

Quantum Mechanics

Offering a detailed account of the key concepts and mathematical apparatus of quantum mechanics, this textbook is an ideal companion to both undergraduate and graduate courses. The formal and practical aspects of the subject are explained clearly alongside examples of modern applications, providing students with the tools required to thoroughly understand the theory and apply it. The authors provide an intuitive conceptual framework that is grounded in a coherent physical explanation of quantum phenomena, established over decades of teaching and research in quantum mechanics and its foundations. The book's educational value is enhanced by the inclusion of examples and exercises, with solutions available online, and an extensive bibliography is provided. Notes throughout the text provide fascinating context on the tumultuous history of quantum mechanics, the people that developed it, and the questions that still remain at its center. This title is also available as Open Access on Cambridge Core.

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A Physical Approach

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Preface

Textbooks are usually designed to introduce students to a new subject, to show them the ways in which it can be learned, and to give them the tools to use and apply it. The purpose of this book is also to provide the student with an adequate physical understanding of the subject – which, in the case of quantum mechanics, is particularly non-trivial.

Quantum physics has come a long way since its inception a little over a century ago. Its applications are expanding every day, many of them with important technological implications. For the new generations, life is unimaginable without the products of quantum physics, which have become part of everyday reality. We are often unaware that many devices that have revolutionized society through optical communications, high-speed internet, high-performance computing, materials by design, and imaging techniques, to name but a few, are based on quantum mechanical effects.

However, these successes come with a caveat: the difficulty – even for the expert – of obtaining a satisfactory physical understanding of quantum phenomena. The impossibility of direct access to the inside of the atom and other quantum constituents of matter has resulted in a theory that still relies on some abstract or mathematical postulates introduced by its founders, in lack of a causal explanation on physical grounds. Instead, a wide range of interpretations can be found, some of which may even contradict fundamental physical principles. This state of affairs is reflected in research papers, conference debates, online courses, ... and textbooks. It even goes beyond the boundaries of science and contributes to the widespread view that “the quantum world is weird.”

Our aim is to provide the reader with a picture of quantum phenomena that is at once as transparent and internally coherent as possible and compatible with the rest of physics, and that serves as a sound conceptual basis for more advanced courses in quantum theory or its many applications. It is not another interpretation, but a more complete picture that we, together with other colleagues in the field, have arrived at through decades of fruitful research into the physical foundations of the theory. Research that, by considering all the relevant physical ingredients that make up quantum systems, enables us to find a causal explanation for the theory’s postulates.

This book is intended primarily for undergraduate and graduate courses in physics, chemistry, engineering, and materials science. Students in quantum information and communication, nanotechnology, quantum chemistry, and other physics-related fields, particularly philosophy of physics, may also find it useful as a reference or supplementary textbook. For this reason, the book covers the basic material of the university curriculum and includes material for more advanced courses or specific needs in sections marked with an asterisk in the Contents list. It emphasizes the theoretical and conceptual aspects and illustrates them with applications of current relevance. Where

appropriate, it provides explanations or derivations not found in the conventional literature in order to give a more complete account of the underlying physics.

Some relevant historical, biographical, and bibliographical information is given throughout to allow the reader to appreciate the impressive development of quantum theory since its inception and the contributions of its main pioneers. Where necessary, background elements of classical physics and mathematics are also included. The text includes illustrative exercises throughout, as well as a selection of problems at the end of each chapter. Its educational value is enhanced by an accompanying online book with detailed solutions, which is available to students and instructors at <https://www.cambridge.org/cetto-delapena>. Due to space limitations, the number of solved exercises and problems is limited; the student is encouraged to take advantage of the wide variety of exercises and applications of quantum mechanics available on the internet.

The book may be of particular help to those interested in quantum theory who have been dissatisfied or confused by the conventional literature on the subject. We hope that our perspective on quantum mechanics can appeal to the curious mind, young or otherwise, to delve with confidence into the normally complex and mysterious depths of quantum nature, and to see the quantum world as part of the world that can be understood and explained.

We invite you to join us on this exciting journey of understanding.

This book is the product of many years of research and teaching on the subject. We are grateful to Fondo de Cultura Económica for allowing us to use material from an earlier textbook, *Introducción a la Mecánica Cuántica*. We would also like to thank everyone who supported us in preparing this volume. Special thanks go to Alejandro Aguilar Sierra, who generously shared his Metagráfica program to draw the figures, and to Carlos López Nataren, who provided consistent, timely technical assistance. We are grateful to Cristo Sánchez Nicolás and Eduardo Varela Ortuño, our students, for their work on the figures and tables, extended figure descriptions, and thoughtful review of the manuscript. We would also like to thank our Ph.D. student and young colleague, José Francisco Pérez Barragán, for carefully reading the final proofs. Our colleague and friend John Bush from MIT provided helpful comments and suggestions on the topic of hydrodynamic quantum analogs. We are likewise grateful to the Instituto de Física for providing working conditions that enabled us to devote ample time to writing and preparing this volume. Lastly, we thank the staff at Cambridge University Press for their encouragement and valuable support throughout the entire editorial process.