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Excerpt
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PART I

# PREPARING TO STUDY FLOW

# 1 Why Study Fluid Mechanics?

## 1.1 Getting motivated

Flows are beautiful and complex. A swollen creek tumbles over rocks and through crevasses, swirling and foaming. A child plays with sticky taffy, stretching and reshaping the candy as she pulls and twists it in various ways. Both the water and the taffy are fluids, and their motions are governed by the laws of nature. Our goal is to introduce readers to the analysis of flows using the laws of physics and the language of mathematics. On mastering this material, readers can harness flow to practical ends or create beauty through fluid design.

In this text we delve into the mathematical analysis of flows; however, before beginning, it is reasonable to ask if it is necessary to make this significant mathematical effort. After all, we can appreciate a flowing stream without understanding why it behaves as it does. We also can operate machines that rely on fluid behavior—drive a car, for example—without understanding the fluid dynamics of the engine. We can even repair and maintain engines, piping networks, and other complex systems without having studied the mathematics of flow. What is the purpose, then, of learning to mathematically describe fluid behavior?

The answer is quite practical: Knowing the patterns that fluids form and why they are formed, and knowing the stresses that fluids generate and why they are generated, is essential to designing and optimizing modern systems and devices. The ancients designed wells and irrigation systems without calculations, but we can avoid the waste and tedium of the trial-and-error process by using mathematical models. Some inventions, such as helicopters and lab-on-a-chip reactors, are sufficiently complex that they never would have been designed without mathematical models. Once a system is modeled accurately, it is then straightforward to calculate operating variables such as flow rates and pressures or to evaluate proposed design or operating changes. A mathematical understanding of fluids is important in fields such as airplane and space flight, biomedicine, plastics processing, volcanology, enhanced oil recovery, pharmaceuticals, environmental remediation, green energy, and astrophysics. Although a trial-and-error approach can get us started in fluids-related problems, significant progress requires formal mathematical analysis.

We seek, then, to understand and model flows. As we begin, one advantage we have is that we already know much about flow: We interact daily with fluids, from throwing balls through the air to watering the lawn (Figure 1.1). We can

Figure 1.1 Reducing the cross-sectional area of the nozzle of a garden hose increases the fluid velocity, causing the water to travel farther before gravity pulls the stream to the ground. The upstream pressure is approximately constant at the pressure supplied by the municipal water system.

build on this familiarity (Chapter 2) and add tools from calculus and physics (Chapters 3–6) to arrive at sensible modeling and engineering results and insights (Chapters 7–10).

We cover the basics, one of which is the use of the continuum model to describe flow. The continuum model treats fluids not as molecules but rather as a deformable whole with properties that can be described by continuous functions of space and time (Chapter 3). Another basic we must master is understanding how molecular stress is generated and diffused in flowing materials. This is a complex topic, and we use two chapters to discuss it (Chapters 4 and 5). We will see that a systematic approach to fluid-stress modeling can make this challenging topic accessible. The stress constitutive equation (Chapter 5) connects fluid stress and motion in a way that leads directly to predictions of flow behavior in subsequent chapters.

We ultimately solve flow problems with momentum balances, which we introduce in Chapter 3 and learn to apply to flows in subsequent chapters. The flows we consider are divided into internal and external flows (Chapters 7 and 8). In both internal and external flows, we consider two regimes of flow: laminar and turbulent. As shown in a water jet in Figure 1.2, in the slow-flow regime, called *laminar flow*, small pieces of fluid move in an orderly fashion in smooth and more-or-less straight lines. At higher flow rates (or at other times when conditions are right), the flow becomes disordered and fluid particles move along seemingly random paths, causing substantial mixing; this is called *turbulent flow*. Another classic behavior exhibited by fluids is the formation of *boundary layers* in rapid flows (Figure 1.3). Boundary layers, both laminar and turbulent (Chapter 8), form in rapid flows as a result of the interaction of fluid momentum with solid boundaries. Knowledge of the mechanisms of laminar flow, turbulent flow, and boundary layers provides the background we need to understand the intricate momentum exchanges in complex flows.

Once the basics are established, we move to a more advanced study of fluids (Chapters 9 and 10). The purpose of advanced study varies among individuals, but the ability to innovate and invent new technologies rests on having an advanced understanding of physical systems, including flowing systems

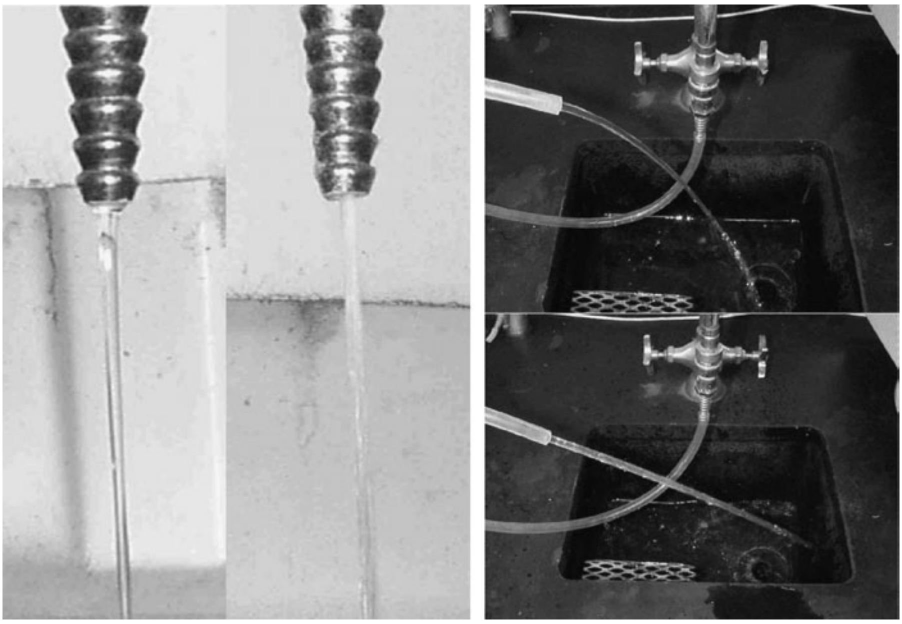


Figure 1.2 There are two basic flow regimes: a smooth slow flow-rate regime (i.e., laminar flow) and a rough, rapid flow-rate regime (i.e., turbulent flow).

(Figures 1.4 and 1.5). Advanced study may take the form of exploring: hemodynamics (i.e., the study of blood flow) [53]; non-Newtonian fluid mechanics, also called rheology [12, 104]; aeronautics [11, 76]; magnetohydrodynamics, which is important in astrophysics and metallurgy [35]; and microfluidics, a new field that explores the behavior of liquids confined in small spaces (Figure 1.5)

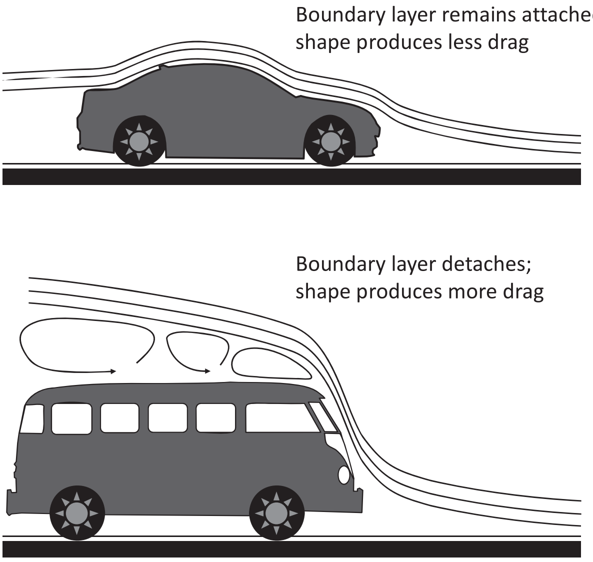



Figure 1.3 Schematic of an attached boundary layer flowing over a streamlined vehicle versus a detached boundary layer flowing over a blunt object such as a van.

Figure 1.4 The human body relies on fluid flow to provide the necessary functions of life. The circulatory system, with blood as the transport medium, keeps nutrients and oxygen flowing to every part of the body as needed and also transports waste back to the lungs and kidneys for disposal. Blood responds as a Newtonian fluid when flowing in arteries and larger veins but, in smaller regions, it displays non-Newtonian behavior [53]. Different flow behaviors are covered in Chapter 4. Shown here is an artificial heart. Detailed knowledge of blood-flow dynamics (i.e., hemodynamics) is required to contribute to the design and manufacture of such devices. Photo courtesy of Abiomed.

[2, 51, 75]. The last of these, microfluidics, is contributing to the development of new biological processing devices (e.g., sensors or lab-on-a-chip devices) that carry out molecular separations in microscopic channels. In this text we touch briefly on some advanced topics of fluid mechanics but, more important, we lay the groundwork needed for the study of such subjects.

The equations that govern flow are nonlinear, second-order, partial differential equations (PDEs); thus, they are complex. In this text we study solutions of PDEs, but we also study simple algebraic equations based on mass, energy, and momentum conservation that tell us a great deal about flows. In fact, the first step in a detailed system analysis usually is to perform algebraic macroscopic balances. In the next section, we introduce the macroscopic mass and energy balances for flow; we use these balances throughout the text, especially in the analysis of pumps and other fluid-driven machinery (Figure 1.6 and Chapter 9).

For detailed flow analysis, we must set up and solve partial differential equations (Chapters 6–8). For complex flows, although we know the PDEs that govern the flow, we cannot always solve them, even with modern methods and computers. When the complete solution of flow equations is not possible, an effective approach is to divide the flow domain into separate regions, where the equations may be simplified and therefore solved. This "divide-and-conquer" approach to

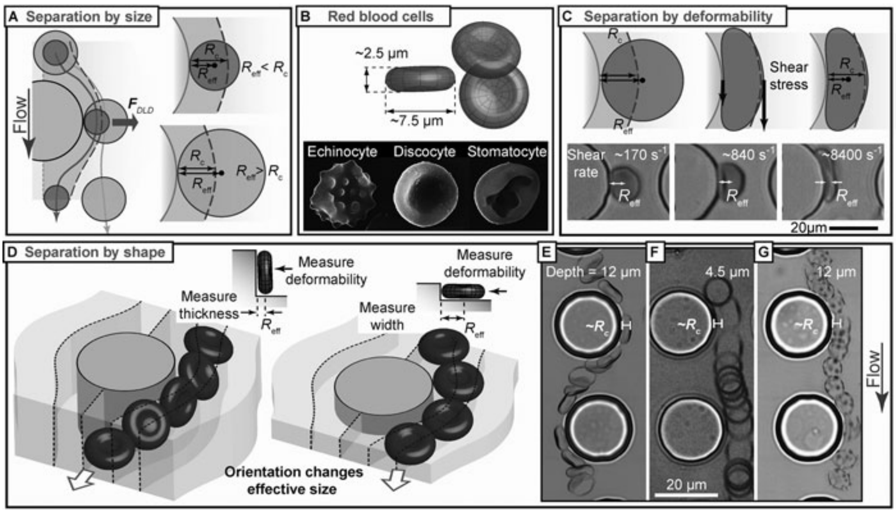
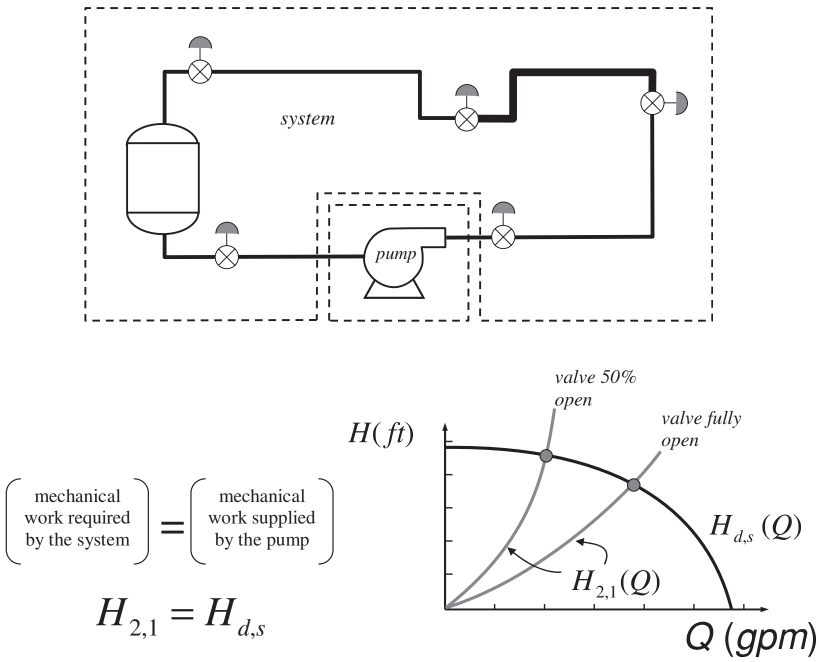



Figure 1.5 *Deterministic lateral displacement* (DLD) is a fluid-mechanics based mechanism for separating blood cells (RBC, see (B)) by (A) size, (C) deformability, or (D) by shape. In (A) particles with effective size $R_{eff}$ smaller than a critical size $R_c$ follow the flow streamlines which pass close to the obstacle, while larger particles cannot approach the obstacle and are forced onto a new path. In (C) shear forces deform particles, and flow at various shear rates is used to measure deformability. In (D) variation of the channel geometry allows researchers to investigate particle shape since different shapes respond to the geometry in specific ways. In (E), (F), and (G) cells are shown in the DLD device. From J. P. Beech et al. *Lab on a Chip*, vol. 12, 1048 (2012). Reproduced by permission of the authors and The Royal Society of Chemistry.

fluid mechanics includes the boundary-layer approach, in which regions close to solid boundaries are handled separately from the main flow (Chapter 8). Dimensional analysis, discussed throughout the text, helps to quantify which forces dominate in which regions of complex flow, thereby helping to address such problems. At the end of the book, we introduce *vorticity*, a physical quantity associated with a flow field that helps track momentum exchange in rapid, curling, twisting flows.

In this book, we explain fluid mechanics. The subjects and type of discussion presented here have been chosen to bring you to a real understanding of how fluids work. We explain the techniques that experts have discovered to model flows. More than just teaching students to pass a fluids course, our goal is to produce a competency with fluid-mechanics modeling that will allow students to contribute to the field and to apply their knowledge to engineering applications. We present many examples that build this understanding as well as competence and confidence in solving problems in fluid mechanics. End-of-chapter problems are provided, and we also direct readers to several published volumes of solved problems to supplement their efforts with this text [46, 56].

We proceed now to the study of elementary fluid mechanics. We begin with a quick-start section in which we show what type of fluid mechanics can be understood with a simple energy balance, without the detailed understanding of momentum exchange that is the primary topic of this text. We introduce




$$H_{2,1} = H_{d,s}$$

Figure 1.6 The performance of a centrifugal pump may be understood through pumping-head curves (see Section 9.2.4.1). These curves of head (i.e., mechanical energy per unit weight) versus capacity (i.e., flow rate) give the operating point of a pump as the intersection between the curve that is characteristic of the pump and the curve that is characteristic of the system through which the fluid is moving. When the system changes (e.g., a valve is closed somewhat), the system curve shifts as does the operating point. Both system and pump curves are derived from the mechanical energy balance.

the mechanical energy balance (MEB) and its no-friction, no-work version—the macroscopic Bernoulli equation—and we solve some basic problems. To proceed beyond the mechanical energy balance to an understanding of the patterns that fluids create and the stresses that fluids generate, we must consider momentum balances. Momentum balances concern us for the majority of this book.

The last section of this chapter discusses mathematical methods used in fluid mechanics. This overview connects mathematics in the abstract to the specific topic of fluid mechanics.

## 1.2 Quick start: The mechanical energy balance

In flowing systems, the laws of conservation of mass, momentum, and energy allow us to calculate how systems behave. For a detailed understanding of flows, we study the versions of conservation laws that apply to microscopic systems called control volumes (Figure 1.7, top). The equations that result from microscopic balances are nonlinear partial differential equations. It is an involved process to develop these equations and to learn to apply them; we start this task in Chapter 2.

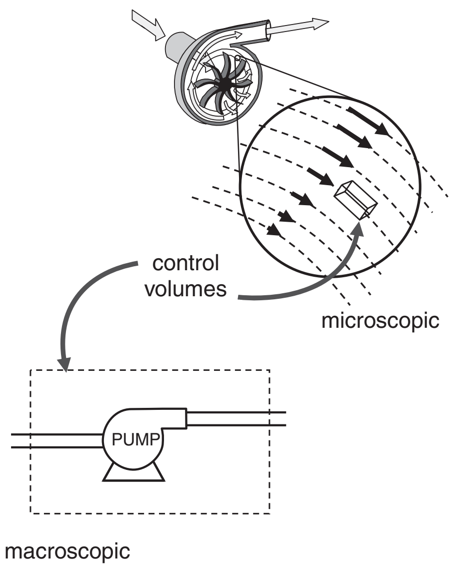



Figure 1.7 Conservation equations applied to small regions in a flow result in partial differential equations that can provide detailed information about the flow field. Conservation equations applied to entire devices or piping systems result in algebraic equations that give relationships among process variables such as average velocity, pressure, and frictional losses.

If a detailed understanding is not required, the conservation laws can be applied to larger-scaled systems rather than microscopic control volumes. Flow systems studied with macroscopic equations can be an entire pumping flow loop, for example (Figure 1.6; Figure 1.7, bottom), or a power station generating electricity at a waterfall. The balance equations in these cases are algebraic rather than differential equations, making them easier to apply and to solve. The drawback to macroscopic analysis is that we must make many assumptions and, because the assumptions sacrifice accuracy, we must supplement theoretical calculations with experiments. Another drawback of macroscopic analysis is that many of the flow details are not determined using such methods. Both microscopic and macroscopic analyses are useful, depending on the information that is sought.

We derive the macroscopic conservation laws later in the book (Chapter 9). In this quick-start section, we present the macroscopic conservation equations without derivation, and we show how they sometimes may be used to calculate and relate flow rates, pressure drops, frictional losses, and work. Practice with these elementary macroscopic calculations is good background for our primary task, which is the detailed study of fluid patterns and fluid stresses in complex flows.

The topic of this section is the mechanical energy balance (MEB), an energy balance applicable to a narrow class of flows that nevertheless are common and practical. We consider the special case of a single-input, single-output flow system such as a liquid pushed through a piping system by a pump (Figure 1.8). The fluid moves through the system at a mass flow rate, $m$, which corresponds to a particular volumetric flow rate $Q$ and average velocity $\langle v \rangle$

$$\text{Volumetric flow rate:} \quad Q = \frac{\mathrm{m}}{\rho} \tag{1.1}$$

$$\text{Average fluid velocity:} \quad \langle v \rangle = \frac{Q}{A} \tag{1.2}$$

where $\rho$ is the density and $A$ is the cross-sectional area of the pipe (see following discussion). There are pressure changes along the flow path as well as velocity and elevation changes. In addition, friction due to fluid contact with the wall or jumbled flow through fittings or other apparatuses causes energy to be converted

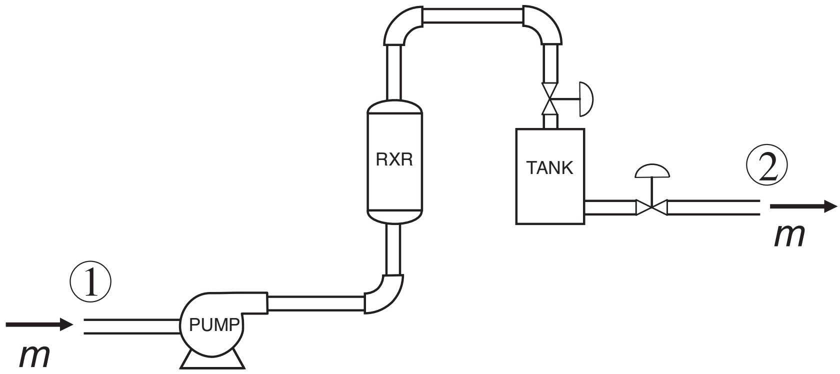



Figure 1.8 A very common system is one with a single-input stream (1), a single-output stream (2), and in which an incompressible ($\rho$ = constant), nonreacting, nearly isothermal fluid is flowing.

to heat and essentially lost. Finally, mechanical devices put energy into or extract energy from the system in the form of *shaft work*, which refers to work associated with devices such as pumps, turbines, and mixers that interact with the fluid through a rotating shaft (see Chapter 9).

A macroscopic energy balance that may be applied to a single-input, single-output system with no reaction, no phase change, and little heat loss or heat generation is the mechanical energy balance, which is derived in Chapter 9 (Figure 1.9).

Mechanical energy balance (single-input, single-output, steady, no phase change, incompressible, $\Delta T \approx 0$, no reaction)

$$\boxed{\frac{\Delta p}{\rho} + \frac{\Delta \langle v \rangle^2}{2\alpha} + g\Delta z + F = -\frac{W_{s,by}}{\text{m}}} \tag{1.3}$$

$$\frac{p_2 - p_1}{\rho} + \frac{\langle v \rangle_2^2 - \langle v \rangle_1^2}{2\alpha} + g(z_2 - z_1) + F_{2,1} = -\frac{W_{s,by,21}}{\text{m}} \tag{1.4}$$

**Definition of Terms in the Mechanical Energy Balance**

| | |
|---|---|
| $\Delta$ | out–in |
| $F$ | friction in system (always positive) |
| $W_{s,by}$ | shaft work done, by fluid (negative for pumps and mixers; positive for turbines) |
| $\alpha$ (velocity profile shape parameter) | $\alpha = \frac{1}{2}$ laminar flow; $\alpha \approx 1$ turbulent flow |

Figure 1.9 The mechanical energy balance relates changes in key energy properties to the friction and work associated with the fluid in the system.