusion systems, 158
 - and normal subsystems, 279
 - not normal, 271
 - transitivity, 157
- weakly subnormal, **286**
- well placed, **224**
- wreathed 2-group, **176**