

INTRODUCTION TO STELLAR DYNAMICS

The study of stellar dynamics is experiencing an exciting new wave of interest thanks to observational campaigns and the ready availability of powerful computers. While its relevance includes many areas of astrophysics, from the structure of the Milky Way to dark matter halos, few texts are suited to advanced students. This volume provides a broad overview of the key concepts beyond the elementary level, bridging the gap between the standard texts and specialist literature. The author reviews Newtonian gravity in depth before examining the dynamical properties of collisional and collisionless stellar-dynamical systems that result from gravitational interactions. Guided examples and exercises ensure a thorough grounding in the mathematics, while discussions of important practical applications give a complete picture of the subject. Readers are given a sound working knowledge of the fundamental ideas and techniques employed in the field and the conceptual background needed to progress to more advanced graduate-level treatises.

LUCA CIOTTI is Professor of Astronomy and Astrophysics at the University of Bologna, where he has also served for many years as Director of the Collegio Superiore. Since 1992, he has been a long-term visitor of the Princeton University Observatory. His work developed in several fields of theoretical astrophysics, with main interests in stellar dynamics, fluid dynamics, and black hole accretion. This is his second book in the field, having previously written *Lecture Notes on Stellar Dynamics* based on the lectures given at the Scuola Normale Superiore in Pisa.

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Luca Ciotti
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Preface

This book aims to provide an introductory yet solid background in stellar dynamics to astronomy students who later will attend more advanced courses (e.g., at the graduate level) and to researchers in other fields of astrophysics looking for an accessible introduction to the subject. It should provide the reader with sufficiently broad (yet rigorous) coverage of some of the basic topics of stellar dynamics, opening the way to the study of specialized texts such as *Dynamics of Galaxies* (Bertin 2014), *Galactic Dynamics* (Binney and Tremaine 2008), *Dynamics and Evolution of Galactic Nuclei* (Merritt 2013), and *Dynamical Evolution of Globular Clusters* (Spitzer 1987), and to a fruitful reading of important reviews such as those of Binney (1982a), Cappellari (2016), and de Zeeuw and Franx (1991). However, it is assumed that the student already knows the basics of extragalactic astronomy (e.g., the classification, morphology, kinematics, and phenomenology of stellar systems such as open and globular clusters, galaxies, and clusters of galaxies); from this point of view, *Galactic Astronomy* (Binney and Merrifield 1998) and *Spiral Galaxies* (Bertin and Lin 1996) are excellent complementary works. From the mathematical point of view, a working knowledge of geometry, linear algebra, and calculus in one and several variables is required.

This book does not cover the whole content of my courses (Extragalactic Astrophysics and Dynamics of Stellar Systems for third- and fourth-year undergraduate astronomy students) taught at the University of Bologna. In fact, as any judicious student knows by experience, no book can substitute for the understanding of a subject obtained from direct interaction with a teacher and from doing first-hand exercises (sometimes erroneously!) at the blackboard. I devoted some care to the addition of arguments and illustrative examples that are not usually found in other presentations but that I found quite effective in my classes for clarifying important conceptual points. Informal interactions with colleagues lead me to expect that this book will also be useful to professional astronomers not working directly on stellar dynamics who are looking for a simple but sufficiently rigorous exposition of the most important concepts and techniques used in the field. At the end of each chapter, the student will find worked-out exercises: they should *not* be considered optional, but as complementary material supporting the arguments discussed in the main text. The student will notice that several important topics of stellar dynamics are not discussed in detail, or

are not even mentioned at all; examples include instabilities and resonances, the evolution of collisional systems such as globular and open clusters, the kinematics and dynamics of spiral arms, the dynamical evolution of galaxy centers under the effects of single and multiple supermassive black holes, and the field of numerical simulations. The absence of these topics is motivated by the lack of sufficient expertise of the author, or because a proper treatment would be beyond the scope of this introductory book, or, finally (but not less importantly), simply due to personal taste.

I am especially grateful to the many students who attended my classes over the years, both in Bologna and at the Scuola Normale Superiore of Pisa, where I taught in 1998–2003 and 2006–2008, during which time I wrote a little collection of *Lecture Notes on Stellar Dynamics* (Ciotti 2000): their questions often helped to reveal weak points in my understanding of the subject. Discussions and exchanges of opinions with several colleagues have also deeply influenced (directly or indirectly) the writing of this book. Among them, special acknowledgment is due to G. Bertin, J. Binney, A. D’Ercole, T. de Zeeuw, D. Lynden-Bell, J. Ostriker, S. Pellegrini, A. Renzini, R. Sancisi, M. Stiavelli, and T. S. van Albada. I also wish to thank W. Dehnen, W. Evans, O. Gerhard, J. Goodman, L. Hernquist, D. Merritt, L. Ossipkov, D. Pfenniger, E. Remiddi, D. Spergel, A. Toomre, S. Tremaine, and my coauthors and students, who are too numerous to be listed by name. Finally, B. Franchi, A. Parmeggiani, and M. C. Tesi of the Department of Mathematics of the University of Bologna are acknowledged for having been generous with their time spent on clarifying my understanding of particular mathematical issues; of course, I alone am responsible for any error in this book.

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On a more personal note, I warmly thank G. Bertin for his friendship: without his constant encouragement, this book simply would not exist. Finally, I wish to thank my wife, Silvia, and our daughters, Anna Magdalena and Sofia Elisabetta, for their patience and help over the years of writing: this book is dedicated to them, and to my parents, Marino and Enrica.

La Scienza è ultima perfezione de la nostra anima.

Dante Alighieri (1265–1321), *Convivio*, Trattato Primo, Capitolo Primo