

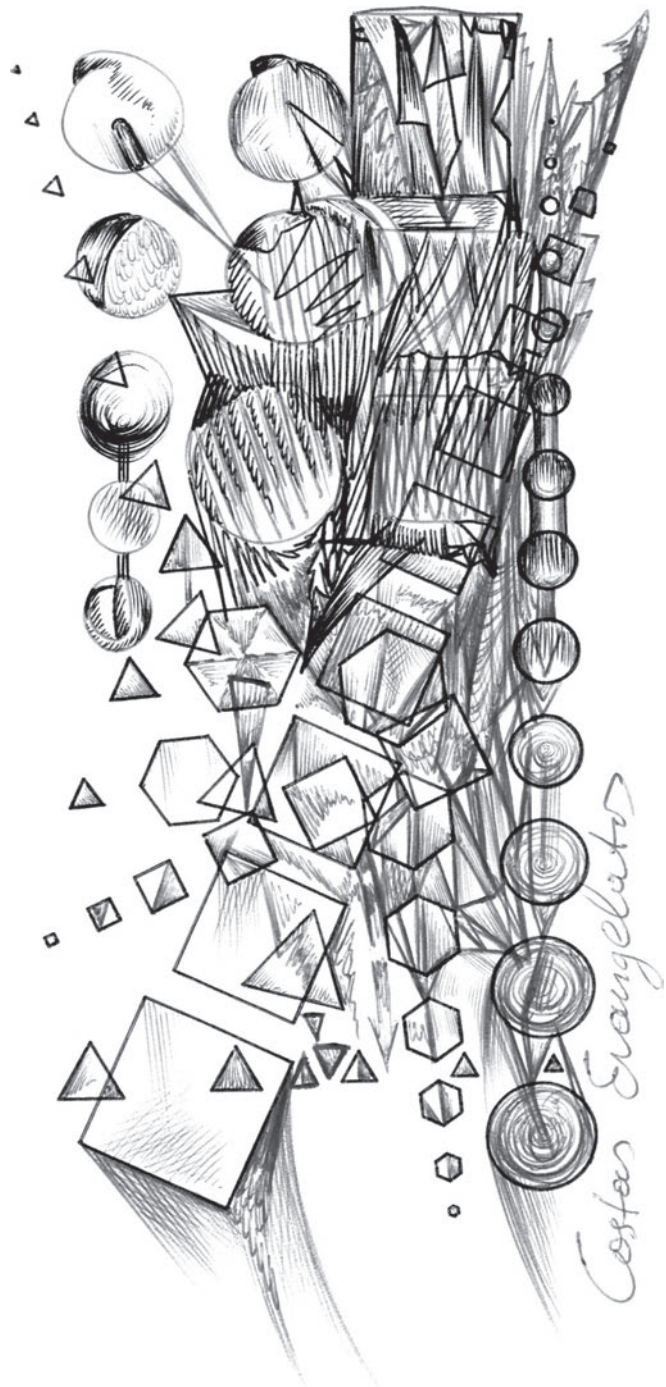
Introduction to Topological Quantum Computation

Combining physics, mathematics and computer science, topological quantum computation is a rapidly expanding research area focused on the exploration of quantum evolutions that are immune to errors. In this book, the author presents a variety of different topics developed together for the first time, forming an excellent introduction to topological quantum computation.

The makings of topological systems, their properties and their computational power are presented in a pedagogical way. Relevant calculations are fully explained, and numerous worked examples and exercises support and aid understanding. Special emphasis is given to the motivation and physical intuition behind every mathematical concept.

Demystifying difficult topics by using accessible language, this book has broad appeal and is ideal for graduate students and researchers from various disciplines who want to get into this new and exciting research field.

Giannis K. Pachos is a Reader in the School of Physics and Astronomy at the University of Leeds, UK. He works on a variety of research topics, ranging from quantum field theory to quantum optics. Dr Pachos is a University Research Fellow of the Royal Society.



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Cambridge University Press

978-1-107-00504-4 - Introduction to Topological Quantum Computation

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CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town,
Singapore, São Paulo, Delhi, Tokyo, Mexico City

Cambridge University Press
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9781107005044

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First published 2012

Printed in the United Kingdom at the University Press, Cambridge

A catalogue record for this publication is available from the British Library

ISBN 978-1-107-00504-4 Hardback

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To Almut and Sevi

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Acknowledgements

I am grateful to Almut Beige and Ville Lahtinen for their continuous support and guidance during the writing of this book. I would also like to thank several people for their direct or indirect support, such as Miguel Aguado, Abbas Al-Shimary, Gavin Brennen, Michael Freedman, Silvano Garnerone, Sofyan Iblisdir, Roman Jackiw, Petr Jizba, Louis Kauffman, Nikolai Kiesel, Alexei Kitaev, Lauri Lehman, Samuel Lomonaco, Mark Mitchison, David Perez-Garcia, So-Young Pi, John Preskill, Nicholas Read, Renato Renner, Emily Riley, Christian Schmid, Ady Stern, David Tong, Zhenghan Wang, Harald Weinfurter, Witłef Wicczorek, James Wootton, Paolo Zanardi and Václav Zatloukal.