Black holes, entropy, and information

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Black holes are a continuing source of mystery. Although their classical properties have been understood since the 1970's, their quantum properties raise some of the deepest questions in theoretical physics. Some of these questions have recently been answered using string theory. I will review these fundamental questions, and the aspects of string theory needed to answer them. I will then explain the recent developments and new insights into black holes that they provide. Some remaining puzzles are mentioned in the conclusion.

1. Introduction

General properties of black holes were studied extensively in the early 1970's, and the basic theory was developed. One of the key results was Hawking's proof that the area of a black hole cannot decrease (Hawking 1971). This led Bekenstein (1973) to suggest that a black hole should have an entropy proportional to its horizon area. This suggestion of a connection between black holes and thermodynamics was strengthened by the formulation of the laws of black-hole mechanics (Bardeen et al. 1973). In addition to the total mass M, angular momentum J, and horizon area A of the black holes, these laws are formulated in terms of the angular velocity of the horizon Ω, and its surface gravity κ. Recall that the surface gravity is the force at infinity required to hold a unit mass stationary near the horizon of a black hole. Of course, the force near the horizon diverges, but there is a redshifting effect so that the force at infinity remains finite. The laws of black-hole mechanics are the following:

0) For stationary black holes, the surface gravity is constant on the horizon

1) Under a small perturbation:

$$dM = \frac{\kappa}{8\pi G} dA + \Omega dJ \quad , \tag{1.1}$$

2) The area of the event horizon always increases.

The zeroth law is obvious for nonrotating black holes which are spherically symmetric, but it is also true for rotating black holes which are not. If κ is like a temperature, and A is like an entropy, then there is a striking similarity to the ordinary laws of thermodynamics:

0) The temperature of an object in thermal equilibrium is constant

1) Under a small perturbation:

$$dE = TdS - PdV \quad , \tag{1.2}$$

2) Entropy always increases.

At the time it seemed clear that the analogy between black holes and thermodynamics should not be taken too seriously, since if black holes really had a temperature, they would have to radiate, and everyone knew that nothing could come out of a black hole. Two years later, everything changed.

Hawking (1975) coupled quantum matter fields to a classical black hole, and showed that they emit black-body radiation with a temperature

$$kT = \frac{\hbar\kappa}{2\pi} \quad . \tag{1.3}$$

So adding quantum mechanics in this limited way (which was all that was known how to do) made the analogy complete. Black holes really are thermodynamic objects. For a solar-mass black hole, the temperature is very low ($T \sim 10^{-7}$ K) so it is astrophysically negligible. But $T \sim 1/M$ so if a black hole starts evaporating, it gets hotter and eventually explodes. A black hole would start evaporating if it is a small primordial black hole formed in the early universe, or if we wait a very long time until the three degree background radiation redshifts to less than 10^{-7} K.

Hawking's determination of the temperature, together with the first law (Eq. 1.1), fixed the coefficient in Bekenstein's formula for the black-hole entropy:

$$S_{BH} = \frac{k}{4\hbar G} A \quad . \tag{1.4}$$

This is an enormous amount of entropy. A solar-mass black hole has $S_{BH} \sim 10^{77}k$. This is much greater than the entropy of the matter that collapsed to form it: Thermal radiation has the highest entropy of ordinary matter, but a ball of thermal radiation has $M \sim T^4R^3$, $S \sim T^3R^3$. When it forms a black hole $R \sim M$, so $T \sim M^{-1/2}$ and hence $S \sim M^{3/2}$. On the other hand, $S_{\rm BH} \sim M^2$. So $S_{\rm BH}$ grows much faster with M than the entropy of a ball of thermal radiation of the same size. Since we have suppressed all physical constants, the two entropies are equal only when M is of order the Planck mass (10^{-5} gms). We will continue to set $c = k = \hbar = 1$ in the following.

The discovery that black holes are thermodynamic objects raised the following fundamental questions:

(1) What is the origin of black-hole entropy? In all other contexts, thermodynamics is just an approximation to a more fundamental statistical description in which the entropy is the log of the number of microstates. The large entropy indicates that black holes have an enormous number of microstates. What are they?

(2) Does black-hole evaporation lose information? Does it violate quantum mechanics? Hawking argued for three decades that it did.

To understand Hawking's argument, recall that another classical property of black holes established in the 1970's was the uniqueness theorem (Robinson 1975): *The only stationary (vacuum) black-hole solution is the Kerr solution.* You can form a black hole by collapsing all kinds of different matter with different multipole moments. However, after it settles down, the black hole is completely described by only two parameters M, J. Wheeler described this by saying "black holes have no hair." The spacetime outside the horizon retains no memory of what was thrown into the black hole. Now the radiation emitted by a black hole is essentially thermal. It cannot depend on the matter inside without violating causality or locality. When the black hole evaporates, M and J are recovered, but the detailed information about what was thrown in is lost. In the language of quantum theory, pure states appear to evolve into mixed states. This would violate unitary evolution and hence one of the basic principles of quantum mechanics.

Hawking argued that the formation and evaporation of a black hole is very different from burning a book. This may seem like it is destroying information, but quantum mechanically, it can be described by unitary evolution of one quantum state into another. In principle, all the information in the book can be recovered from the ashes and emitted radiation.

2. String theory

String theory is a promising candidate for both a quantum theory of gravity and a unified theory of all the known forces and matter. One of the main successes of string theory is that it has been able to provide answers to the two fundamental questions above. To understand these answers, one needs a few basic facts about string theory.

(For more detail, see e.g., Zwiebach 2004). The first is that when one quantizes a string in flat spacetime, there are an infinite tower of massive states. For every integer N there are states with

$$M^2 \sim N/l_s^2 \quad , \tag{2.1}$$

where l_s is a new length scale in the theory set by the string tension. These states are highly degenerate, and one can show that the number of string states at excitation level $N \gg 1$ is e^S where

$$S_s \sim \sqrt{N} \quad , \tag{2.2}$$

i.e., the string entropy is proportional to the mass in string units. One can understand this in terms of a simple model of the string as a random walk with step size l_s. As a result of the string tension, the energy in the string after n steps is proportional to its length: $E \sim n/l_s$. If one can move in k possible directions at each step, the total number of configurations is k^n, so the entropy for large n is proportional to n, i.e., proportional to the energy.

String interactions are governed by a string coupling constant g (which is determined by a scalar field called the *dilaton*). Newton's constant G is related to g and the string length l_s by $G \sim g^2 l_s^2$ in four spacetime dimensions. It is sometimes convenient to use string units where $l_s = 1$, and sometimes to use Planck units where $G = l_p^2 = 1$. It is important to distinguish them, especially when g changes. Since g is in fact determined by a dynamical field, one can imagine that it changes as a result of a physical process, e.g., a wave of dilaton passing by. However, it will often be convenient to assume the dilaton is constant and treat g as just a parameter in the theory. In general, physical properties of a state can change when g is varied. But we will see that in some cases, certain properties remain unchanged.

The classical spacetime metric is well defined in string theory only when the curvature is less than the string scale $1/l_s^2$. This follows from the fact that fundamentally, the metric is unified with all the other modes of the string. This is easily seen in perturbation theory where the graviton is just one of the massless excitations of the string. When the curvature is small compared to $1/l_s^2$, one can integrate out the massive modes and obtain an effective low energy equation of motion which takes the form of Einstein's equation with an infinite number of correction terms consisting of higher powers of the curvature multiplied by powers of l_s. When curvatures approach the string scale, this low energy approximation breaks down.

String theory includes supersymmetry. Although this symmetry has not yet been seen in nature, there is hope that it will soon be discovered by the Large Hadron Collider being built at CERN. An important consequence of this new symmetry is the following. Supersymmetric theories have a bound on the mass of all states given by their charge, which roughly says $M \geqslant Q$. This is called the *BPS bound*. States which saturate this bound are called *BPS*. They have the special property that the mass does not receive any quantum corrections.

Quantizing a string also leads to a prediction that space has more than three dimensions. This is because a symmetry of the classical string action is preserved in the quantum theory only in ten spacetime dimensions. The idea that spacetime may have more than four dimensions was first proposed in the 1920's by Kaluza and Klein. Their motivation was to create a unified theory of the two known forces: gravity and electromagnetism. It turns out that a theory of pure gravity in five dimensions reduces to gravity plus electromagnetism (plus a scalar field) in four dimensions. The standard explanation for why we do not see the extra dimensions is that they are curled up into a small ball. However recently, it has been suggested that the extra dimensions might be large, but we do not see them because we are confined to live on a 3+1 dimensional submanifold called a *brane*.

In fact, it was realized about ten years ago that string theory is not just a theory of strings. There are other extended objects called *D-branes*; these are generalizations of membranes. They are nonperturbative objects with mass $M \sim 1/gl_s$. But the gravitational field of a D-brane is proportional to $GM \sim gl_s$ and hence goes to zero at weak coupling. This means that there is a flat spacetime description of these nonperturbative objects. Indeed, they are simply surfaces on which open strings can end. The strings we have been discussing so far have been topological circles with no endpoints. The dynamics of D-branes at weak coupling is described by open strings (topological line segments) in which the two endpoints are stuck on certain surfaces. Indeed, the "D" stands for Dirichlet boundary conditions on the ends of the string keeping it on the surface, and the surface itself is the *brane*. All the particles of the standard model (e.g., the quarks, leptons, and gauge bosons) are believed to come from these open strings and are confined to these branes. Only the graviton comes from the closed string and is free to move in the bulk spacetime.

Many types of D-branes exist, of various dimensions, and each carries a charge. If the branes are flat (or, more generally, form an extremal surface) and have no open strings attached, they are BPS states. Excited D-branes (with open strings added) lose energy when two such strings combine to form a closed string. Since the closed string has no ends, it can leave the brane.

3. Application to black holes

We now wish to apply string theory to black holes, and answer the two fundamental questions raised in the Introduction. We start with the question: What is the origin of black-hole entropy?

For two decades after Bekenstein and Hawking showed that black holes have an entropy, people tried to answer this question with limited success. The breakthrough came in a paper by Strominger & Vafa (1996). They considered a charged black hole. Charged black holes are not interesting astrophysically, but they are interesting theoretically since they satisfy a bound just like the BPS bound $M \geqslant Q$. Black holes with $M = Q$ are called *extremal* and have zero Hawking temperature. They are stable, even quantum mechanically. In string theory, extremal black holes are strong coupling analogs of BPS states. One can now do the following calculation: Start with an extremal black hole and compute its entropy $S_{\rm BH}$. Imagine reducing the string coupling g. When g is very small, one obtains a weakly coupled system of strings and branes with the same charge. Strominger and Vafa count the number of BPS states in this system at weak coupling and find

$$N_{BPS} = e^{S_{\rm BH}} \quad . \tag{3.1}$$

This is a microscopic explanation of black-hole entropy! Unlike previous attempts to explain $S_{\rm BH}$, one counts quantum states of a system in flat spacetime where there is no horizon. One obtains a number which, remarkably, is related to the area of the black hole which forms at strong coupling.

The idea of decreasing the string coupling should be viewed as a (very useful) thought experiment in string theory. In the real world, g is fixed to some value which is difficult to change. The actual value of the string coupling depends on many details about how string theory is connected to the standard model of particle physics and is not yet known. It is likely to be of order unity.

After the initial breakthrough, the agreement between black holes and a weakly coupled system of strings and D-branes was extended in many directions (for a review, see Peet 2000). It was shown that the entropy agrees for extremal charged black holes with rotation. The entropy also agrees for near extremal black holes with nonzero Hawking temperature. Since the entropy agrees as a function of energy, it is not surprising that the radiation from the D-branes has the same temperature as the black hole. What was surprising was that the total rate of radiation from black holes agrees with D-branes. (The analog of Hawking radiation for D-branes is just the process of two open strings combining to form a closed string, which leaves the branes.) What was truly remarkable was that the deviations from black-body spectrum also agree! Neither side is exactly thermal. On the black-hole side, these deviations arise since the radiation has to propagate through the curved spacetime outside the black hole. This produces potential barriers which give rise to frequency-dependent greybody factors. On the D-brane side, there are deviations since the modes come from separate left and right moving sectors on the D-branes. The calculations of these deviations could not look more different. On the black-hole side, one solves a wave equation in a black-hole background. The solutions involve hypergeometric functions. On the D-brane side, one does a calculation in D-brane perturbation theory. Remarkably, the answers agree.

More recently, there has been further progress in counting the microstates of charged black holes. A small black hole in string theory has an entropy which is not exactly given by the Bekenstein-Hawking formula (Eq. 1.4). There are subleading corrections coming from higher curvature terms in the action. Wald (1993) derived the form of these corrections to black-hole entropy in any theory of gravity. Recently, it has been shown that for certain extremal black holes the counting of microstates in string theory reproduces the black-hole entropy *including these subleading corrections* (Dabholkar 2006). The corrections are of order the string scale divided by the Schwarzschild radius to some power.

What about neutral black holes? Susskind (1998) suggested that there should be a one-to-one correspondence between ordinary excited string states and black holes. Start with a highly excited string with mass (Eq. 2.1) and imagine increasing the string coupling g. Since $G \sim g^2 l_s^2$, two effects take place. First, the gravitational attraction of one part of the string on the other causes the string size to decrease. Second, since G increases, the gravitational field produced by the string becomes stronger and the effective Schwarzschild radius GM increases in string units. Clearly, for a sufficiently large value of the coupling, the string forms a black hole.

Conversely, suppose one starts with a black hole and decreases the string coupling. Then the Schwarzschild radius shrinks in string units and eventually becomes of order the string scale. At this point the metric is no longer well defined near the horizon. Susskind suggested that the black hole becomes an excited string state.

When I first heard this, I didn't believe it. The first half of the argument sounded plausible enough, but the second half seemed to contradict the well known fact that the string entropy is proportional to the mass while the black-hole entropy is proportional to the mass squared. It turns out that there is a simple resolution of this apparent contradiction (Horowitz & Polchinski 1997). If one changes the string coupling g, the string mass is constant in string units, while the black-hole mass is constant in Planck units. Thus $M_s/M_{\rm BH}$ depends on g. We expect the transition to occur when the curvature at the horizon of the black hole reaches the string scale. This implies that the Schwarzschild radius r_0 is of order the string scale. Setting $M_s \sim M_{\rm BH}$ when $r_0 \sim l_s$ we find:

$$S_{BH} \sim r_0 M_{BH} \sim l_s M_s \sim S_s \quad . \tag{3.2}$$

So the entropies agree at this *correspondence point*. This agreement between the string entropy and black-hole entropy applies to essentially all black holes, including higher dimensional Schwarzschild black holes, and charged black holes that are far from extremality.

This leads to a simple picture for the endpoint of black-hole evaporation. In Hawking's picture, the black hole evaporated down to the Planck scale where the semiclassical approximations being used broke down. In string theory, the black hole evaporates until it reaches the string scale, at which point it turns into a highly excited string. The excited string continues to radiate until it becomes an unexcited string, i.e., just another elementary particle. The timescale for black-hole evaporation is modified slightly. In the black-hole phase, $dM/dt \sim T^2$ and $T \sim 1/M$, so the time to evaporate most of the mass is of order M^3 (in Planck units). When the temperature reaches the string scale, the black hole turns into a highly excited string. After this transition, the temperature stays at the string scale as the string radiates.

The above argument shows that strings have enough states to reproduce the entropy of all black holes, but the argument is not precise enough to reproduce the entropy exactly, including the factor of 1/4. More recently, Emparan & Horowitz (2006) showed that one can exactly reproduce the entropy of a class of neutral black holes. These are rotating black holes in five dimensions which have a translational symmetry around one compact direction (as in Kaluza-Klein theory). If one rewrites the solution as a four-dimensional black hole, there are charges associated with the Maxwell field coming from the higher dimensional metric. Using various symmetries of string theory, one can map these charges into D-brane charges and count the microstates in the same way that was done for BPS black holes.

In fact, a slight extension of this argument yields a precise calculation of the entropy of an extremal Kerr black hole (Horowitz & Roberts 2007). This black hole has an entropy which is just given in terms of its angular momentum

$$S = 2\pi|J| \quad . \tag{3.3}$$

Since J is naturally quantized, this is like the entropy of the extremal charged black holes in which the entropy is again just a function of the quantized charges. It turns out that one can lift an extremal Kerr black hole to five dimensions and map it into the class of neutral black holes who entropy was counted precisely.

We now return to the second fundamental question raised earlier: Do black holes lose information? For charged, near extremal black holes, the weak coupling limit provides a quantum mechanical system with the same entropy and radiation. This was a good indication that black-hole evaporation would not violate quantum mechanics. However, the case soon became much stronger.

By studying the black-hole entropy calculations, Maldacena (1998) was led to a remarkable conjecture now called the *gauge/gravity correspondence*: *Under certain boundary conditions, string theory (which includes gravity) is completely equivalent to a (nongravitational) gauge theory living at infinity.* At first sight this conjecture seems unbelievable. How could an ordinary field theory describe all of string theory? I don't have time to describe the impressive body of evidence in favor of this correspondence which has accumulated over the past few years. The obvious differences between string theory and gauge theory are explained by the fact that our intuition about both theories is largely based on weak coupling analyses. Under the gauge/gravity correspondence, when string theory is weakly coupled, gauge theory is strongly coupled, and vice versa.

This conjecture provides a "holographic" description of quantum gravity in that the fundamental degrees of freedom live on a lower dimensional space. The idea that quantum gravity might be holographic was first suggested by 't Hooft and Susskind, motivated by the fact that black-hole entropy is proportional to its horizon area.

The gauge/gravity correspondence has an immediate consequence: The formation and evaporation of small black holes can be described by ordinary Hamiltonian evolution in the gauge theory. It does not violate quantum mechanics. After 30 years, Hawking (2005) finally conceded this point (although his reasons were not directly related to string theory).

Let me conclude with a few open questions:

(1) Can we count the entropy of Schwarzschild black holes precisely? The recent calculation of the extremal Kerr entropy in terms of microstates gives one hope that this may soon be possible.

(2) How does the information get out of the black hole? What is wrong with Hawking's original argument? It appears that we will need some violation of locality. In other words, when one reconstructs the string theory from the gauge theory, physics may not be local on all length scales.

However, perhaps the most important open question is

(3) What is the origin of spacetime? How is it reconstructed from the gauge theory? How does a black-hole horizon know to adjust itself to have area $A = 4GS_{\text{BH}}$?

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Gravitational waves from black-hole mergers

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Coalescing black-hole binaries are expected to be the strongest sources of gravitational waves for ground-based interferometers, as well as the space-based interferometer *LISA*. Recent progress in numerical relativity now makes it possible to calculate the waveforms from the strong-field dynamical merger, and is revolutionizing our understanding of these systems. We review these dramatic developments, emphasizing applications to issues in gravitational wave observations. These new capabilities also make possible accurate calculations of the recoil or kick imparted to the final remnant black hole when the merging components have unequal masses, or unequal or unaligned spins. We highlight recent work in this area, focusing on results of interest to astrophysics.

1. Introduction

Gravitational wave astronomy will open a new observational window on the universe. Since large masses concentrated in small volumes and moving at high velocities generate the strongest, and therefore most readily detectable waves, the final coalescence of black-hole binaries is expected to be one of the strongest sources. During the last century, the opening of the full electromagnetic spectrum to astronomical observation greatly expanded our understanding of the cosmos. In this new century, observations across the gravitational wave spectrum will provide a wealth of new knowledge, including accurate measurements of binary black-hole masses and spins.

The high frequency part of the gravitational wave spectrum, $\sim$10 Hz $\lesssim f \lesssim 10^3$ Hz, is being opened today through the pioneering efforts of first-generation ground-based interferometers such as the Laser Interferometer Gravitational-Wave Observatory (LIGO), currently operating at design sensitivity. Such instruments can detect gravitational waves from coalescing stellar-mass ($M \lesssim 10^2\ M_\odot$) and intermediate-mass ($10^2\ M_\odot \lesssim M \lesssim 10^3\ M_\odot$) black-hole binaries. While detections from this first generation of detectors are likely to be rare, the advanced LIGO (adLIGO) upgrade may detect the coalescence of several stellar-mass and tens of intermediate-mass black-hole binaries per year. Other high-frequency sources include binary neutron-star coalescences, supernovae, and rotating neutron stars.

The low-frequency gravitational-wave window, 3×10^{-5} Hz $\lesssim f \lesssim 1$ Hz, is especially rich in astrophysical sources and will be opened by the space-based *Laser Interferometer Space Antenna* (*LISA*) detector, currently in the formulation stage. *LISA* will be sensitive to the coalescence of massive black-hole binaries with total masses in the range $10^4\ M_\odot \lesssim M \lesssim 10^7\ M_\odot$ to large redshifts $z \gtrsim 10$ at relatively high signal-to-noise ratios (SNRs), and may detect 10 or more such events per year. Using such observations, the black-hole masses, spins and luminosity distances can be determined to very good precision, with errors <1% in some cases (Lang & Hughes 2006). In addition, *LISA* will detect gravitational waves from the inspiral of compact stars into central massive black holes out to $z \sim 1$, as well as tens of thousands of compact binaries in the Galaxy.

The actual merger of two comparable-mass black holes that plunge together and form a common event horizon takes place in the strong-field dynamical regime of general relativity. For many years, we were unable to calculate the expected waveforms from these very energetic events due to severe problems with the large-scale computer codes needed to simulate the mergers. Recently, however, a series of stunning breakthroughs has occurred in numerical relativity, resulting in stable, robust, and accurate simulations of black-hole mergers, as well as applications to astrophysics. In Section 2 we review these developments and present examples of the resulting gravitational waveforms. Applications of these signals to issues in gravitational-wave observations are discussed in Section 3. When the merging black holes have unequal masses, or unequal or unaligned spins, the final remnant black hole suffers a recoil; recent progress in calculating these "kicks" and their applications to astrophysics are presented in Section 4. We conclude with a summary in Section 5.

2. Calculating black-hole binary coalescence

The final coalescence of a black-hole binary is driven by gravitational wave emission, and proceeds in three stages: an adiabatic inspiral, a dynamical merger, and a final ring-down (Flanagan & Hughes 1998). During the inspiral, the black holes are well separated and can be approximated as point particles. The black holes spiral together on quasi-circular trajectories, and the resulting gravitational waveforms are *chirps*, i.e., sinusoids that increase in both frequency and amplitude as the black holes get closer together. The inspiral can be treated analytically using the post-Newtonian (PN) approach, which is an expansion in v/c, where v is the characteristic orbital velocity (see Blanchet 2006 for a review of PN results). The inspiral is followed by a dynamical merger in which the black holes plunge together to form a highly distorted single black hole, producing a powerful burst of gravitational radiation. Since the merger stage occurs in the regime of very strong gravity, a full understanding of this process requires numerical-relativity simulations of the Einstein equations. After merger, the remnant black hole then settles down, evolving towards a quiescent Kerr state by shedding its non-axisymmetric modes as gravitational waves. The late part of this ringdown stage can be treated analytically using black-hole perturbation theory, and the resulting gravitational waveforms are superpositions of exponentially damped sinusoids of constant frequency (Leaver 1986; Echeverría 1989).

In numerical relativity, the full set of Einstein's equations are solved on a computer in the dynamical, nonlinear regime. This is typically accomplished by slicing 4-D spacetime into a stack of 3-D space-like hypersurfaces, each labeled by time t (Arnowitt et al. 1962; Misner et al. 1973). The Einstein equations split into two sets. The constraints give a set of relationships that must hold on each slice, and in particular constrain the initial data for a black-hole binary simulation. This data is then propagated forward in time using the evolution equations. Four freely specifiable coordinate, or gauge, conditions give the development of the spatial and temporal coordinates during the evolution.

Simulating the merger of a black-hole binary using numerical relativity has proved to be very challenging. The first attempt to evolve a head-on collision in 2-D axisymmetry dates back to 1964 (Hahn & Lindquist 1964). In the mid-1970s, the head-on collision of two equal mass, nonspinning black holes was first successfully simulated, along with the extraction of some information about the gravitational radiation (Smarr et al. 1976; Smarr 1977, 1979). In the 1990s, fully 3-D numerical relativity codes were developed and used to evolve grazing collisions of black holes (Brügmann 1999; Brandt et al. 2000; Alcubierre et al. 2001). However, the codes were plagued by a host of instabilities that caused them to crash before any significant portion of a black-hole binary orbit could be evolved. For many years, progress was slow and incremental.

Recently, a series of dramatic developments has led to major progress in black-hole binary simulations across a broad front. The first complete orbit of a black-hole binary was achieved in 2004 (Brügmann et al. 2004). This was followed by the first full simulation of a black-hole binary through an orbit, plunge, merger and ringdown in 2005 (Pretorius 2005). In late 2005, the development of new coordinate conditions produced a breakthrough in the ability to carry out accurate and stable long-term evolutions of black-hole binaries (Campanelli et al. 2006a; Baker et al. 2006c; van Meter et al. 2006). These novel but simple "moving puncture" techniques proved highly effective. They were quickly adopted by a broad segment of the numerical relativity community, leading to stunning advances in black-hole binary modeling, starting with evolutions of equal mass, nonspinning black holes and moving quickly to include unequal masses and spins; (see, e.g., Campanelli et al. 2006b; Baker et al. 2006b; Campanelli et al. 2006d; Gonzalez et al. 2007b; Baker et al. 2007b; Herrmann et al. 2007a,b; Campanelli et al. 2007a,b; Koppitz et al. 2007; Gonzalez et al. 2007a; Baker et al. 2007a; Tichy & Marronetti 2007).

The most rapid advances in modeling black-hole binary coalescences cover the previously least understood part of the gravitational waveform, i.e., the final few cycles of radiation generated from near the "innermost stable circular orbit" (ISCO) and afterward, which we call the "merger ringdown." There is already considerable progress toward a full understanding of this important "burst" portion of the waveform, through which the frequency sweeps by a factor of ~ 3 up to ringdown, and during which the gravitational wave luminosity is $\sim 10^{23}\ L_\odot$, more than the luminosity of the combined starlight in the visible universe.

A particularly significant development was the demonstration of initial data-independence of merger-ringdown waveforms for equal-mass, nonspinning black holes (Baker et al. 2006b), as summarized in Figure 1. Results from four runs with successively larger initial separations are shown; the waveforms have been aligned so that the moment of peak radiation amplitude in each simulation occurs at time $t = 0$. Here we show the gravitational wave strain from the dominant ($l = 2$, $m = 2$) mode; this represents an observation made on the equatorial plane of the system, where only a single polarization component contributes to the measured strain. The upper panel of Figure 1 shows the full simulation waveforms, while the lower panel focuses on the final merger-ringdown burst. Note that here and elsewhere in this paper, we use *geometrical units*, $G = c = 1$, to measure time, distance and mass in the same units. In particular, one solar mass $M_\odot$ is equivalent to $\sim 5 \times 10^{-6}$ sec, or ~ 1.5 km.† In Figure 1, our timescale is the final mass m_f, of the post-merger hole; this will be less than the initial total binary mass M because of gravitational radiation. In the shortest run (solid line), the black holes are placed on initial orbits close to the ISCO and undergo a brief plunge followed by a merger and ringdown (Baker et al. 2006c). At the next-largest initial separation (dashed line), the black holes complete ~ 1.8 orbits before merging (Baker et al. 2006b). The waveforms from the two runs with successively larger initial separations (dot-dashed and dotted lines, respectively) then lock on to the merger-ringdown part of this shorter (dashed) run. In fact, the waveforms from the three longer simulations show very strong agreement for $t \gtrsim -50\, m_f$, with differences among these waveforms $\lesssim 1\%$ in this regime.

† Since the simulation results scale with the masses of the black holes, they are equally applicable to *LISA* and ground-based detectors.