

Discrete Models of Financial Markets

This book explains in simple settings the fundamental ideas of financial market modelling and derivative pricing, using the No Arbitrage Principle. Relatively elementary mathematics leads to powerful notions and techniques – such as viability, completeness, self-financing and replicating strategies, arbitrage and equivalent martingale measures – which are directly applicable in practice. The general methods are applied in detail to pricing and hedging European and American options within the Cox–Ross–Rubinstein (CRR) binomial tree model. A simple approach to discrete interest rate models is included, which, though elementary, has some novel features. All proofs are written in a user-friendly manner, with each step carefully explained, and following a natural flow of thought. In this way the student learns how to tackle new problems.

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Mastering Mathematical Finance

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Discrete Models of Financial Markets

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To Ewa and Margaret

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Preface

In this first volume of the series ‘Mastering Mathematical Finance’ we present discrete-time mathematical models for the pricing and hedging of derivative securities, as well as an initial analysis of fixed income securities. Throughout, the sample space of possible scenarios is assumed to be finite, and there are finitely many trading dates. This greatly reduces the need for sophisticated mathematical tools, while providing sufficient complexity to highlight the key aspects of arbitrage pricing techniques.

Keeping the mathematical requirements to a minimum makes the text accessible to students from a wide variety of backgrounds, while the large number of exercises, which should be regarded as integral to the text, include routine numerical examples and test understanding of basic techniques as well as providing more challenging problems. Solutions and additional exercises are available on the linked website www.cambridge.org/9781107002630, where, if necessary, a list of errata will be updated regularly.

While most of the material is well known, we have sought to develop ideas gradually through simple examples, leading to careful proofs of the key results for option pricing in finite discrete models. While the setting of the final chapter is standard, the discussion of binomial term structure models, though close to the Ho–Lee model, contains features we believe to be novel.