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The Shrikhande Graph

A Window on Discrete Mathematics

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Preface

The *Petersen graph*, a strongly regular graph on 10 vertices, is so famous, and its occurrences within graph theory as an example or a counterexample are so many and varied, that it has a whole book devoted to it [59].

Our subject here is the *Shrikhande graph*, discovered by the Indian mathematician Sharadchandra Shankar Shrikhande in 1959. Our motivation is a little different to that of Derek Holton and John Sheehan. The Shrikhande graph occupies a pivotal position within discrete mathematics, at the meeting point of algebraic and topological graph theory, Latin squares, root systems, graph eigenvalues, Seidel switching and other topics. We invite the reader to a gentle introduction to these topics.

In this book, the unadorned term ‘the Graph’ will sometimes be used to refer to the Shrikhande graph.

If you look at the picture of the Shrikhande graph on the torus (Figure 7.1), you will see that from a vertex there are six different directions you can travel. In this book, we are going to give six different constructions of the Shrikhande graph:

(a) a direct construction (Section 3.5), which can be used to calculate the number of automorphisms (Section 4.4);
(b) as a Cayley graph (Sections 2.6 and 4.3);
(c) as the complement of the Latin square graph from the Cayley table of the cyclic group of order 4 (Section 6.7);
(d) as a regular map on the torus (Section 7.3);
(e) embedded in the exceptional root system E_7 (Section 9.3);
(f) and by Seidel switching from the 4×4 grid graph, the line graph of $K_{4,4}$ (Section 10.2).

Each construction is embedded in a detailed account of the relevant part of discrete mathematics.

The book begins with a short biography of Shrikhande. He is best known as one of the 'Euler spoilers', the trio of mathematicians who disproved the conjecture of Euler on the existence of Graeco-Latin squares. (For the novice, we remark that the name 'Euler' is pronounced to rhyme with 'spoiler'.) In keeping with this, we observe that there are two Latin squares of order 4, the Cayley tables of the two groups of order 4; the complements of their Latin square graphs are $L_2(4)$ (the line graph of the complete bipartite graph $K_{4,4}$) and the Shrikhande graph.

This is followed by two chapters giving an introduction to the notation and terminology of graph theory and to the class of strongly regular graphs (including the Shrikhande graph and several other beautiful graphs). The concepts introduced are illustrated in Chapters 1 and 2 by showing how they work out in the case of the Shrikhande graph.

The final part of the book was one of our main reasons for writing it: a discussion of the various contexts in which the Shrikhande graph arises, treating Latin squares, triangulations of the torus, root systems, Seidel switching and others. Many of these contexts throw further light on the properties of the Shrikhande graph and its special place in graph theory.

Thus, we use the Shrikhande graph as a window for viewing a number of topics in discrete mathematics, including strongly regular and distance-regular graphs, Latin squares, generalized line graphs, error-correcting codes and several others.

Shrikhande's theorem states that a graph with the same spectrum as the line graph of the complete bipartite graph $K_{n,n}$ is isomorphic to $L(K_{n,n})$ unless $n = 4$, in which case there is just one further graph, the Shrikhande graph. Two extensions of this theorem we discuss are classifications of connected graphs with smallest eigenvalue -2 (where the Shrikhande graph is one of the finitely many exceptions to the statement that these are generalized line graphs) and distance-regular graphs with the same parameters as Hamming graphs (where the Doob graphs, Cartesian products of 4-cliques and copies of the Shrikhande graph, are the exceptions when the alphabet size is 4). Also, a set of mutually orthogonal Latin squares containing two fewer than a complete set can be completed if the order is not 4; the exception for order 4 is explained by the Shrikhande graph.

Acknowledgements

Shrikhande's discovery of his graph was not a one-off event, but was part of a much wider project at the time, to characterize certain graphs by the spectra of their adjacency matrices. The American mathematician Alan Hoffman was a leading figure in this project, but it also involved mathematicians from many countries, including Belgium, China, India, the Netherlands, the United Kingdom and the United States. It will also be made clear from this book that the Shrikhande graph is connected to a wide range of topics in discrete mathematics, a worldwide language and enterprise.

The subject has a rich history. Some of this, such as Euler's conjecture on Graeco-Latin squares, we treat in detail; but we also give references to other interesting byways, including Mesner's discovery of the Higman–Sims graph, Paley's non-discovery of the Paley graphs and the story of Hoffman's bound, for which we recommend the papers [65], [61] and [55], respectively, by Klin and Woldar, Jones and Haemers, respectively. We are grateful to all these authors for their papers and discussion.

We have attempted to give references for further reading on the topics we cover, including the surprising occurrence of the Coxeter–Dynkin diagrams of type ADE in the theory of graph spectra.

We are grateful to many colleagues and friends who have provided us with the information which we are passing on in these pages. In particular, we thank Sharad Sane and Mohan Shrikhande for some biographical information about S. S. Shrikhande, Andries Brouwer for the construction of a locally Shrikhande graph, Wilfried Imrich for results about the primality of the Shrikhande graph and R. A. Bailey for the anecdote in Chapter 1.

The pictures in Chapter 6 and Section 10.6 are from photographs by the first author, while those in Sections 6.2 and 10.7 are © Neill Cameron. In other cases, we have attempted to use pictures in the public domain.