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978-1-107-00558-7 - Compressed Sensing: Theory and Applications

Edited by Yonina C. Eldar and Gitta Kutyniok

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Compressed Sensing

Compressed sensing is an exciting, rapidly growing field which has attracted considerable attention in electrical engineering, applied mathematics, statistics, and computer science. Since its initial introduction several years ago an avalanche of results have been obtained both of a theoretical as well as practical nature, and various conferences, workshops, and special sessions have been dedicated to this growing research field. This book provides the first detailed introduction to the subject, highlighting recent theoretical advances and a range of applications, as well as outlining numerous remaining research challenges. After a thorough review of the basic theory, many cutting-edge advances in the field are presented, including advanced signal modeling, sub-Nyquist sampling of analog signals, hardware prototypes, non-asymptotic analysis of random matrices, adaptive sensing, greedy algorithms, the use of graphical models, and the separation of morphologically distinct data components. Each chapter is written by leading researchers in the field, and consistent style and notation are utilized throughout. An extended introductory chapter summarizes the basics of the field so that no prior knowledge is required. Key background information and clear definitions make this book an ideal resource for researchers, graduate students, and practitioners wanting to join this exciting research area. It can also serve as a supplementary textbook for courses on computer vision, coding theory, signal processing, image processing, and algorithms for efficient data processing.

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Theory and Applications

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Preface

Compressed sensing (CS) is an exciting, rapidly growing field that has attracted considerable attention in electrical engineering, applied mathematics, statistics, and computer science. Since its initial introduction several years ago, an avalanche of results have been obtained, both of theoretical and practical nature, and various conferences, workshops, and special sessions have been dedicated to this growing research field. This book provides the first comprehensive introduction to the subject, highlighting recent theoretical advances and a range of applications, as well as outlining numerous remaining research challenges.

CS offers a framework for simultaneous sensing and compression of finite-dimensional vectors, that relies on linear dimensionality reduction. Quite surprisingly, it predicts that sparse high-dimensional signals can be recovered from highly incomplete measurements by using efficient algorithms. To be more specific, let x be a length- n vector. In CS we do not measure x directly, but rather acquire $m < n$ linear measurements of the form $y = Ax$ using an $m \times n$ CS matrix A . Ideally, the matrix is designed to reduce the number of measurements as much as possible while allowing for recovery of a wide class of signals from their measurement vectors y . Thus, we would like to choose $m \ll n$. However, this renders the matrix A rank-deficient, meaning that it has a nonempty nullspace. This implies that for any particular signal x_0 , an infinite number of signals x will yield the same measurements $y = Ax = Ax_0$ for the chosen CS matrix. In order to enable recovery, we must therefore limit ourselves to a special class of input signals x .

The most prevalent signal structure used in CS is that of sparsity. In its simplest form, sparsity implies that x has only a small number of nonzero values. More generally, CS ideas can be applied when a suitable representation of x is sparse. The surprising result at the heart of CS is that if x (or a suitable representation of x) is k -sparse, i.e., it has at most k nonzero elements, then it can be recovered from $y = Ax$ using a number of measurements m that is on the order of $k \log n$. Furthermore, recovery is possible using simple, polynomial-time algorithms. In addition, these methods can be shown to be robust to noise and mismodelling of x . Many of the first research papers in CS were devoted to the analysis of theoretical guarantees on the CS matrix A in order to enable stable recovery, as well as the development of accompanying efficient algorithms.

This basic discovery has led to a fundamentally new approach to signal processing, image recovery, and compression algorithms, to name a few areas that have benefited from CS. Interestingly, the research field of CS draws from a variety of other areas

such as approximation theory, Banach space theory, convex optimization, frame theory, numerical linear algebra, random matrix theory, and signal processing. The combined efforts of mathematicians, computer scientists, and engineers have led to a deluge of significant contributions to theory and applications of CS. This includes various constructions of efficient sensing matrices, fast algorithms for sparse recovery, extension of the notion of sparsity to more general signal structures including low-rank matrices and analog signal models, hardware designs of sub-Nyquist converters that rely on ideas of CS, as well as applications to radar analysis, face recognition, image processing, biomedical imaging, and many more. CS also holds promise for increasing resolution by exploiting the signal structure. This can potentially revolutionize many applications such as radar and microscopy by making efficient use of the available degrees of freedom in these settings. Consumer electronics, microscopy, civilian and military surveillance, medical imaging, radar and many other applications rely on efficient sampling and are resolution-limited. Reducing the sampling rate in these applications and increasing resolution can improve the user experience, increase data transfer, improve imaging quality and reduce exposure time.

This book is the first monograph in the literature to provide a comprehensive survey of compressed sensing. The potential reader of this book could be a researcher in the areas of applied mathematics, computer science, and electrical engineering, or a related research area, or a graduate student seeking to learn about CS. The particular design of this volume ensures that it can serve as both a state-of-the-art reference for researchers as well as a textbook for students.

The book contains 12 diverse chapters written by recognized leading experts from all over the world covering a large variety of topics. The book begins with a comprehensive introduction to CS which serves as a background for the remaining chapters, and also sets the notation to be used throughout the book. It does not assume any prior knowledge in the field. The following chapters are then organized into 4 categories: Extended signal models (Chapters 2–4), sensing matrix design (Chapters 5–6), recovery algorithms and performance guarantees (Chapters 7–9), and applications (Chapters 10–12). The chapters are self-contained, covering the most recent research results in the respective topic, and can all be treated independent of the others. A brief summary of each chapter is given next.

Chapter 1 provides a comprehensive introduction to the basics of CS. After a brief historical overview, the chapter begins with a discussion of sparsity and other low-dimensional signal models. The authors then treat the central question of how to accurately recover a high-dimensional signal from a small set of measurements and provide performance guarantees for a variety of sparse recovery algorithms. The chapter concludes with a discussion of some extensions of the sparse recovery framework.

Chapter 2 goes beyond traditional sparse modeling, and addresses collaborative structured sparsity to add stability and prior information to the representation. In structured sparse modeling, instead of considering the dictionary atoms as singletons, the atoms are partitioned in groups, and a few groups are selected at a time for the signal encoding. Further structure is then added via collaboration, where multiple signals, which are known to follow the same model, are allowed to collaborate in the coding. The authors discuss applications of these models to image restoration and source separation.

Chapter 3 generalizes CS to reduced-rate sampling of analog signals. It introduces Xampling, a unified framework for low rate sampling and processing of signals lying in a union of subspaces. A hardware-oriented viewpoint is advocated throughout, addressing practical constraints and exemplifying hardware realizations of sub-Nyquist systems. A variety of analog CS applications are reviewed within the unified Xampling framework including multiband communications with unknown carrier frequencies, ultrasound imaging, and wideband radar.

Chapter 4 considers reduced-rate sampling of finite rate of innovation (FRI) analog signals such as streams of pulses from discrete measurements. Exploiting the fact that only a small number of parameters per unit of time are needed to fully describe FRI signals allows to sample them at rates below Nyquist. The authors provide an overview of the theory and algorithms along with a diverse set of applications in areas such as superresolution, radar and ultrasound.

Chapter 5 considers constructions of random CS matrices with proven performance guarantees. The author provides an overview of basic non-asymptotic methods and concepts in random matrix theory. Several tools from geometric functional analysis and probability theory are put together in order to analyze the extreme singular values of random matrices. This then allows deducing results on random matrices used for sensing in CS.

Chapter 6 investigates the advantages of sequential measurement schemes that adaptively focus sensing using information gathered throughout the measurement process. This is in contrast to most theory and methods for sparse recovery which are based on an assumption of non-adaptive measurements. In particular, the authors show that adaptive sensing can be significantly more powerful when the measurements are contaminated with additive noise.

Chapter 7 introduces a unified high dimensional geometric framework for analyzing the phase transition phenomenon of ℓ_1 minimization in sparse recovery. This framework connects studying the phase transitions of ℓ_1 minimization with computing the Grassmann angles in high dimensional convex geometry. The authors further demonstrate the broad applications of this Grassmann angle framework by giving sharp phase transitions for related recovery methods.

Chapter 8 presents an overview of several greedy methods and explores their theoretical properties. Greedy algorithms are very fast and easy to implement and often have similar theoretical performance guarantees to convex methods. The authors detail some of the leading greedy approaches for sparse recovery, and consider extensions of these methods to more general signal structures.

Chapter 9 surveys recent work in applying ideas from graphical models and message passing algorithms to solve large scale regularized regression problems. In particular, the focus is on CS reconstruction via ℓ_1 penalized least-squares. The author discusses how to derive fast approximate message passing algorithms to solve this problem and shows how the analysis of such algorithms allows to prove exact high-dimensional limit results on the recovery error.

Chapter 10 considers compressed learning, where learning is performed directly in the compressed domain. The authors provide tight bounds demonstrating that the linear

kernel SVM’s classifier in the measurement domain, with high probability, has true accuracy close to the accuracy of the best linear threshold classifier in the data domain. It is also shown that for a family of well-known CS matrices, compressed learning is provided on the flight. The authors then demonstrate these results in the context of texture analysis.

Chapter 11 surveys methods for data separation by sparse representations. The author considers the use of sparsity in problems in which the data is composed of two or more morphologically distinct constituents. The key idea is to choose a deliberately over-complete representation made of several frames, each one providing a sparse expansion of one of the components to be extracted. The morphological difference between the components is then encoded as incoherence conditions of those frames which allows for separation using CS algorithms.

Chapter 12 applies CS to the classical problem of face recognition. The authors consider the problem of recognizing human faces in the presence of real-world nuisances such as occlusion and variabilities in pose and illumination. The main idea behind the proposed approach is to explain any query image using a small number of training images from a single subject category. This core idea is then generalized to account for various physical variabilities encountered in face recognition. The authors demonstrate how the resulting system is capable of accurately recognizing subjects out of a database of several hundred subjects with state-of-the-art accuracy.