Fundamentals of Numerical Weather Prediction

Numerical models have now become essential tools in environmental science, particularly in weather forecasting and climate prediction. This book provides a comprehensive overview of the techniques used in these fields, with emphasis on the design of the most recent numerical models of the atmosphere. It presents a short history of numerical weather prediction and its evolution, before providing step-by-step descriptions of the various model equations and how to solve them numerically. It outlines the main elements of a meteorological forecast suite, and the theory is illustrated throughout with practical examples of operational models and parameterizations of the main physical processes.

This book is founded on the author's many years of experience, working as a scientist at Météo-France and teaching university-level courses. It is a practical and accessible textbook for graduate courses and is a handy resource for researchers and professionals in atmospheric physics, meteorology, and climatology, as well as the related disciplines of fluid dynamics, hydrology, and oceanography.

Jean Coiffier is now retired from Météo-France, where he was Ingénieur en Chef des Ponts et Chaussées, and is a member of the Société Météorologique de France. His involvement in meteorological science began in 1968 at the new Algerian Meteorological Service, implementing elements of a modest meteorological forecast suite on a small computer, before joining the Direction de la Météorologie Nationale (later Météo-France) where he took part in the development and implementation of operational models. In 1989, he became the head of the General Forecast Office. He worked there until his retirement, also giving regular lectures on numerical weather prediction to students of the École Nationale de la Météorologie and training courses to professional forecasters. He also played an active role in realizing Computer Aided Learning modules devoted to numerical modelling and forecasting methods.

Fundamentals of Numerical Weather Prediction

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formerly at Météo-France

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Foreword to the French Edition

It is a pleasure to present this book by Jean Coiffier. After many years of teaching at the *École Nationale de la Météorologie*, he wanted to give a lasting form to the work at which he had laboured so long. The result is what is probably the first book (at any rate it is the first in French) to present the fundamentals and the current methods of numerical weather prediction in a comprehensive and consistent way.

Numerous books, some of them excellent, deal with dynamic meteorology, the science of the moving atmosphere. But although numerical modelling has become a fundamental tool not just for weather forecasting but also for dynamic meteorology itself as well as for oceanography and climate studies, there is no one book that presents the methods and techniques in a systematic manner. New users of numerical models of the atmosphere, whether students on placement or experienced scientists making career changes, find it hard to make their way into the world of numerical modelling. It is not that documentation is lacking; most models now come with detailed guides to help users take their first steps; and copious academic literature including some outstanding papers is also readily available.

What is lacking is an overview of the techniques and methods; and that is what *Fundamentals of Numerical Weather Prediction* provides. Without requiring from readers more knowledge than the standard science graduate should have acquired, Jean Coiffier guides them with pragmatism and pedagogy. The main points are explained in simple but, at the same time, clear and rigorous terms. A few years ago, I proposed to a group of students at the *École Polytechnique* to develop a general circulation model of the atmosphere from scratch as a shared coursework project. It was Jean Coiffier's lecture notes (those on which this book is based) that appeared to me the most suitable document to take them straight to the crux of what they needed to learn.

Numerical weather prediction is now more than 50 years old. One might wonder what its pioneers would make of what it has become. Perhaps they would be surprised at how the definition of initial conditions and data assimilation have grown steadily more important over the years. Jean Coiffier deals with all these aspects of numerical modelling, introducing the essential points of assimilation methods (although so very different from what he describes elsewhere) with the same precision and the same rigour as he presents the discretization algorithms.

Jean Coiffier is perhaps a little too modest. He confines the title of his book to 'numerical weather prediction,' but the methods set out in it are the same as are used for numerical modelling of the climate, which is now so crucial for forecasting the changes in store for us and for preventing or at least mitigating their harmful effects.

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This book is aimed at students and it will be very helpful to them. But I am sure that a number of engineers and experienced scientists too will be using it, for they can be sure to find in it clear answers to precise questions. I am confident that many will join me in thanking Jean Coiffier for his *Fundamentals of Numerical Weather Prediction*.

Olivier Talagrand – 25 September 2007

Foreword to the English Edition

I am delighted to have been given the opportunity to write a foreword to this excellent textbook by Jean Coiffier on the mathematical and physical basis for NWP (Numerical Weather Prediction). It is an English translation of one originally published in French by Météo-France entitled '*Les bases de la prévision numérique du temps*'. To reflect the increasing use and importance of nonhydrostatic models in NWP, both for research and for operational forecasting, a valuable new appendix has been added in the English version. This gives an overview of two currently used nonhydrostatic models, one developed in Europe and the other in the United States: whilst both models are based on the same underlying continuous equation set, by way of contrast the first has an implicit time discretization, whereas the second has an (almost) explicit one with a smaller time step.

I fully endorse Olivier Talagrand's assessment, reproduced herein after translation, of the virtues of the original French version of this book. Assuming only a basic knowledge of physics and mathematics, the student is provided with an elegantly written synthesis of the essentials of NWP. Particularly noteworthy is the logical progression from simple equation sets, and their properties and discretization, to increasingly complex ones.

Whilst this book has been primarily written with the student in mind, it is also a valuable reference book for the experienced practitioner wishing to refresh his memory on specific aspects of the subject and to locate associated journal papers from the extensive references. To date, only French-speaking readers have been able to benefit from Jean's well-balanced, clear, and rigorous exposition of the subject. With this in mind, he is to be congratulated for finding further time during his retirement years to collaborate with Cambridge University Press on an English translation of his book, thereby making it available to a much wider audience.

It remains only for me to highly recommend this book to anyone, student or experienced practitioner, who is interested in the scientific foundation of weather prediction and climate simulation.

Andrew Staniforth – 15 October 2010

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Preface

Fundamentals of Numerical Weather Prediction is intended to introduce students to current techniques for developing numerical weather prediction models. It is based on lecture notes for the course I taught on numerical weather prediction at the *École Nationale de la Météorologie* in the 1990s.

Numerical weather prediction consists of automatically performing meteorological forecasts and involves implementing a series of clearly identified processes: data collection and control, determination of the initial state of the atmosphere (analysis), computation of the final state at a given range (forecast), computation of the characteristic weather parameters at the local scale, tailoring and dissemination of results. This book does not purport to describe exhaustively all the techniques used to implement all of these processes in practice but focuses instead on the forecasting process proper. This consists in determining, with the help of numerical computation techniques, the solutions of a system of equations describing the behaviour of the atmosphere. The choice of an appropriate system of equations and of the series of numerical calculations to be performed to determine approximate solutions for this system defines what is commonly called a numerical prediction model. This basic tool is used both for weather forecasting and for climate simulation.

This book is for students looking to take their first steps in the techniques of numerical prediction and does not require any particular mathematical knowledge beyond what is expected of science graduates. However, it does assume knowledge of the fundamentals of dynamic meteorology with regard to developing and justifying the systems of equations and of the main methods of numerical analysis for solving partial differential equations.

Chapter 1 provides an overview of the history of numerical weather prediction from its beginnings in the 1950s until the late 1990s, highlighting the major advances and the evolution of computing tools.

Chapter 2 introduces the systems of equations most widely used for simulating atmospheric motion, and especially the ‘primitive equations’ and the way they are formulated in various coordinate systems.

Chapters 3 and 4 explain the principal techniques for representing meteorological fields for numerical weather prediction: the finite difference method and the spectral method.

Chapter 5 examines the effects of the various finite difference schemes in space and time; the linear shallow water model is used for comparing exact solutions with numerical ones.

Chapter 6 describes a few barotropic models using the numerical techniques studied before with regard to spatial discretization and implementing the various time integration schemes.

Chapter 7 explains the formulation of the primitive equations used in the baroclinic models and specifies their properties with regard to global invariants; a general formulation encompassing the primitive (hydrostatic) equations and the Euler (nonhydrostatic) equations is also presented.

Chapter 8 describes in detail the vertical discretization of the equations and the implementation of baroclinic models using explicit and semi-implicit time integration algorithms, while the fields are represented horizontally using the finite difference technique or the spectral method.

Chapter 9 deals with the parameterization of physical processes. Instead of explaining in detail all the parameterization schemes used in forecasting models (for which an abundant literature is continuously updated in the specialized journals), I have opted to describe a few examples emphasizing how the model dynamics and model physics hang together. Plentiful bibliographic references to other parameterization schemes are also given.

Chapter 10 is an overview of the processes making up an operational forecasting suite, with special emphasis on data assimilation, which has grown ever more important in recent years. The chapter ends with a look at future prospects for NWP.

Appendix A was written especially for the English edition of the book. It describes two types of models based on the nonhydrostatic equations that have been increasingly utilized for operational mesoscale forecasting in recent years.

Last, a copious further reading list completes the text and should enable readers to find more details about the various NWP techniques explained in the book.

It is no accident that many of the examples of NWP methods and techniques presented refer to models developed and operated by Météo-France under various acronyms (Améthyste, Sisyphe, Émeraude, Péridot, Arpège/IFS Aladin, and Arome), since I spent most of my working life with the French Meteorological Service. And it was at Météo-France that I was fortunate enough to work with Jean Lepas, Daniel Rousseau, and Jean-François Geleyn, who constantly helped me to acquire the essential knowledge that has gone into this book.

My hope is, then, that this book will enable students to acquire the basic knowledge and essential techniques of NWP so that they can quickly and effectively become active members of teams engaged in atmospheric modelling.

Jean Coiffier

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I would like to thank here all of those colleagues who were patient enough to work through the chapters of the French version of this book in its various stages of development and so helped to make the whole more readable and intelligible: Frédéric Atger, Éric Bazile, Pierre Bénard, Yves Bouteloup, François Bouttier, François Bouyssel, Jean-Marie Carrière, Gérard De Moor, Michel Déqué, Jean-François Geleyn, Gwenaelle Hello, Jean-Pierre Javelle, Régis Juvanon du Vachat, Sylvie Malardel, Pascal Marquet, Jean Nicolau, Jean Pailleux, Jean-Marcel Piriou, Michel Rochas, Daniel Rousseau, Yann Seity, Joël Stein, and Karim Yessad.

My special thanks to Gérard De Moor and Jacques Siméon, who constantly encouraged me to continue writing this book from my lecture material, and to Claude Sinolecka, who painstakingly read through the entire French manuscript to bring it up to scratch for publication.

In the time that elapsed between completing the French version and writing the English version of the book, I have been able to add an appendix describing two types of models based on nonhydrostatic equations. Pierre Bénard at Météo-France and Stan Benjamin and Jimy Dudhia at the National Center for Atmospheric Research have been kind enough to help me to finalize the parts on the AROME model and WRF/ARW model respectively. The figures depicting the evolution of skill in operational numerical prediction have also been updated to take account of the latest data, thanks to help from Jeff McQueen, Geoff Di Mego, Michelle Mainelli, and Denis Staley at NCEP, Dominique Marbouty and Robert Hine at ECMWF, and Bruno Lacroix and Marc Tardy at Météo-France. I owe special thanks to Jean Pailleux and Peter Lynch for their critical review of the English manuscript, and to Jean-François Geleyn, who has painstakingly amended, improved, and enhanced the chapter on physical parameterizations.

The English version of this book would certainly never have seen the light of day without the efforts made by my former colleagues at Météo-France: Anne Guillaume, who convinced me to buckle down to the job and who helped me to present the project to the publishers; and Michel Hontarrède, who managed to solve sundry administrative headaches. I am grateful to Cambridge University Press and in particular to Susan Francis for trusting me to see the project through, and to Christopher Hudson, Laura Clark, and Christopher Miller, who were always there to help me in preparing this book.

I have nothing but praise for the way Christopher Sutcliffe has cooperated with me on the English translation of this book, and I am most grateful to him for the time spent discussing the relevance of the scientific terms used.

The splendid picture of the Earth captured in its grid on the cover is the meticulous labour of Pascal Lamboley on a Meteosat image provided by the *Centre de Météorologie Spatiale de Météo-France*, thanks to the good offices of Patrick Donguy.

My warm thanks to my friends Olivier Talagrand, Director of Research at the CNRS, and Andrew Staniforth, Met Office scientist, who agreed to write forewords for the French and English versions respectively.

Finally, I am especially grateful to my wife, Florence, for tolerating a husband who has spent most of his time riveted to his computer throughout the making of this book.

Partial list of symbols

Latin letters

a	Mean radius of the Earth; length.
b	Coefficient; length.
c	Wave velocity.
c_R	Rossby wave velocity.
d	Distance.
e	Depth inside the soil.
e	mathematical constant.
e_N	2.71828 is the numerical value of the mathematical constant e. Residue of an expansion containing N terms.
e_T	Turbulent kinetic energy; soil thermal emissivity.
f	Coriolis parameter: $2\Omega \sin \varphi$; function.
g	Gravity acceleration; function.
g^*	Newtonian acceleration.
h	Standard deviation of subgrid-scale orography; weighting coefficient of digital filter; rain/snow discrimination function.
i	$\sqrt{-1}$
i, j	Horizontal location indices.
k	Wave number; vertical level index; time step index.
$\mathbf{k}$	Unit vector in the vertical.
l	Obstacle width; length.
$\boldsymbol{\ell}$	Prandtl length.
m	Map scale factor; zonal wavenumber.
$\tilde{m}$	Map scale factor in the case of variable resolution spectral treatment.
n	Global wavenumber.
p	Pressure; vertical coordinate.
p_s	Surface pressure.
q	Specific humidity of the air; concentration.
q_{sat}	Saturation specific humidity.
q_S	Surface specific humidity.
q_c	Specific concentration of condensed water inside the cloud.
q_d	Specific concentration of dry air.
q_v	Specific concentration of water vapour.
q_l	Specific concentration of suspended liquid water.
q_s	Specific concentration of suspended solid water.
q_r	Specific concentration of precipitating liquid water.

q_i	Specific concentration of precipitating ice.
$\mathbf{r}$	Radius vector.
r	Radial distance.
$\mathrm{r_i}$	Proportion of ice.
r_f	Proportion of snow within a layer.
s	General vertical coordinate; dry static energy.
s_s	Surface dry static energy.
$\dot{s}$	Vertical velocity: ds/dt.
t	Time.
u, v	Horizontal wind components.
u^g	Quantity of gas radiation passes through.
u^*	Friction velocity.
s^*	Scaling dry static energy in the surface boundary layer.
q^*	Scaling specific humidity in the surface boundary layer.
w	Vertical velocity in z-system; weighting coefficient.
w_c	Vertical velocity within the cloud.
$w^g_{\Delta v}$	Equivalent line bandwith relating to the gas g and to the spectral interval Δv.
w_k	Gauss weight at latitude μ_k.
x, y	Horizontal Cartesian coordinates.
z	Height.
z_0	Roughness length.
A	Generic variable identifying a 2-dimensional field $A(\lambda, \mu)$ or $A(x, y)$; coefficient.
A^m_n	Coefficient of series expansion of A in terms of surface spherical harmonics.
$A_m(\mu)$	Fourier coefficient at latitude μ.
A_T	Terrestrial albedo for solar radiation.
B	Planck function; buoyancy.
C	Constant; phase change coefficient for water vapour.
C_p	Specific heat at constant pressure for air.
C_v	Specific heat at constant volume for air.
C_{pd}	Specific heat at constant pressure for dry air.
C_{p_v}	Specific heat at constant pressure for water vapour.
C_l	Specific heat at constant pressure for liquid water.
C_i	Specific heat at constant pressure for ice.
$\hat{C}$	Specific heat for the comprehensive set of non-precipitating phases.
C_{Sol}	Soil characteristic constant.
C_S	Soil specific heat.
C_D	Drag coefficient.
C_H, C_E	Transfer coefficients for dry static energy and specific humidity.
C_N	Net condensation rate.
C_{BCC}	Buoyant convective condensation rate.
C_{UCC}	Non-buoyant convective condensation rate.

D	Discretized divergence of the horizontal wind; raindrop diameter.
$\mathfrak{D}$	Thermal diffusion coefficient.
$\tilde{D}_k$	Numerical divergence of the momentum at level k.
E	Surface evaporation flux.
E_c	Kinetic energy.
E_p	Potential energy.
F	Inverse Froude number: NH/U.
$\mathbf{F}$	Friction force.
$F_{\downarrow}$	Downward radiation flux.
$F_{\uparrow}$	Upward radiation flux.
$F^*_{\downarrow}$	Modified downward flux.
$F^*_{\uparrow}$	Modified upward flux.
G	Radiation flux absorbed by the soil.
H	Effective obstacle height.
H_L	Latent heat flux.
H_s	Sensible heat flux.
Hu	Relative humidity.
$H_n^m(x, y)$	Product of complex exponential functions with wavenumbers n and m.
I_0	Solar constant.
J	Turbulent flux; cost function.
J, K, L, M	Number of grid points.
K^c_{abs}	Absorption coefficient for the element c.
K	Kinetic energy.
K_s	Tuning coefficient.
K_M	Exchange coefficient for momentum.
K_H, K_E	Exchange coefficient for dry static energy and specific humidity.
K_{DH}	Horizontal diffusion coefficient.
K_R	Relaxation factor.
$\mathbf{K}$	Wave vector.
L	Wavelength.
L_M	Monin-Obukhov length.
L_R	Rossby radius of deformation.
$L_l(T_0)$	Latent heat of vaporization of water at $T_0 = 0$ K.
$L_i(\mathrm{T}_0)$	Latent heat of melting of ice at $T_0 = 0$ K.
M	Source of water vapour by mass unit and time unit; mass of water; maximum zonal wavenumber.
M_i	Mass of snow.
M_c	Mass flux.
N	Cloud cover; cost function; Brunt-Vaïsälä frequency; number of levels in the vertical; number of time steps; maximum global wavenumber; number of raindrops.
P	Term introduced to express the linearized pressure force in the semi-implicit algorithm; upper air precipitation flux.

P_l	Rain flux.
P_i	Snow flux.
P_L	Surface rain flux.
P_I	Surface snow flux.
P_T	Total precipitation flux at the surface: $P_L + P_I$
P_S	Precipitation flux at the surface.
$P_n^m(\mu)$	Associated Legendre function.
Q	Heat source per mass unit and time unit; wave velocity reduction factor.
Q_1	Heat source per unit time.
Q_2	Energy loss per unit time corresponding to the water vapour deficit.
Q_R	Heat source due to radiation.
Q_1^c	Static energy tendency: $Q_I - Q_R$ due to convection within the mesh.
Q_2^c	Specific humidity tendency: $-Q_2/L$ due to convection within the mesh.
Q_3^c	Momentum tendency due to convection within the mesh.
R	Specific ideal-gas constant relative to the air; influence radius; wave amplitude reduction factor.
Ri	Richardson number.
R_d, R_v	Ideal-gas constants relative to dry air, to water vapour.
R_T	Terrestrial radiation.
R_A	Atmospheric radiation.
R_G	Global radiation.
S	Solar radiation flux.
S_0	Solar radiation flux at the top of the atmosphere.
T	Thermodynamic temperature; truncation.
T_S	Surface temperature.
T_{00}	Temperature of the water triple point.
U, V	Reduced components of the horizontal wind; velocities.
V	Volume.
$\mathbf{V}$	Horizontal wind (general case).
$\mathbf{V}_H$	2-dimensional horizontal wind.
$\mathbf{V}_3$	3-dimensional wind velocity.
Veg	Vegetation proportion.
w	Velocity of falling raindrops.
w_p	Relative deep layer water content.
w_s	Relative surface water content.
$Y_n^m(\lambda, \mu)$	Surface spherical harmonic.
Z	Natural logarithm of surface pressure; height; length.
Z_s	Height of topography.

Gothic letters

$\mathfrak{M}$	Total mass.
$\mathfrak{E}$	Total energy.
$\mathfrak{I}$	Total angular momentum.
$\mathfrak{K}$	Total kinetic energy.

$\mathfrak{Z}$	Total potential absolute vorticity.
$\mathfrak{H}$	Total absolute potential enstrophy.
$\mathfrak{Q}$	Total internal energy.

Greek letters

α	Angle; coefficient; length.
β	Rossby parameter: $\partial f/\partial y$; fractional part of backscattered radiation; coefficient; length.
γ	Length; coefficient; binary variable.
δ	Discretized divergence; optical path; length; binary indicator; amplitude of an orographic wave; root mean square (RMS) error.
δp_k	Thickness of layer k in p-system.
$\delta\sigma_k$	Thickness of layer k in sigma-system.
δ_x	Mesh size along x-axis.
δ_y	Mesh size along y-axis.
ε	Entrainment rate of water vapour inside the cloud; coefficient; infinitesimal quantity.
ε_T	Truncation error.
ζ	Vorticity.
η	Divergence.
ζ'	Reduced value of vorticity.
η'	Reduced value of divergence.
θ	Co-latitude; potential temperature.
κ	Ratio R/C_p; Von Karman's constant.
λ	Longitude; wavelength; asymptotic mixing length; coefficient; variable.
λ_S	Soil thermal conductivity factor.
μ	Sine of latitude.
μ_k	Gaussian latitude.
μ_0	Cosine of the zenith angle.
v, v'	Numerical coefficients.
ξ	Absolute vorticity: $\zeta+f$.
ξ^*	Absolute potential vorticity ξ/Φ.
π	value of π is 3.14159…, mathematical constant.
π	Mass-type vertical coordinate or hydrostatic pressure when the pressure at the top of the working domain vanishes.
$\dot{\pi}$	Vertical velocity: $d\pi/dt$.
ρ	Air density.
ρ_w	Water vapour density.
ρ_r	Rain density.
ρ_s	Snow density.
σ	Vertical coordinate sigma; Stefan-Boltzman constant; partition coefficient; frequency; standard deviation.
$\dot{\sigma}$	Vertical velocity: $d\sigma/dt$.
$\dot{\varsigma}_{\tilde{k}}$	Vertical velocity term $\dot{s}\,\partial p/\partial s$.

τ	Radiation transmission factor; e-folding time.
$\boldsymbol{\tau}$	Momentum flux.
φ	Latitude.
χ	Velocity potential function.
ψ	Stream function; generic function.
ω	Vertical velocity: *dp*/*dt*; frequency (angular frequency).
ω^*	Mass flux.
ϖ	Simple scattering albedo.
Γ	Gamma function; transfer coefficient of momentum in the vertical.
Δt	Time step.
Δx	Mesh size along x-axis.
Δy	Mesh size along y-axis.
$\Delta\delta$	Optical depth.
$\Delta\nu$	Absorption bandwidth.
Θ	Angle.
Φ	Geopotential.
Φ_s	Surface geopotential.
Φ_1,Φ_2	Universal functions for the surface boundary layer.
Ψ	Angle.
Ω	Angular velocity of the Earth; angular velocity.
$\boldsymbol{\Omega}$	Angular velocity vector of the Earth.

Generalized vectors, matrices, and operators

$\mathbf{A}$	Linearized energy conversion term matrix; analysis error covariance matrix.
$\mathbf{B}$	Linearized hydrostatic relation matrix; background field error covariance matrix.
$\tilde{\mathbf{D}}$	Column vector of momentum.
$\mathbf{H}$	Linearized observation operator matrix.
$\mathbf{E}$	Error vector.
$\mathbf{H}_r$	Linearized observation operator matrix for the incremental method.
$\mathbf{K}$	Optimal interpolation weight matrix.
$\mathbf{M}$	Linearized barotropic system matrix; vertical structure matrix of the baroclinic linearized system for the semi-implicit algorithm.
$\mathbf{P}$	Column vector of linearized pressure force term.
$\mathbf{P}_f$	Background field error covariance matrix.
$\mathbf{P}_a$	Analysis error covariance matrix.
$\mathbf{Q}$	Matrix of the eigenvectors of the operator defined by the matrix $\mathbf{M}$.
$\mathbf{Q}(t_k)$	Model error at time step t_k.
$\mathbf{R}$	Observation error covariance matrix.
$\mathbf{R}^*$	Column vector describing the linearization profile $R\mathbf{T}^*$.
${}^t\mathbf{S}_k$	Row vector of the layer depths in the vertical up to level k followed by zeroes.
$\mathbf{T}$	Column vector of temperature.
$\mathbf{U}, \mathbf{V}$	Column vector of reduced wind components U and V.
$\mathbf{X}$	Generic vector; state vector.

$\mathbf{X}_\mathrm{a}$	Analysis state vector.
$\mathbf{X}_\mathrm{b}$	Background field state vector.
$\mathbf{X}_\mathrm{t}$	State vector of the actual atmosphere at time t.
$\mathbf{X}_F^*$	Resulting state vector after application of the digital filter.
$\hat{\mathbf{Z}}$	Eigenmode of the linearized model $\mathcal{L}$.
$\delta\mathbf{X}'$	State vector deviation from an approximate value.
$\boldsymbol{\varepsilon}$	Error vector.
$\bar{\boldsymbol{\varepsilon}}$	Mean error vector.
$\boldsymbol{\zeta}$	Column vector of vorticity.
$\boldsymbol{\eta}$	Column vector of divergence.
$\dot{\boldsymbol{\varsigma}}$	Column vector of vertical velocity.
$\boldsymbol{\Phi}$	Column vector of geopotential.
$\boldsymbol{\psi}$	Column vector of streamfunction.
$\boldsymbol{\chi}$	Column vector of velocity potential.
$\boldsymbol{\Lambda}$	Diagonal matrix of the implicit system obtained with the baroclinic model.
$\mathcal{S}$	Nonlinear general operator.
$\mathcal{L}$	Linear operator.
$\mathcal{N}$	Nonlinear operator.
$\mathcal{F}$	Nonlinear operator acting on the state vector at time t.
$\nabla_x\mathcal{F}$	Tangent linear operator of $\mathcal{F}$.
$\mathcal{H}$	Observation operator.
$\mathcal{M}_k$	Model operator acting between times t_k and t_{k+1}.

Various mathematical notations

$\cdot$	Scalar or dot product.
$\times$	Vector product.
$\lvert\ \rvert$	Absolute value.
$\lVert\ \rVert$	Norm of a vector.
$<\,,\,>$	Scalar product.
∇	Gradient of a scalar function.
$\nabla\cdot$	Divergence.
$\nabla\times$	Curl.
∇^2	Laplacian.
∇'^2	Reduced Laplacian.
$J(\,,\,)$	Jacobian.
∇_x	Gradient of a scalar function of a vector **X** with respect to the components of this vector.
$\partial/\partial l$	Partial derivative of a multivariable function with respect to the variable l.
$d\,/dt$	Total derivative.
$\overline{A}^x$	Mean value: $[A(x + \Delta x/2) + A(x + \Delta x/2)]/2$.
A_x	Finite difference: $[A(x + \Delta x/2) - A(x + \Delta x/2)]/\Delta x$.