1 Introduction

Motivation: This chapter is intended to introduce the class of systems addressed in this volume – the so-called Linear Plant/Nonlinear Instrumentation (LPNI) systems – and to characterize the control methodology developed in this book – Quasilinear Control (QLC).

Overview: After introducing the notions of LPNI systems and QLC and listing the problems addressed, the main technique of this book – the method of stochastic linearization – is briefly described and compared with the usual, Jacobian, linearization. In the framework of this comparison, it is shown that the former provides a more accurate description of LPNI systems than the latter, and the controllers designed using the QLC result, generically, yield better performance than those designed using linear control (LC). Finally, the content of the book is outlined.

1.1 Linear Plant/Nonlinear Instrumentation Systems and Quasilinear Control

Every control system contains nonlinear instrumentation – actuators and sensors. Indeed, the actuators are ubiquitously saturating; the sensors are often quantized; deadzone, friction, hysteresis, and so on are also encountered in actuator and sensor behavior.

Typically, the plants in control systems are nonlinear as well. However, if a control system operates effectively, that is, maintains its operation in a desired regime, the plant may be linearized and viewed as locally linear. The instrumentation, however, can not: to reject large disturbances, to respond to initial conditions sufficiently far away from the operating point, or to track large changes in reference signals – all may activate essential nonlinearities in actuators and sensors, resulting in fundamentally nonlinear behavior. These arguments lead to a class of systems that we refer to as *Linear Plant/Nonlinear Instrumentation* (LPNI).

The controllers in feedback systems are often designed to be linear. The main design techniques are based on root locus, sensitivity functions, LQR/LQG, H_∞, and so on, all leading to linear feedback. Although for LPNI systems both linear and nonlinear controllers may be considered, to transfer the above-mentioned techniques to the LPNI case, we are interested in designing *linear* controllers. This leads to *closed loop LPNI systems*.

This volume is devoted to methods for analysis and design of closed loop LPNI systems. As it turns out, these methods are quite similar to those in the linear case. For example, root locus can be extended to LPNI systems, and so can LQR/LQG, H_∞, and so on. In each of them, the analysis and synthesis equations remain practically the same as in the linear case but coupled with additional transcendental equations, which account for the nonlinearities. That is why we refer to the resulting methods as *Quasilinear Control* (QLC) *Theory*. Since the main analysis and design techniques of QLC are not too different from the well-known linear control theoretic methods, QLC can be viewed as a simple addition to the standard toolbox of control engineering practitioners and students alike.

Although the term “LPNI systems” may be new, such systems have been considered in the literature for more than 50 years. Indeed, the theory of absolute stability was developed precisely to address the issue of global asymptotic stability of linear plants with linear controllers and sector-bounded actuators. For the same class of systems, the method of harmonic balance/describing functions was developed to provide a tool for limit cycle analysis. In addition, the problem of stability of systems with saturating actuators has been addressed in numerous publications. However, the issues of performance, that is, disturbance rejection and reference tracking, have been addressed to a much lesser extent. These are precisely the issues considered in this volume and, therefore, we use the subtitle *Performance Analysis and Design of Feedback Systems with Nonlinear Actuators and Sensors*.

In view of the above, one may ask a question: If all feedback systems include nonlinear instrumentation, how have controllers been designed in the past, leading to a plethora of successful applications in every branch of modern technology? The answer can be given as follows: In practice, most control systems are, indeed, designed ignoring the actuator and sensor nonlinearities. Then, the resulting closed loop systems are evaluated by computer simulations, which include nonlinear instrumentation, and the controller gains are readjusted so that the nonlinearities are not activated. Typically, this leads to performance degradation. If the performance degradation is not acceptable, sensors and actuators with larger linear domains are employed, and the process is repeated anew. This approach works well in most cases, but not in all: the Chernobyl nuclear accident and the crash of a YF-22 airplane are examples of its failures. Even when this approach does work, it requires a lengthy and expensive process of simulation and design/redesign. In addition, designing controllers so that the nonlinearities are not activated (e.g., actuator saturation is avoided) leads, as is shown in this book, to performance losses. Thus, developing methods in which the instrumentation nonlinearities are taken into account from the very beginning of the design process, is of significant practical importance. The authors of this volume have been developing such methods for more than 15 years, and the results are summarized in this volume.

As a conclusion for this section, it should be pointed out that modern Nonlinear Control Theory is not applicable to LPNI systems because it assumes that the control signals enter the system equations in a linear manner, thereby excluding saturation and other nonlinearities in actuators. Model Predictive Control may also be undesirable, because it is computationally extensive and, therefore, complex in implementation.

1.2 QLC Problems

Consider the closed loop LPNI system shown in Figure 1.1. Here the transfer functions $P(s)$ and $C(s)$ represent the plant and controller, respectively, and the nonlinear functions $f(\cdot)$ and $g(\cdot)$ describe, respectively, the actuator and sensor. The signals r, d, e, u, v, y, and y_m are the reference, disturbance, error, controller output, actuator output, plant output, and measured output, respectively. These notations are used throughout this book. In the framework of the system of Figure 1.1, this volume considers the following problems (rigorous formulations are given in subsequent chapters):

P1. **Performance analysis:** Given $P(s)$, $C(s)$, $f(\cdot)$, and $g(\cdot)$, quantify the performance of the closed loop LPNI system from the point of view of reference tracking and disturbance rejection.

P2. **Narrow sense design:** Given $P(s)$, $f(\cdot)$, and $g(\cdot)$, design, if possible, a controller so that the closed loop LPNI system satisfies the required performance specifications.

P3. **Wide sense design:** Given $P(s)$, design a controller $C(s)$ and select the instrumentation $f(\cdot)$ and $g(\cdot)$ so that the closed loop LPNI system satisfies the required performance specifications.

P4. **Partial performance recovery:** Assume that a controller, $C_l(s)$, is designed so that the closed loop system meets the performance specifications if the actuator and sensor were linear. Select $f(\cdot)$ and $g(\cdot)$ so that the performance degradation of the closed loop LPNI system with $C_l(s)$ does not exceed a given bound, as compared with the linear case.

P5. **Complete performance recovery:** As in the previous problem, let $C_l(s)$ be a controller that satisfies the performance specifications of the closed loop system with linear instrumentation. For given $f(\cdot)$ and $g(\cdot)$, redesign $C_l(s)$ so that the closed loop LPNI exhibits, if possible, no performance degradation.

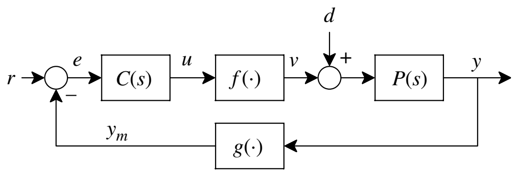

Figure 1.1. Closed loop LPNI system.

The first two of the above problems are standard in control theory, but are considered here for the LPNI case. The last three problems are specific to LPNI systems and have not been considered in linear control (LC). Note that the last problem is reminiscent of anti-windup control, whereby $C_l(s)$ is augmented by a mechanism that prevents the so-called windup of integral controllers in systems with saturating actuators.

1.3 QLC Approach: Stochastic Linearization

The approach of QLC is based on a quasilinearization technique referred to as stochastic linearization. This method was developed more than 50 years ago and since then has been applied in numerous engineering fields. Applications to feedback control have also been reported. However, comprehensive development of a control theory based on this approach has not previously been carried out. This is done in this volume.

Stochastic linearization requires exogenous signals (i.e., references and disturbances) to be random. While this is often the case for disturbances, the references are assumed in LC to be deterministic – steps, ramps, or parabolic signals. Are these the only references encountered in practice? The answer is definitely in the negative: in many applications, the reference signals can be more readily modeled as random than as steps, ramps, and so on. For example, in the hard disk drive control problem, the read/write head in both track-seeking and track-following operations is affected by reference signals that are well modeled by Gaussian colored processes. Similarly, the aircraft homing problem can be viewed as a problem with random references. Many other examples of this nature can be given. Thus, along with disturbances, QLC assumes that the reference signals are random processes and, using stochastic linearization, provides methods for designing controllers for both reference tracking and disturbance rejection problems. The standard, deterministic, reference signals are also used, for example, to develop the notion of LPNI system types and to define and analyze the notion of the so-called trackable domain.

The essence of stochastic linearization can be characterized as follows: Assume that the actuator is described by an odd piecewise differentiable function $f(u(t))$, where $u(t)$ is the output of the controller, which is assumed to be a zero-mean wide sense stationary (wss) Gaussian process. Consider the problem: approximate $f(u(t))$ by $Nu(t)$, where N is a constant, so that the mean-square error is minimized. It turns out (see Chapter 2) that such an N is given by

$$N = E\left[\left. \frac{df(u)}{du} \right|_{u=u(t)} \right], \tag{1.1}$$

where E denotes the expectation. This is referred to as the *stochastically linearized gain* or *quasilinear gain* of $f(u)$. Since the only free parameter of $u(t)$ is its standard deviation, σ_u, it follows from (1.1) that the stochastically linearized gain depends on a single variable – the standard deviation of its argument; thus,

$$N = N(\sigma_u). \tag{1.2}$$

Note that stochastic linearization is indeed a quasilinear, rather than linear, operation: the quasilinear gains of $\alpha f(\cdot)$ and $f(\cdot)\alpha$, where α is a constant, are not the same, the former being $\alpha N(\sigma_u)$ the latter being $N(\alpha\sigma_u)$.

In the closed loop environment, σ_u depends not only on $f(u)$ but also on all other components of the system (i.e., the plant and the controller parameters) and on all exogenous signals (i.e., references and disturbances). This leads to transcendental equations that define the quasilinear gains. The study of these equations in the framework of various control-theoretic problems (e.g., root locus, sensitivity functions, LQR/LQG, H_∞) is the essence of the theory of QLC.

As in the open loop case, a stochastically linearized closed loop system is also not linear: its output to the sum of two exogenous signals is not equal to the sum of the outputs to each of these signals, that is, superposition does not hold. However, since, when all signals and functional blocks are given, the system has a constant gain N, we refer to a stochastically linearized closed loop system as *quasilinear*.

1.4 Quasilinear versus Linear Control

Consider the closed-loop LPNI system shown in Figure 1.2(a). If the usual Jacobian linearization is used, this system is reduced to that shown in Figure 1.2(b), where all signals are denoted by the same symbols as in Figure 1.2(a) but with a ~. In this

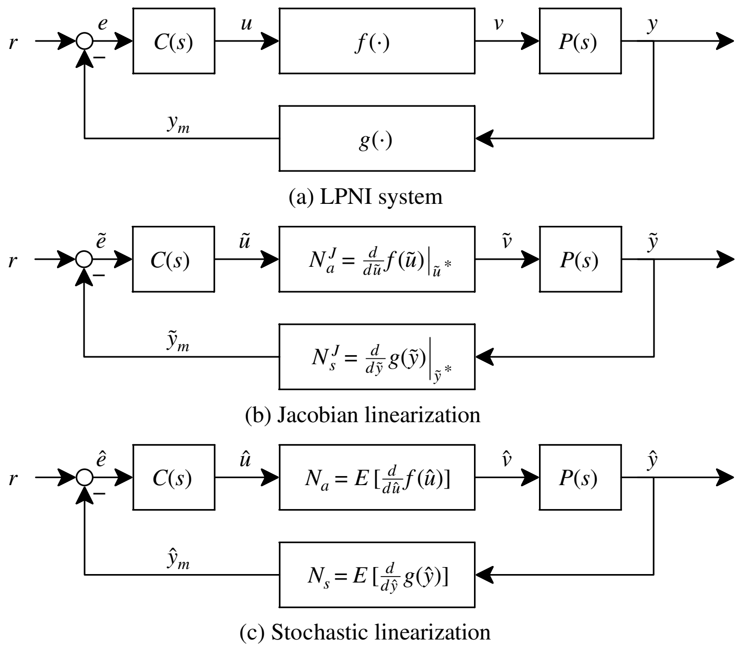

Figure 1.2. Closed loop LPNI system and its Jacobian and stochastic linearizations.

system, the actuator and sensor are represented by constant gains evaluated as the derivatives of $f(\cdot)$ and $g(\cdot)$ at the operating point:

$$N_a^J = \left.\frac{df(\tilde{u})}{d\tilde{u}}\right|_{\tilde{u}=\tilde{u}^*}, \tag{1.3}$$

$$N_s^J = \left.\frac{dg(\tilde{y})}{d\tilde{y}}\right|_{\tilde{y}=\tilde{y}^*}. \tag{1.4}$$

Clearly, this system describes the original LPNI system of Figure 1.2(a) only locally, around the fixed operating point.

If stochastic linearization is used, the system of Figure 1.2(a) is reduced to the quasilinear one shown in Figure 1.2(c), where all signals are again denoted by the same symbols as in Figure 1.2(a) but with a ˆ; these notations are used throughout this book. As it is indicated above and discussed in detail in Chapter 2, here the actuator and sensor are represented by their quasilinear gains:

$$N_a(\sigma_{\hat{u}}) = E\left[\frac{df(\hat{u})}{d\hat{u}}|_{\hat{u}=\hat{u}(t)}\right], \tag{1.5}$$

$$N_s(\sigma_{\hat{y}}) = E\left[\frac{dg(\hat{y})}{d\hat{y}}|_{\hat{y}=\hat{y}(t)}\right]. \tag{1.6}$$

Since $N_a(\sigma_{\hat{u}})$ and $N_s(\sigma_{\hat{y}})$ depend not only on $f(\cdot)$ and $g(\cdot)$ but also on all elements of the system in Figure 1.2(c), the quasilinearization describes the closed loop LPNI system globally, with "weights" defined by the statistics of $\hat{u}(t)$ and $\hat{y}(t)$.

The LC approach assumes the reduction of the original LPNI system to that of Figure 1.2(b) and then rigorously develops methods for closed loop system analysis and design. In contrast, the QLC approach assumes that the reduction of the original LPNI system to that of Figure 1.2(c) takes place and then, similar to LC, develops rigorous methods for quasilinear closed loop systems analysis and design. In both cases, of course, the analysis and design results are supposed to be used for the actual LPNI system of Figure 1.2(a).

Which approach is better, LC or QLC? This may be viewed as a matter of belief or a matter of calculations. *As a matter of belief*, we think that QLC, being global, provides a more faithful description of LPNI systems than LC. To illustrate this, consider the disturbance rejection problem for the LPNI system of Figure 1.2(a) with

$$P(s) = \frac{1}{s^2+s+1},\ C(s) = 1,\ f(u) = \mathrm{sat}_\alpha(u),\ g(y) = y,\ r(t) = 0 \tag{1.7}$$

and with a standard white Gaussian process as the disturbance at the input of the plant. In (1.7), $\mathrm{sat}_\alpha(u)$ is the saturation function given by

$$\mathrm{sat}_\alpha(u) = \begin{cases} \alpha, & u > +\alpha, \\ u, & -\alpha \le u \le \alpha, \\ -\alpha, & u < -\alpha. \end{cases} \tag{1.8}$$

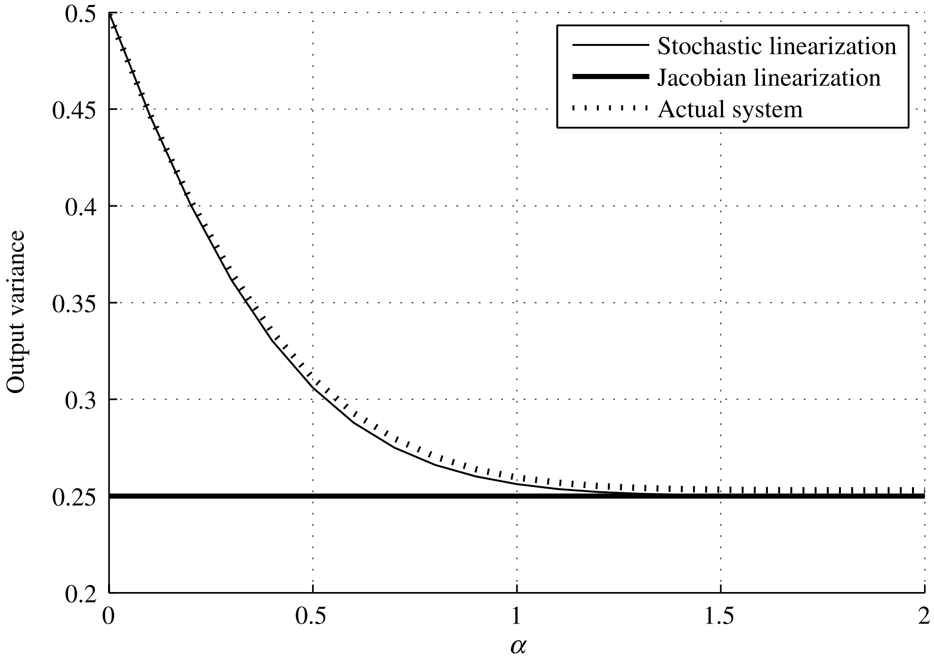

Figure 1.3. Comparison of stochastic linearization, Jacobian linearization, and actual system performance.

For this LPNI system, we construct its Jacobian and stochastic linearizations and calculate the variances, $\sigma_{\tilde{y}}^2$ and $\sigma_{\hat{y}}^2$, of the outputs $\tilde{y}(t)$ and $\hat{y}(t)$ as functions of α. (Note that $\sigma_{\tilde{y}}^2$ is calculated using the usual Lyapunov equation approach and $\sigma_{\hat{y}}^2$ is calculated using the stochastic linearization approach developed in Chapter 2.) In addition, we simulate the actual LPNI system of Figure 1.2(a) and numerically evaluate σ_y^2. All three curves are shown in Figure 1.3. From this figure, we observe the following:

- The Jacobian linearization of $\text{sat}_\alpha(u)$ is independent of α, thus, the predicted variance is constant.
- When α is large (i.e., the input is not saturated), Jacobian linearization is accurate. However, it is highly inaccurate for small values of α.
- Stochastic linearization accounts for the nonlinearity and, thus, predicts an output variance that depends on α.
- Stochastic linearization accurately matches the actual performance for *all* values of α.

We believe that a similar situation takes place for any closed loop LPNI system: Stochastic linearization, when applicable, describes the actual LPNI system more faithfully than Jacobian linearization. (As shown in Chapter 2, stochastic linearization is applicable when the plant is low-pass filtering.)

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Excerpt
More Information

As a matter of calculations, consider the LPNI system of Figure 1.2(a) defined by the following state space equations:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \mathrm{sat}_\alpha(u) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} w$$
$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \tag{1.9}$$

with the feedback

$$u = Kx, \tag{1.10}$$

where $x = [x_1, x_2]^T$ is the state of the plant and w is a standard white Gaussian process. The problem is to select a feedback gain K so that the disturbance is rejected in the best possible manner, that is, σ_y^2 is minimized. Based on Jacobian linearization, this can be accomplished using the LQR approach with a sufficiently small control penalty, say, $\rho = 10^{-5}$. Based on stochastic linearization, this can be accomplished using the method developed in Chapter 5 and referred to as SLQR (where the "S" stands for "saturating") with the same ρ. The resulting controllers, of course, are used in the LPNI system. Simulating this system with the LQR controller and with the SLQR controller, we evaluated numerically σ_y^2 for both cases. The results are shown in Figure 1.4 as a function of the saturation level. From this figure, we conclude the following:

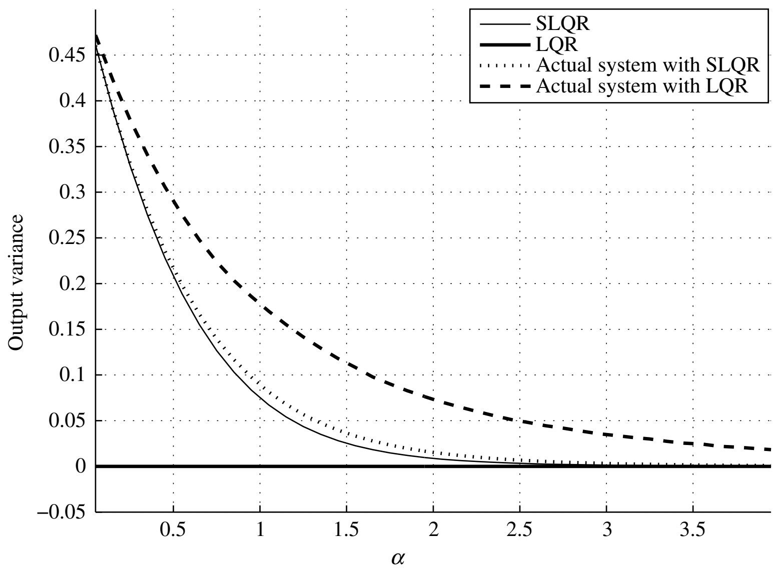

Figure 1.4. Comparison of LQR, SLQR, and actual system performance.

- Since ρ is small and the plant is minimum phase, LQR provides a high gain solution that renders the output variance close to zero. Due to the underlying Jacobian linearization, this solution is constant for all α.
- Due to the input saturation, the performance of the actual system with the LQR controller is significantly worse than the LQR design, even for relatively large values of α.
- The SLQR solution explicitly accounts for α and, thus, yields a nonzero output variance.
- The performance of the actual system with an SLQR controller closely matches the intended design.
- The actual SLQR performance exceeds the actual LQR performance for all values of α.

As shown, using LQR in this situation is deceiving since the actual system can never approach the intended performance. In contrast, the SLQR solution is highly representative of the actual system behavior (and, indeed, exceeds the actual LQR performance). In fact, it is possible to prove that QLC-based controllers (e.g., controllers designed using SLQR) generically ensure better performance of LPNI systems than LC-based controllers (e.g., based on LQR).

These comparisons, we believe, justify the development and utilization of QLC.

1.5 Overview of Main QLC Results

This section outlines the main QLC results included in this volume.

Chapter 2 describes the method of stochastic linearization in the framework of LPNI systems. After deriving the expression for quasilinear gain (1.1) and illustrating it for typical nonlinearities of actuators and sensors, it concentrates on closed loop LPNI systems (Figure 1.2(a)) and their stochastic linearizations (Figure 1.2(c)). Since the quasilinear gain of an actuator, N_a, depends on the standard deviation of the signal at its input, $\sigma_{\hat{u}}$ and, in turn, $\sigma_{\hat{u}}$ depends on N_a, the quasilinear gain of the actuator is defined by a *transcendental equation*. The same holds for the quasilinear gain of the sensor. Chapter 2 derives these transcendental equations for various scenarios of reference tracking and disturbance rejection. For instance, in the problem of reference tracking with a nonlinear actuator and linear sensor, the quasilinear gain of the actuator is defined by the equation

$$N_a = \mathcal{F}\left(\left\| \frac{F_{\Omega_r}(s)C(s)}{1+P(s)N_aC(s)} \right\|_2\right), \tag{1.11}$$

where

$$\mathcal{F}(\sigma) = \int_{-\infty}^{\infty} \left[\frac{d}{dx}f(x)\right] \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{x^2}{2\sigma^2}\right) dx. \tag{1.12}$$

Here, $F_{\Omega_r}(s)$ is the reference coloring filter, $f(x)$ is the nonlinear function that describes the actuator, and $||\cdot||_2$ is the 2-norm of a transfer function. Chapter 2 provides a sufficient condition under which this and similar equations for other performance problems have solutions and formulates a bisection algorithm to find them with any desired accuracy. Based on these solutions, the performance of closed loop LPNI systems in problems of reference tracking and disturbance rejection is investigated. Finally, Chapter 2 addresses the issue of accuracy of stochastic linearization and shows (using the *Fokker-Planck equation* and the *filter hypothesis*) that the error between the standard deviation of the plant output and its quasilinearization (i.e., σ_y and $\sigma_{\hat{y}}$) is well within 10%, if the plant is low-pass filtering. The equations derived in Chapter 2 are used throughout the book for various problems of performance analysis and design.

Chapter 3 is devoted to analysis of reference tracking in closed loop LPNI systems. Here, the notion of *system type* is extended to feedback control with saturating actuators, and it is shown that the type of the system is defined by the plant poles at the origin (rather than the loop transfer function poles at the origin, as it is in the linear case). The controller poles, however, also play a role, but a minor one compared with those of the plant. In addition, Chapter 3 introduces the notion of *trackable domains*, that is, the ranges of step, ramp, and parabolic signals that can be tracked by LPNI systems with saturating actuators. In particular, it shows that the trackable domain (TD) for step inputs, $r(t) = r_0\mathbf{1}(t)$, where r_0 is a constant and $\mathbf{1}(t)$ is the unit step function, is given by

$$TD = \{r_0 : |r_0| < \left|\frac{1}{C_0} + P_0\right|\alpha\}, \tag{1.13}$$

where C_0 and P_0 are d.c. gains of the controller and plant, respectively, and α is the level of actuator saturation. Thus, TD is finite, unless the plant has a pole at the origin.

While the above results address the issue of tracking deterministic signals, Chapter 3 investigates also the problem of random reference tracking. First, linear systems are addressed. As a motivation, it is shown that the standard deviation of the error signal, σ_e, is a poor predictor of tracking quality since for the same σ_e track loss can be qualitatively different. Based on this observation, the so-called *tracking quality indicators*, similar to gain and phase margins in linear systems, are introduced. The main instrument here is the so-called *random sensitivity function* (RS). In the case of linear systems, this function is defined by

$$RS(\Omega) = ||F_\Omega(s)S(s)||_2, \tag{1.14}$$

where, as before, $F_\Omega(s)$ is the reference signal coloring filter with 3dB bandwidth Ω and $S(s)$ is the usual sensitivity function. The bandwidth of $RS(\Omega)$, its d.c. gain, and the resonance peak define the tracking quality indicators, which are used as specifications for tracking controller design.

Finally, Chapter 3 transfers the above ideas to tracking random references in LPNI systems. This development is based on the so-called *saturating random*