

# 1 Measurement of wireless transceivers

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## 1.1 Introduction

This book is entitled *Microwave and Wireless Measurement Techniques*, since the objective is to identify and understand measurement theory and practice in wireless systems.

In this book, the concept of a wireless system is applied to the collection of sub-systems that are designed to behave in a particular way and to apply a certain procedure to the signal itself, in order to convert a low-frequency information signal, usually called the baseband signal, to a radio-frequency (RF) signal, and transmit it over the air, and vice versa.

Figure 1.1 presents a typical commercial wireless system architecture. The main blocks are amplifiers, filters, mixers, oscillators, passive components, and domain converters, namely digital to analog and vice versa.

In each of these sub-systems the measurement instruments will be measuring voltages and currents as in any other electrical circuit. In basic terms, what we are measuring are always voltages, like a voltmeter will do for low-frequency signals. The problem here is stated as how we are going to be able to capture a high-frequency signal and identify and quantify its amplitude or phase difference with a reference signal. This is actually the problem throughout the book, and we will start by identifying the main figures of merit that deserve to be measured in each of the identified sub-systems.

In order to do that, we will start by analyzing a general sub-system that can be described by a network. In RF systems it can be a single-port, two-port, or three-port network. The two-port network is the most common.

## 1.2 Linear two-port networks

### 1.2.1 Microwave description

A two-port network, Fig. 1.2, is a network in which the terminal voltages and currents relate to each other in a certain way.

The relationships between the voltages and currents of a two-port network can be given by matrix parameters such as  $Z$ -parameters,  $Y$ -parameters, or ABCD parameters. The reader can find more information in [1, 2].

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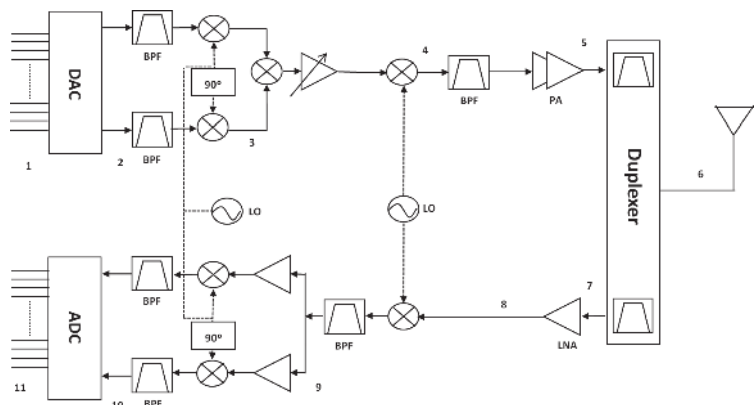


Figure 1.1 A typical wireless system architecture, with a full receiver and transmitter stage.

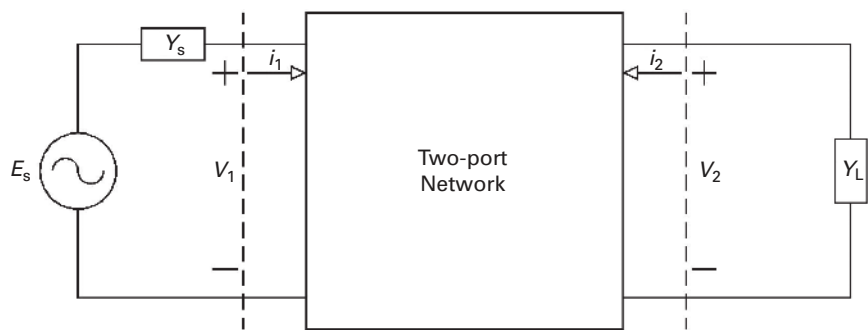


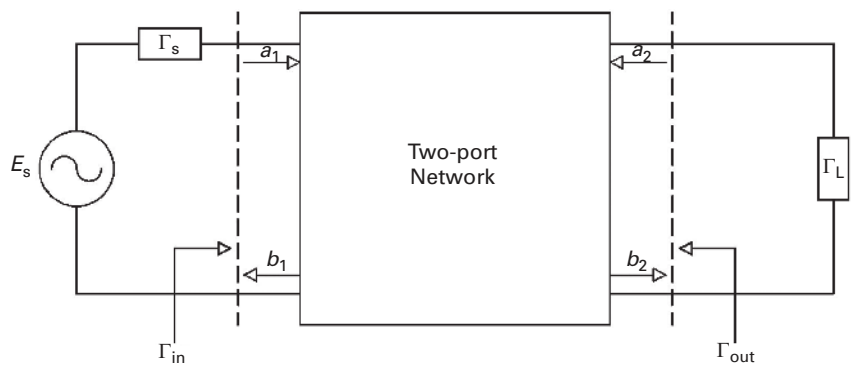
Figure 1.2 A two-port network, presenting the interactions of voltages and currents at its ports.

The objective is always to relate the input and output voltages and currents by using certain relationships. One of these examples using  $Y$ -parameters is described by the following equation:

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \tag{1.1}$$

where

$$\begin{aligned} y_{11} &= \left. \frac{i_1}{v_1} \right|_{v_2=0} \\ y_{12} &= \left. \frac{i_1}{v_2} \right|_{v_1=0} \\ y_{21} &= \left. \frac{i_2}{v_1} \right|_{v_2=0} \\ y_{22} &= \left. \frac{i_2}{v_2} \right|_{v_1=0} \end{aligned}$$



**Figure 1.3** Two-port scattering parameters, where the incident and reflected waves can be seen in each port.

As can be seen, these  $Y$ -parameters can be easily calculated by considering the other port voltage equal to zero, which means that the other port should be short-circuited. For instance,  $y_{11}$  is the ratio of the measured current at port 1 and the applied voltage at port 1 by which port 2 is short-circuited.

Unfortunately, when we are dealing with high-frequency signals, a short circuit is not so simple to realize, and in that case more robust high-frequency parameters should be used.

In that sense some scientists started to think of alternative ways to describe a two-port network, and came up with the idea of using traveling voltage waves [1, 2]. In this case there is an incident traveling voltage wave and a scattered traveling voltage wave at each port, and the network parameters become a description of these traveling voltage waves, Fig. 1.3.

One of the most well-known matrices used to describe these relations consists of the scattering parameters, or  $S$ -parameters, by which the scattered traveling voltage waves are related to the incident traveling voltage waves in each port.

In this case each voltage and current in each port will be divided into an incident and a scattered traveling voltage wave,  $V^+(x)$  and  $V^-(x)$ , where the  $+$  sign refers to the incident traveling voltage wave and the  $-$  sign refers to the reflected traveling voltage wave. The same can be said about the currents, where  $I^+(x) = V^+(x)/Z_0$  and  $I^-(x) = V^-(x)/Z_0$ ,  $Z_0$  being the characteristic impedance of the port. The value  $x$  now appears since we are dealing with waves that travel across the space, being guided or not, so  $V^+(x) = Ae^{-\gamma x}$  [1, 2].

These equations can be further simplified and normalized to be used efficiently:

$$\begin{aligned} v(x) &= \frac{V(x)}{\sqrt{Z_0}} \\ i(x) &= \sqrt{Z_0} I(x) \end{aligned} \tag{1.2}$$

Then each normalized voltage and current can be decomposed into its incident and scattered wave. The incident wave is denoted  $a(x)$  and the scattered one  $b(x)$ :

$$\begin{aligned} v(x) &= a(x) + b(x) \\ i(x) &= a(x) - b(x) \end{aligned} \tag{1.3}$$

where

$$\begin{aligned} a(x) &= \frac{V^+(x)}{\sqrt{Z_0}} \\ b(x) &= \frac{V^-(x)}{\sqrt{Z_0}} \end{aligned} \tag{1.4}$$

with

$$\begin{aligned} V &= \sqrt{Z_0}(a + b) \\ I &= \frac{1}{\sqrt{Z_0}}(a - b) \end{aligned}$$

Fortunately, we also know that in a load the reflected wave can be related to the incident wave using its reflection coefficient  $\Gamma(x)$ :

$$b(x) = \Gamma(x)a(x)$$

or

$$\Gamma(x) = \frac{b(x)}{a(x)} \tag{1.5}$$

In this way it is then possible to calculate and use a new form of matrix parameter to describe these wave relationships in a two-port network, namely the scattering parameters:

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \tag{1.6}$$

where

$$S_{ij} = \left. \frac{b_i(x)}{a_j(x)} \right|_{a_k=0 \text{ to } k \neq j} \tag{1.7}$$

As can be deduced from the equations, and in contrast to the  $Y$ -parameters, for the calculation of each parameter, the other port should have no reflected wave. This corresponds to matching the other port to the impedance of  $Z_0$ . This is easier to achieve at high frequencies than realizing a short circuit or an open circuit, as used for  $Y$ - and  $Z$ -parameters, respectively.

Moreover, using this type of parameter allows us to immediately calculate a number of important parameters for the wireless sub-system. On looking at the next set of equations, it is possible to identify the input reflection coefficient immediately from  $S_{11}$ , or, similarly, the output reflection coefficient from  $S_{22}$ :

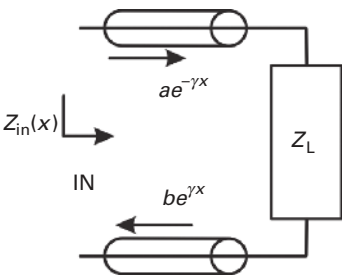


Figure 1.4 Power waves traversing a guided structure.

$$\begin{aligned} S_{11} &= \left. \frac{b_1(x)}{a_1(x)} \right|_{a_2=0} \\ &= \frac{Z_1 - Z_0}{Z_1 + Z_0} \end{aligned} \tag{1.8}$$

$$\begin{aligned} S_{22} &= \left. \frac{b_2(x)}{a_2(x)} \right|_{a_1=0} \\ &= \frac{Z_2 - Z_0}{Z_2 + Z_0} \end{aligned} \tag{1.9}$$

The same applies to the other two parameters,  $S_{21}$  and  $S_{12}$ , which correspond to the transmission coefficient and the reverse transmission coefficient, respectively. The square of their amplitude corresponds to the forward and reverse power gain when the other port is matched.

Note that in the derivation of these parameters it is assumed that the other port is matched. If that is not the case, the values can be somewhat erroneous. For instance,  $\Gamma_{in}(x) = S_{11}$  only if the other port is matched or either  $S_{12}$  or  $S_{21}$  is equal to zero. If this is not the case, the input reflection should be calculated from

$$\Gamma_{in} = S_{11} + \frac{S_{12} S_{21} \Gamma_L}{1 + S_{22} \Gamma_L} \tag{1.10}$$

More information can be found in [1, 2].

With the parameters based on the wave representation that have now been defined, several quantities can be calculated. See Fig. 1.4.

For example, if the objective is to calculate the power at terminal IN, then

$$P = VI^* = aa^* - bb^* = |a|^2 - |b|^2 \tag{1.11}$$

Here  $|a|^2$  actually corresponds to the incident power, while  $|b|^2$  corresponds to the reflected power.

Important linear figures of merit that are common to most wireless sub-systems can now be defined using the  $S$ -parameters.

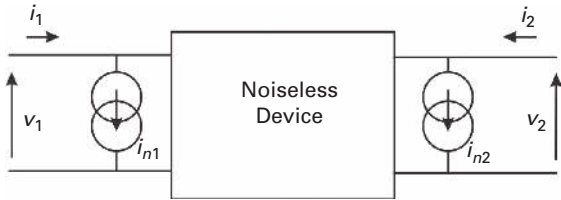


Figure 1.5 A noisy device, Y-parameter representation, including noise sources.

1.2.2 Noise

Another very important aspect to consider when dealing with RF and wireless systems is the amount of introduced noise. Since for RF systems the main goal is actually to achieve a good compromise between power and noise, in order to achieve a good noise-to-power ratio, the study of noise is fundamental. For that reason, let us briefly describe the noise behavior [3] in a two-port network.

A noisy two-port network can be represented by a noiseless two-port network and a noise current source at each port. An admittance representation can be developed.

The voltages and currents in each port can be related to the admittance matrix:

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = [Y] \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} i_{n1} \\ i_{n2} \end{bmatrix} \tag{1.12}$$

(Fig. 1.5). A correlation matrix  $C_Y$  can also be defined, as

$$[C_Y] = \begin{bmatrix} \langle i_{n1} i_{n1}^* \rangle & \langle i_{n1} i_{n2}^* \rangle \\ \langle i_{n2} i_{n1}^* \rangle & \langle i_{n2} i_{n2}^* \rangle \end{bmatrix} \tag{1.13}$$

The correlation matrix relates the properties of the noise in each port. For a passive two-port network, one has

$$[C_Y] = 4k_B T \Delta f \operatorname{Re}(Y) \tag{1.14}$$

where  $k_B$  is the Boltzmann constant ( $1.381 \times 10^{-23}$  J/K),  $T$  the temperature (typically 290 K),  $\Delta f$  the bandwidth, and  $Y$  the admittance parameter.

Actually these port parameters can also be represented by using scattering parameters. In that case the noisy two-port network is represented by a noiseless two-port network and the noise scattering parameters referenced to a nominal impedance at each port (Fig. 1.6).

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = [S] \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + \begin{bmatrix} b_{n1} \\ b_{n2} \end{bmatrix} \tag{1.15}$$

where  $b_{n1}$  and  $b_{n2}$  can be considered noise waves, and they are related using the correlation matrix,  $C_S$ . The correlation matrix  $C_S$  is defined by

$$[C_S] = \begin{bmatrix} \langle b_{n1} b_{n1}^* \rangle & \langle b_{n1} b_{n2}^* \rangle \\ \langle b_{n2} b_{n1}^* \rangle & \langle b_{n2} b_{n2}^* \rangle \end{bmatrix} \tag{1.16}$$

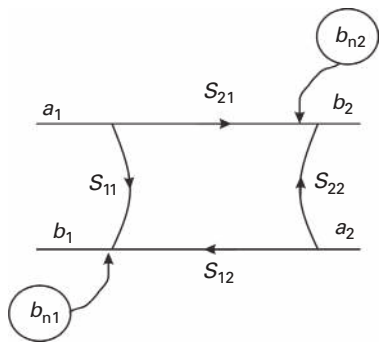


Figure 1.6 A noisy device, *S*-parameter representation.

and, for a passive two-port network,

$$[C_S] = k_B T \Delta f ((I) - (S)(S)^{T*}) \tag{1.17}$$

where  $(I)$  is the unit matrix and  $(S)^{T*}$  denotes transpose and conjugate.

1.3 Linear FOMs

After having described linear networks, we proceed to explain the corresponding figures of merit (FOMs). We make a distinction between FOMs that are defined on the basis of *S*-parameters (Section 1.3.1) and those defined on the basis of noise (Section 1.3.2).

1.3.1 Linear network FOMs

1.3.1.1 The voltage standing-wave ratio

The voltage standing-wave ratio (VSWR) is nothing more than the evaluation of the port mismatch. Actually, it is a similar measure of port matching, the ratio of the standing-wave maximum voltage to the standing-wave minimum voltage. Figure 1.7 shows different standing-wave patterns depending on the load.

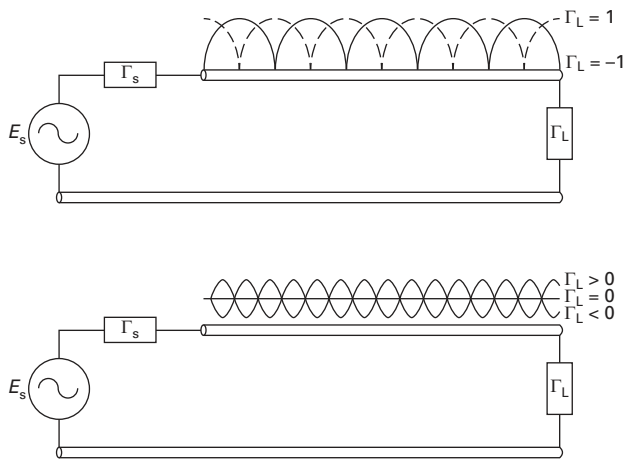
In this sense it therefore relates the magnitude of the voltage reflection coefficient and hence the magnitude of either  $S_{11}$  for the input port or  $S_{22}$  for the output port.

The VSWR for the input port is given by

$$\text{VSWR}_{\text{in}} = \frac{1 + |S_{11}|}{1 - |S_{11}|} \tag{1.18}$$

and that for the output port is given by

$$\text{VSWR}_{\text{out}} = \frac{1 + |S_{22}|}{1 - |S_{22}|} \tag{1.19}$$



**Figure 1.7** The VSWR and standing-wave representation. The standing wave can be seen for different values of the VSWR.

1.3.1.2 Return loss

Other important parameters are the input and output return losses. The input return loss ( $RL_{in}$ ) is a scalar measure of how close the actual input impedance of the network is to the nominal system impedance value, and is given by

$$RL_{in} = |20 \log_{10} |S_{11}|| \text{ dB} \tag{1.20}$$

It should be noticed that this value is valid only for a single-port network, or, in a two-port network, it is valid only if port 2 is matched; if not,  $S_{11}$  should be exchanged for the input reflection coefficient as presented in Eq. (1.10). As can be seen from its definition, the return loss is a positive scalar quantity.

The output return loss ( $RL_{out}$ ) is similar to the input return loss, but applied to the output port (port 2). It is given by

$$RL_{out} = |20 \log_{10} |S_{22}|| \text{ dB} \tag{1.21}$$

1.3.1.3 Gain/insertion loss

Since  $S_{11}$  and  $S_{22}$  have the meaning of reflection coefficients, their values are always smaller than or equal to unity. The exception is the  $S_{11}$  of oscillators, which is larger than unity, because the RF power returned is larger than the RF power sent into the oscillator port.

The  $S_{21}$  of a linear two-port network can have values either smaller or larger than unity. In the case of passive circuits,  $S_{21}$  has the meaning of loss, and is thus restricted to values smaller than or equal to unity. This loss is usually called the insertion loss. In the case of active circuits, there is usually gain, or in other words  $S_{21}$  is larger than unity. In the case of passive circuits,  $S_{12}$  is equal to  $S_{21}$  because passive circuits are reciprocal. The only exception is the case of ferrites. In the case of active circuits,  $S_{12}$  is different from  $S_{21}$  and usually much smaller than unity, since it represents feedback,



which is often avoided by design due to the Miller effect. The gain or loss is typically expressed in decibels:

$$\text{gain/insertion loss} = |20 \log_{10} |S_{21}|| \text{ dB} \tag{1.22}$$

1.3.2 Noise FOMs

1.3.2.1 The noise factor

The previous results actually lead us to a very important and key point regarding noisy devices, that is, the FOM called the noise factor (NF), which characterizes the degradation of the signal-to-noise ratio (SNR) by the device itself.

The noise factor is defined as follows.

**DEFINITION 1.1** *The noise factor (F) of a circuit is the ratio of the signal-to-noise ratio at the input of the circuit to the signal-to-noise ratio at the output of the circuit:*

$$F = \frac{S_I/N_I}{S_O/N_O} \tag{1.23}$$

where

$S_I$  is the power of the signal transmitted from the source to the input of the two-port network

$S_O$  is the power of the signal transmitted from the output of the two-port network to the load

$N_I$  is the power of the noise transmitted from the source impedance  $Z_S$  at temperature  $T_0 = 290 \text{ K}$  to the input of the two-port network

$N_O$  is the power of the noise transmitted from the output of the two-port network to the load

The noise factor can be expressed as

$$F = \frac{N_{ad} + G_A N_{al}}{G_A N_{al}} \tag{1.24}$$

where  $G_A$  is the available power gain of the two-port network (for its definition, see Section 1.8),  $N_{ad}$  is the additional available noise power generated by the two-port network, and  $N_{al}$  is the available noise power generated by the source impedance:

$$N_{al} = 4k_B T_0 \Delta f \tag{1.25}$$

As can be seen from Eq. (1.24),  $F$  is always greater than unity, and it does not depend upon the load  $Z_L$ . It depends exclusively upon the source impedance  $Z_S$ .

Using reference [3], the noise factor can also be related to the  $S$ -parameters by:

$$F = F_{\min} + 4 \frac{R_N}{Z_0} \frac{|\Gamma_{\text{OPT}} - \Gamma_s|^2}{(1 - |\Gamma_s|^2)|1 + \Gamma_{\text{OPT}}|^2} \tag{1.26}$$

where  $F_{\min}$  is the minimum noise factor,  $R_N$  is called the noise resistance,  $\Gamma_{\text{OPT}}$  is the optimum source reflection coefficient for which the noise factor is minimum.

This formulation can also be made in terms of  $Y$ -parameters, and can be expressed as a function of the source admittance  $Y_S$ :

$$F = F_{\min} + \frac{R_N}{\operatorname{Re}(Y_S)} |Y_S - Y_{\text{OPT}}|^2 \quad (1.27)$$

where  $Y_{\text{OPT}}$  is the optimum source admittance for which the noise factor is minimum. The terms  $F_{\min}$ ,  $R_N$ , and  $\Gamma_{\text{OPT}}$  (or  $Y_{\text{OPT}}$ ) constitute the four noise parameters of the two-port network. They can be related to the correlation matrices very easily [3]. The noise figure (NF) is simply the logarithmic version of the noise factor,  $F$ .

### 1.3.2.2 Cascade of noisy two-port components

If we cascade two noisy devices with noise factors  $F_1$  and  $F_2$ , and with available power gains  $G_{A1}$  and  $G_{A2}$ , with a source impedance at temperature  $T_0 = 290$  K, the additional available noise powers are

$$\begin{aligned} N_{\text{ad1}} &= (F_1 - 1)G_{A1}k_B T_0 \Delta f \\ N_{\text{ad2}} &= (F_2 - 1)G_{A2}k_B T_0 \Delta f \end{aligned} \quad (1.28)$$

The available noise power at the output of the second two-port network is

$$N_{\text{aO2}} = k_B T_0 \Delta f G_{A1} G_{A2} + N_{\text{ad1}} G_{A2} + N_{\text{ad2}} \quad (1.29)$$

The total noise factor is thus

$$F = \frac{N_{\text{aO2}}}{k_B T_0 \Delta f G_{A1} G_{A2}} \quad (1.30)$$

This finally leads to the well-known noise Friis formula,

$$F = F_1 + \frac{F_2 - 1}{G_{A1}} \quad (1.31)$$

In this expression the gain is actually the available power gain of the first two-port network, which depends on the output impedance of the first network.  $F_1$  depends on the source impedance, and  $F_2$  depends on the output impedance of the first two-port network.

The general Friis formula is

$$F = F_1 + \frac{F_2 - 1}{G_{A1}} + \frac{F_3 - 1}{G_{A1} G_{A2}} + \dots + \frac{F_N - 1}{G_{A1} G_{A2} \dots G_{A(N-1)}}$$

## 1.4 Nonlinear two-port networks

In order to better understand nonlinear distortion effects, let us start by explaining the fundamental properties of nonlinear systems. Since a nonlinear system is defined as a system that is not linear, we will start by explaining the fundamentals of linear systems.

Linear systems are systems that obey superposition. This means that they are systems whose output to a signal composed by the sum of elementary signals can be given as the sum of the outputs to these elementary signals when taken individually.