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978-1-107-00258-6 - Hadamard Expansions and Hyperasymptotic Evaluation: An Extension of the Method of Steepest Descents

R. B. Paris

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