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978-1-107-00509-9 - Quantum Mechanics and Quantum Field Theory: A Mathematical Primer

Jonathan Dimock

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## Quantum Mechanics and Quantum Field Theory

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Explaining the concepts of quantum mechanics and quantum field theory in a precise mathematical language, this textbook is an ideal introduction for graduate students in mathematics, helping to prepare them for further studies in quantum physics.

The textbook covers topics that are central to quantum physics: non-relativistic quantum mechanics, quantum statistical mechanics, relativistic quantum mechanics, and quantum field theory. There is also background material on analysis, classical mechanics, relativity, and probability. Each topic is explored through a statement of basic principles followed by simple examples. Around 100 problems throughout the textbook help readers develop their understanding.

**Jonathan Dimock** is Professor of Mathematics, SUNY at Buffalo. He has carried out research in various areas of mathematical physics, including constructive quantum field theory, quantum field theory on manifolds, renormalization group methods, and string theory.

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## A Mathematical Primer

JONATHAN DIMOCK

SUNY at Buffalo



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for Benjamin, Christina, Gregory, and Ann

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## Preface

This is a book on mathematical physics for a reader with a good background in mathematics, but possibly a minimal knowledge of physics. The subject matter is quantum physics and includes non-relativistic quantum mechanics, quantum statistical mechanics, relativistic quantum mechanics, and quantum field theory. The book only contains material which meets the twin criteria of being basic physics and being treatable with complete mathematical rigor. For each topic there is a straightforward statement of basic principles followed by simple examples. There is also background material in analysis, classical mechanics, relativity, and probability.

The book does not prove deep mathematical theorems. The book does not consider the complicated models of mathematical physics. The book does not enter into the fascinating speculative topics on the frontiers of physics, for example string theory. Finally the book does not consider questions concerning the foundations or philosophy of quantum physics. However the book does help prepare the reader for a journey in any of these directions.

The book assumes knowledge of elementary analysis, measure theory, linear algebra, some group theory, and some knowledge of differential equations. Some reference is made to manifolds, differential geometry, and Lie groups. Not much knowledge of physics is assumed beyond an introductory course. However one probably needs more than this to really appreciate the material.

The book is suitable for a graduate course in mathematics. In this connection there are problems scattered throughout the text. These serve the dual function of further developing the material and providing a study aid. The level of difficulty is quite variable.

Books which cover similar ground are Gustafson and Sigal (2003) and Takhtajan (2008). The mathematical level is about the same, but they have different points of emphasis.