Stochastic Interest Rates

This volume in the *Mastering Mathematical Finance* series strikes just the right balance between mathematical rigour and practical application.

Existing books on the challenging subject of stochastic interest rate models are often too advanced for Master's students or fail to include practical examples. *Stochastic Interest Rates* covers practical topics such as calibration, numerical implementation and model limitations in detail. The authors provide numerous exercises and carefully chosen examples to help students acquire the necessary skills to deal with interest rate modelling in a real-world setting. In addition, the book's webpage at www.cambridge.org/9781107002579 provides solutions to all of the exercises as well as the computer code (and associated spreadsheets) for all numerical work, which allows students to verify the results.

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Mastering Mathematical Finance is a series of short books that cover all core topics and the most common electives offered in Master's programmes in mathematical or quantitative finance. The books are closely coordinated and largely self-contained, and can be used efficiently in combination but also individually.

The MMF books start financially from scratch and mathematically assume only undergraduate calculus, linear algebra and elementary probability theory. The necessary mathematics is developed rigorously, with emphasis on a natural development of mathematical ideas and financial intuition, and the readers quickly see real-life financial applications, both for motivation and as the ultimate end for the theory. All books are written for both teaching and self-study, with worked examples, exercises and solutions.

[DMFM] *Discrete Models of Financial Markets,*
Marek Capiński, Ekkehard Kopp

[PF] *Probability for Finance,*
Ekkehard Kopp, Jan Malczak, Tomasz Zastawniak

[SCF] *Stochastic Calculus for Finance,*
Marek Capiński, Ekkehard Kopp, Janusz Traple

[BSM] *The Black–Scholes Model,*
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[PTRM] *Portfolio Theory and Risk Management,*
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[NMFC] *Numerical Methods in Finance with C++,*
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[SIR] *Stochastic Interest Rates,*
Daragh McInerney, Tomasz Zastawniak

[CR] *Credit Risk,*
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[SCAF] *Stochastic Control Applied to Finance,*
Szymon Peszat, Tomasz Zastawniak

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To our daughters Teresa, Francesca, Karolina and Klementyna

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Preface

In this volume of the 'Mastering Mathematical Finance' series we relax the assumption of constant interest rates adopted in the binomial or the Black–Scholes market models covered in earlier volumes, in particular [DMFM] and [BSM]. In general, interest rates are time dependent and random. Being closely linked to, and indeed determined by, fixed-income instruments traded in the market, the rates also depend on the maturity dates of the underlying instruments. This gives rise to the notion of term structure, i.e. the family of interest rates parameterised by the maturity date. We are going to study models describing the random evolution through time of the term structure, that is, of the entire family of interest rates for various maturities.

Because the rates for different maturities are related to one another and evolve simultaneously in time, their joint evolution is more intricate than that of a single quantity such as a stock price. There is not a single term structure model universally adopted as a benchmark to play a similar role as the Black–Scholes model does for stock prices. Instead, a range of alternative and to some extent complementary models are in use to capture various aspects of the evolution of the term structure. A selection of such models will be presented along with the associated interest rate derivative securities.

The prerequisites for this book are covered in some other volumes of the 'Mastering Mathematical Finance' series. These include probability theory [PF], stochastic calculus [SCF], and the Black–Scholes model [BSM]. Familiarity with Monte Carlo simulations [NMFC] will also be helpful.

We begin with various fundamental notions and properties associated with fixed-income instruments in Chapter 1 and the basic 'vanilla' interest rate derivatives in Chapter 2. Here we also cover the change of numeraire technique and introduce the notion of forward measure, a very useful alternative to the risk-neutral measure when pricing interest rate derivatives.

A number of short-rate models, in which the evolution of the entire term structure is driven by a single interest rate, namely the short rate, are covered in Chapter 3. In particular, the Merton, Vasiček and one-factor and two-factor Hull–White models are discussed in detail. In Chapter 4 we turn our attention to one-factor and multi-factor models of forward rates within what is known as the Heath–Jarrow–Morton (HJM) framework, and learn how the term structure is driven by the evolution of the family of forward rates.

Chapters 5, 6 and 7 are devoted to the LIBOR market model (LMM) and the swap market model (SMM). These models are presented and analysed in Chapter 5. In particular, Black's formula is derived for caplets and swaptions. This formula is essential for calibration to implied market volatilities, discussed in Chapter 6 along with the implementation of the LMM via Monte Carlo simulation. In Chapter 7 we introduce a range of options that can be valued within the LMM. Chapter 8 on modelling volatility skews and smiles concludes the volume.

The book contains a considerable number of examples and exercises, which are an important part of the course. The code and spreadsheets that were used to compute many of the numerical examples and plot some of the figures, along with the solutions to all exercises in this volume can be downloaded from www.cambridge.org/9781107002579.