Cambridge Tracts in Theoretical Computer Science 52

Advanced Topics in Bisimulation and Coinduction

Coinduction is a method for specifying and reasoning about infinite data types and automata with infinite behaviour. In recent years, it has come to play an ever more important role in the theory of computing. It is studied in many disciplines, including process theory and concurrency, modal logic and automata theory. Typically, coinductive proofs demonstrate the equivalence of two objects by constructing a suitable bisimulation relation between them.

This collection of surveys is aimed at both researchers and Master's students in computer science and mathematics, and deals with various aspects of bisimulation and coinduction, with an emphasis on process theory. Seven chapters cover the following topics: history; algebra and coalgebra; algorithmics; logic; higher-order languages; enhancements of the bisimulation proof method; and probabilities. Exercises are also included to help the reader master new material.

DAVIDE SANGIORGI is Full Professor in Computer Science at the University of Bologna, Italy, and Head of the University of Bologna/INRIA team 'Focus'.

JAN RUTTEN is a senior researcher at Centrum Wiskunde & Informatica (CWI) in Amsterdam and Professor of Theoretical Computer Science at the Radboud University Nijmegen.

Cambridge Tracts in Theoretical Computer Science 52

Titles in the series

A complete list of books in the series can be found at www.cambridge.org/mathematics.
Recent titles include the following:

29. P. Gärdenfors (ed) *Belief Revision*
30. M. Anthony & N. Biggs *Computational Learning Theory*
31. T. F. Melham *Higher Order Logic and Hardware Verification*
32. R. Carpenter *The Logic of Typed Feature Structures*
33. E. G. Manes *Predicate Transformer Semantics*
34. F. Nielson & H. R. Nielson *Two-Level Functional Languages*
35. L. M. G. Feijs & H. B. M. Jonkers *Formal Specification and Design*
36. S. Mauw & G. J. Veltink (eds) *Algebraic Specification of Communication Protocols*
37. V. Stavridou *Formal Methods in Circuit Design*
38. N. Shankar *Metamathematics, Machines and Gödel's Proof*
39. J. B. Paris *The Uncertain Reasoner's Companion*
40. J. Desel & J. Esparza *Free Choice Petri Nets*
41. J.-J. Ch. Meyer & W. van der Hoek *Epistemic Logic for AI and Computer Science*
42. J. R. Hindley *Basic Simple Type Theory*
43. A. S. Troelstra & H. Schwichtenberg *Basic Proof Theory*
44. J. Barwise & J. Seligman *Information Flow*
45. A. Asperti & S. Guerrini *The Optimal Implementation of Functional Programming Languages*
46. R. M. Amadio & P.-L. Curien *Domains and Lambda-Calculi*
47. W.-P. de Roever & K. Engelhardt *Data Refinement*
48. H. Kleine Büning & T. Lettmann *Propositional Logic*
49. L. Novak & A. Gibbons *Hybrid Graph Theory and Network Analysis*
50. J. C. M. Baeten, T. Basten & M. A. Reniers *Process Algebra: Equational Theories of Communicating Processes*
51. H. Simmons *Derivation and Computation*
52. D. Sangiorgi & J. Rutten (eds) *Advanced Topics in Bisimulation and Coinduction*
53. P. Blackburn, M. de Rijke & Y. Venema *Modal Logic*
54. W.-P. de Roever et al. *Concurrency Verification*
55. Terese *Term Rewriting Systems*
56. A. Bundy et al. *Rippling: Meta-Level Guidance for Mathematical Reasoning*

Advanced Topics in Bisimulation and Coinduction

Edited by

DAVIDE SANGIORGI
University of Bologna (Italy)
and INRIA (France)

JAN RUTTEN
Centrum Wiskunde & Informatica (CWI), Amsterdam
and the Radboud University Nijmegen

CAMBRIDGE UNIVERSITY PRESS

CAMBRIDGE
UNIVERSITY PRESS

Shaftesbury Road, Cambridge CB2 8EA, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India

103 Penang Road, #05–06/07, Visioncrest Commercial, Singapore 238467

Cambridge University Press is part of Cambridge University Press & Assessment,
a department of the University of Cambridge.

We share the University's mission to contribute to society through the pursuit of
education, learning and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org/9781107004979

First published 2012

A catalogue record for this publication is available from the British Library

ISBN 978-1-107-00497-9 Hardback

Additional resources for this publication at www.cs.unibo.it/~sangio/Book_Bis_Coind.html

Contents

Contributors

Luca Aceto
School of Computer Science, Reykjavik University, Menntavegur 1,
101 Reykjavik, Iceland.
email: luca@ru.is
web: www.ru.is/faculty/luca/

Anna Ingolfsdottir
School of Computer Science, Reykjavik University, Menntavegur 1,
101 Reykjavik, Iceland.
email: annai@ru.is
web: www.ru.is/faculty/annai/

Bart Jacobs
Institute for Computing and Information Sciences (ICIS), Radboud University
Nijmegen, Heyendaalseweg 135, 6525 AJ Nijmegen, The Netherlands.
email: bart@cs.ru.nl
web: www.cs.ru.nl/B.Jacobs/

Prakash Panangaden
McGill University, 3480 rue University, Room 318 Montreal, Quebec,
H3A 2A7 Canada.
email: prakash@cs.mcgill.ca
web: www.cs.mcgill.ca/∼prakash/

Andrew M. Pitts
University of Cambridge, Computer Laboratory, William Gates Building,
15 JJ Thomson Ave, Cambridge CB3 0FD, UK.
email: Andrew.Pitts@cl.cam.ac.uk
web: www.cl.cam.ac.uk/∼amp12/

Damien Pous
CNRS, Team Sardes, INRIA Rhône-Alpes, 655, avenue de l'Europe, Montbonnot, 38334 Saint Ismier, France.
email: Damien.Pous@inria.fr
web: http://sardes.inrialpes.fr/∼pous/

Jan Rutten
CWI, P.O. Box 94079, 1090 GB Amsterdam, The Netherlands.
Also: Radboud University Nijmegen.
email: janr@cwi.nl
web: http://homepages.cwi.nl/∼janr/

Davide Sangiorgi
Università di Bologna/INRIA Team Focus, Dipartimento di Scienze dell'Informazione, Università di Bologna, Mura Anteo Zamboni, 7 40126 Bologna, Italy.
email: Davide.Sangiorgi@cs.unibo.it
web: www.cs.unibo.it/∼sangio/

Jiri Srba
Department of Computer Science, University of Aalborg, Selma Lagerlöfs Vej 300, 9220 Aalborg East, Denmark.
email: srba@cs.aau.dk
web: www.brics.dk/∼srba/

Colin Stirling
School of Informatics, Edinburgh University, Informatics Forum, 10 Crichton Street, Edinburgh EH8 9AB.
email: cps@staffmail.ed.ac.uk
web: http://homepages.inf.ed.ac.uk/cps/

Preface

This book is about bisimulation and coinduction. It is the companion book of the volume *An Introduction to Bisimulation and Coinduction*, by Davide Sangiorgi (Cambridge University Press, 2011), which deals with the basics of bisimulation and coinduction, with an emphasis on labelled transition systems, processes, and other notions from the theory of concurrency.

In the present volume, we have collected a number of chapters, by different authors, on several advanced topics in bisimulation and coinduction. These chapters either treat specific aspects of bisimulation and coinduction in great detail, including their history, algorithmics, enhanced proof methods and logic. Or they generalise the basic notions of bisimulation and coinduction to different or more general settings, such as coalgebra, higher-order languages and probabilistic systems. Below we briefly summarise the chapters in this volume.

- *The origins of bisimulation and coinduction, by Davide Sangiorgi*

  In this chapter, the origins of the notions of bisimulation and coinduction are traced back to different fields, notably computer science, modal logic, and set theory.
- *An introduction to (co)algebra and (co)induction, by Bart Jacobs and Jan Rutten*

  Here the notions of bisimulation and coinduction are explained in terms of coalgebras. These mathematical structures generalise all kinds of infinite-data structures and automata, including streams (infinite lists), deterministic and probabilistic automata, and labelled transition systems. Coalgebras are formally dual to algebras and it is this duality that is used to put both induction and coinduction into a common perspective. This generalises the treatment in the companion introductory volume, where induction and coinduction were explained in terms of least and greatest fixed points.

- *The algorithmics of bisimilarity, by Luca Aceto, Anna Ingolfsdottir and Jiří Srba*

  This chapter gives an overview of the solutions of various algorithmic problems relating bisimilarity and other equivalences and preorders on labelled transition systems. Typical questions that are addressed are: How can one compute bisimilarity? What is the complexity of the algorithms? When is bisimilarity decidable?
- *Bisimulation and logic, by Colin Stirling*

  This chapter discloses the strong and beautiful ties that relate bisimulation and modal logics. Various logical characterisations of bisimilarity are discussed. The main results are the characterisations of bisimilarity via a simple modal logic, the Hennessy–Milner logic, and the characterisation of this modal logic as the fragment of first-order logic that is bisimulation invariant. The results are then extended to modal logic with fixed points and to second-order logic.
- *Howe's Method for higher-order languages, by Andrew Pitts*

  In programming languages, an important property of bisimulation-based equivalences is whether they are a congruence, that is, compatible with the language constructs. This property may be difficult to prove if such languages involve higher-order constructs, that is, ones permitting functions and processes to be data that can be manipulated by functions and processes. This chapter presents a method for establishing compatibility of coinductively defined program equalities, originally due to Howe.
- *Enhancements of the bisimulation proof method, by Damien Pous and Davide Sangiorgi*

  This chapter discusses enhancements of the bisimulation proof method, with the goal of facilitating the proof of bisimilarity results. The bisimulation proof method is one of the main reasons for the success of bisimilarity. According to the method, to establish the bisimilarity between two given objects one has to find a bisimulation relation containing these objects as a pair. This means proving a certain closure property for each pair in the relation. The amount of work needed in proofs therefore depends on the size of the relation. The enhancements of the method in the chapter allow one to reduce such work by using relations that need only be *contained* in bisimulation relations. The chapter shows that it is possible to define a whole theory of enhancements, which can be very effective in applications.
- *Probabilistic bisimulation, by Prakash Panangaden*

  Here notions of bisimulation are introduced for probabilistic systems. These differ from non-deterministic ones in that they take quantitative data into account on the basis of which they make quantitative predictions about a system's behaviour. The chapter first discusses the basic example of discrete systems, called labelled Markov chains. After a rapid introductory section on measure theory, the more general continuous case, of so-called labelled Markov processes, is treated. For both the discrete and the continuous case, logical characterisations of bisimilarity are given.

The chapters on probabilities and higher-order linguistic constructs deal with two important refinements of bisimulation. While these are certainly not the only interesting refinements of bisimulation (one could mention, for instance, the addition of time or of space constraints), probabilities and higher-order constructs have strong practical relevance (e.g. in distributed systems and other complex systems such as biological systems, and in programming languages) and offer technical challenges that make them one of the most active research topics in the area of coinduction and bisimulation.

Each chapter is a separate entity, therefore notations among chapters may occasionally differ. We are very grateful to the colleagues who contributed the chapters for the time, effort, and enthusiasm that they put into this book project.

We recall the Web page with general information and auxiliary material about the two volumes, including solutions to exercises in some of the chapters. At the time of writing, the page is

`www.cs.unibo.it/~sangio/Book_Bis_Coind.html`

Davide Sangiorgi and Jan Rutten