

Cambridge University Press
978-1-107-00249-4 - Probability for Finance
Ekkehard Kopp, Jan Malczak and Tomasz Zastawniak
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Probability for Finance

Students and instructors alike will benefit from this rigorous, unfussy text, which keeps a clear focus on the basic probabilistic concepts required for an understanding of financial market models, including independence and conditioning. Assuming only some calculus and linear algebra, the text applies key results of measure and integration to probability spaces and random variables, culminating in Central Limit Theory. Consequently it provides essential pre-requisites to graduate-level study of modern finance and, more generally, to the study of stochastic processes.

Results are proved carefully and the key concepts are motivated by concrete examples drawn from financial market models. Students can test their understanding through the large number of exercises that are integral to the text.

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Mastering Mathematical Finance

Mastering Mathematical Finance is a series of short books that cover all core topics and the most common electives offered in Master's programmes in mathematical or quantitative finance. The books are closely coordinated and largely self-contained, and can be used efficiently in combination but also individually.

The MMF books start financially from scratch and mathematically assume only undergraduate calculus, linear algebra and elementary probability theory. The necessary mathematics is developed rigorously, with emphasis on a natural development of mathematical ideas and financial intuition, and the readers quickly see real-life financial applications, both for motivation and as the ultimate end for the theory. All books are written for both teaching and self-study, with worked examples, exercises and solutions.

- [DMFM] *Discrete Models of Financial Markets*,
Marek Capiński, Ekkehard Kopp
- [PF] *Probability for Finance*,
Ekkehard Kopp, Jan Malczak, Tomasz Zastawniak
- [SCF] *Stochastic Calculus for Finance*,
Marek Capiński, Ekkehard Kopp, Janusz Traple
- [BSM] *The Black–Scholes Model*,
Marek Capiński, Ekkehard Kopp
- [PTRM] *Portfolio Theory and Risk Management*,
Maciej J. Capiński, Ekkehard Kopp
- [NMFC] *Numerical Methods in Finance with C++*,
Maciej J. Capiński, Tomasz Zastawniak
- [SIR] *Stochastic Interest Rates*,
Daragh McInerney, Tomasz Zastawniak
- [CR] *Credit Risk*,
Marek Capiński, Tomasz Zastawniak
- [FE] *Financial Econometrics*,
Marek Capiński
- [SCAF] *Stochastic Control Applied to Finance*,
Szymon Peszat, Tomasz Zastawniak

Series editors Marek Capiński, *AGH University of Science and Technology, Kraków*; Ekkehard Kopp, *University of Hull*; Tomasz Zastawniak, *University of York*

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Contents

	<i>Preface</i>	page vii
1	Probability spaces	1
	1.1 Discrete examples	1
	1.2 Probability spaces	6
	1.3 Lebesgue measure	11
	1.4 Lebesgue integral	13
	1.5 Lebesgue outer measure	33
2	Probability distributions and random variables	39
	2.1 Probability distributions	39
	2.2 Random variables	46
	2.3 Expectation and variance	56
	2.4 Moments and characteristic functions	62
3	Product measure and independence	66
	3.1 Product measure	67
	3.2 Joint distribution	73
	3.3 Iterated integrals	75
	3.4 Random vectors in \mathbb{R}^n	81
	3.5 Independence	83
	3.6 Covariance	96
	3.7 Proofs by means of d -systems	98
4	Conditional expectation	106
	4.1 Binomial stock prices	106
	4.2 Conditional expectation: discrete case	112
	4.3 Conditional expectation: general case	119
	4.4 The inner product space $L^2(P)$	130
	4.5 Existence of $\mathbb{E}(X \mathcal{G})$ for integrable X	137
	4.6 Proofs	142
5	Sequences of random variables	147
	5.1 Sequences in $L^2(P)$	147
	5.2 Modes of convergence for random variables	156
	5.3 Sequences of i.i.d. random variables	167
	5.4 Convergence in distribution	170
	5.5 Characteristic functions and inversion formula	174

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Frontmatter
[More information](#)

vi

Contents

5.6	Limit theorems for weak convergence	176
5.7	Central Limit Theorem	180
	<i>Index</i>	187

Preface

Mathematical models of financial markets rely in fundamental ways on the concepts and tools of modern probability theory. This book provides a concise but rigorous account of the probabilistic ideas and techniques most commonly used in such models. The treatment is self-contained, requiring only calculus and linear algebra as pre-requisites, and complete proofs are given – some longer constructions and proofs are deferred to the ends of chapters to ensure the smooth flow of key ideas.

New concepts are motivated through examples drawn from finance. The selection and ordering of the material are strongly guided by the applications we have in mind. Many of these applications appear more fully in later volumes of the ‘Mastering Mathematical Finance’ series, including [SCF], [BSM] and [NMFC]. This volume provides the essential mathematical background of the financial models described in detail there.

In adding to the extensive literature on probability theory we have not sought to provide a comprehensive treatment of the mathematical theory and its manifold applications. We focus instead on the more limited objective of writing a fully rigorous, yet concise and accessible, account of the basic concepts underlying widely used market models. The book should be read in conjunction with its partner volume [SCF], which describes the properties of stochastic processes used in these models.

In the first two chapters we introduce probability spaces, distributions and random variables from scratch. We assume a basic level of mathematical maturity in our description of the principal aspects of measures and integrals, including the construction of the Lebesgue integral and the important convergence results for integrals. Beginning with discrete examples familiar to readers of [DMFM], we motivate each construction by means of specific distributions used in financial modelling. Chapter 3 introduces product measures and random vectors, and highlights the key concept of independence, while Chapter 4 is devoted to a thorough discussion of conditioning, moving from the familiar discrete setting via the properties of inner product spaces and the Radon–Nikodym theorem to the construction of general conditional expectations for integrable random variables. The final chapter explores key limit theorems for sequences of random variables, beginning with orthonormal sequences of square-integrable functions, fol-

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Frontmatter
[More information](#)

viii

Preface

lowed by a discussion of the relationships between various modes of convergence, and concluding with an introduction to weak convergence and the Central Limit Theorem for independent identically distributed random variables of finite mean and variance.

Concrete examples and the large number of exercises form an integral part of this text. Solutions to the exercises and further material can be found at www.cambridge.org/9781107002494.