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Excerpt
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1 The Quantum World and Its Description

Quantum mechanics (QM) is just over a century old. This is a short time for a fundamental theory of physics when you consider that it is expected to describe an important part of the world for centuries to come. Geometry, for example, is over two thousand years old and still going strong. Physical theories are much more recent, even Newtonian mechanics – the oldest branch of modern physics – is only three centuries old. Yet it is a well-established theory, so well established that it has successfully survived, one after the other, the unexpected birth of two mechanics, relativity and quantum mechanics, two whole new chapters of physics, both of which emerged in the twentieth century. The effect of the new theories on classical mechanics has essentially been to define its scope: we now know that it applies in general to systems that are not too small – that is, not atomic in size or less – and to speeds that are not too close to the speed of light. Newtonian mechanics is still used to describe the behavior of all sorts of objects and situations around us, as well as to design the most gigantic airplanes and ships, even spaceships, huge bridges, and all sorts of machines.

Quantum mechanics, for its part, has grown impressively in the first hundred years of its life, reaping success after success. It works flawlessly well in thousands of laboratories and millions of personal computers around the world, although its predictions are statistical in nature. However, like any scientific theory, QM is certainly not infallible; we just haven't reached the limits of its applicability yet.

This should not be a cause for concern: recognizing that there are limits to the application of a physical theory is neither unexpected nor extraordinary. On the contrary, identifying such limits is an important part of the daily task of science. When a new physical theory is born (and this applies to virtually all scientific endeavors), its limits of applicability – its *range*, in the language of T. A. Brody (1993)[1] – are unknown. In practice, much of the day-to-day work of research is, often unconsciously, to determine the range of the theory. Will it still work in my new application, at such high speeds, at low or high temperatures, over short or long distances? The scope of QM is not yet clearly defined, but its expansion into new domains and applications to more extreme scales will ultimately determine it.

Quantum mechanics is currently the most valuable tool we have for the study and understanding of the atomic and subatomic world. Quantum mechanics and the theories based on it have enabled us to explore not only the atomic nucleus, but also the structure of its constituent parts and their elementary components on the one hand,

[1] Thomas Alexander Brody (1922–1988) was a remarkable German–Mexican physicist, polymath, and polyglot. He was a central figure in the development of Mexican activity in physics, especially nuclear and statistical physics, as well as in computation and philosophy of science, always with a pronounced critical attitude.

and, seemingly paradoxically, the structure of matter from the nanoscale to the scale of the universe. In fact, today's cosmology recognizes that physics is ubiquitous and determines the large-scale dynamics of the cosmos, which emerges bottom-up from local dynamics at the small scale; there is no separate large-scale physics of the cosmos. Hence the importance of the continuing search for a quantum theory of gravity.

1.1 The Quantum World, Part of the Physical World

The three current branches of mechanics: classical mechanics, quantum mechanics, and the theory of relativity, each have to some extent their own domain of knowledge and application. The domain of QM is essentially (but not exclusively) the submicroscopic world, that is, the world of atoms, molecules, and their constituent elements, known globally as quantum particles, where the term "particle" must be used with due (quantum) care. The most important quantum particles are those that make up ordinary matter: electrons, protons, and neutrons, to which we should add – among others – the photon, the "particle" that represents the smallest exchangeable quantity of electromagnetic radiation of a given frequency. The first known submicroscopic particles, the electron and the proton, have gradually been joined by many others. Today, according to the so-called Standard Model, the list of fundamental quantum particles consists of the quarks and the gluons, the photon, the Higgs particle, and the leptons, as well as their antiparticles in all cases. If we leave the details to the specialists, we can consider about 30 particles – some elementary, the rest composite – and several hundred resonances. Quantum mechanics, in its diversity, applies to all of these particles and their possible combinations, although it is usually restricted to atoms, molecules, and systems composed of not too many particles, without excluding, however, some larger systems.

In fact, in our everyday world, QM extends very far beyond submicroscopic systems; there are even huge physical or technological systems whose basic functioning is quantum in nature. Well-known examples are the laser, a large crystal, a volume of superfluid, or even a superconducting magnet that can weigh tons. Quantum mechanics is a central theoretical corpus for a wide range of contemporary subjects and applications, starting at the level of today's most fundamental theory, quantum chromodynamics (QCD), with its "colored" quarks and gluons – the constituents of protons and neutrons, along with the rest of baryons and mesons – which in turn are the constituents of atomic nuclei, which along with electrons are the constituents of atoms, which in turn combine to form molecules. Quantum phenomena at the nanoscale are the focus of increasing attention and may even become a key to understanding phenomena as complex as the functioning of the human brain. Quantum mechanics is also having an astonishing impact on current developments in astrophysics, where it plays an important role in the study of the evolution of the universe.

In recent decades, QM has opened its doors to *information theory*, which as a field of knowledge dedicated to the study of the quantification, storage, and communication of digital information, is at the crossroads of probability theory, statistics, statistical mechanics, computer science, information engineering, and even electrical and electronic engineering. With the support of quantum theory, quantum information has found very important and extensive applications, of which quantum cryptography is the best known.

A word of caution, however: it is true that we are made up of protons, neutrons, and electrons, which are quantum particles, but that does not make us quantum persons. When systems made up of millions of particles reach the macroscopic scale (our scale), various factors intervene that cause their behavior to be governed by the laws of classical physics. A prime example of the need to make this distinction is the popular Schrödinger cat, discussed in Section 14.5.3.

So although QM has a relatively well-defined domain of knowledge, its applicability depends on the nature of the system, its size, its temperature, and so on, and even on the purpose of the study. Take a small piece of matter: in general, the energies involved in quantum processes are very small, so that as the temperature rises, thermal fluctuations may be sufficient to disrupt proper quantum behavior. For a system of macroscopic dimensions, on the other hand, classical physics is usually the natural choice; however, only a quantum treatment gives the correct optical and electronic properties of a crystal. The very existence of nanophysics warns us against the idea of a dichotomy with the quantum world in opposition to and separate from the classical world: there is a continuous spectrum in the physical world where different theories coexist, each with its own laws, overlapping here and there and contrasting in their demands.

1.1.1 The quantum toolkit

As with other branches of physics, dealing with the quantum world has required the development of a range of theoretical, experimental, and computational tools and techniques. Much of the success of QM lies precisely in these tools, which have reached a high degree of sophistication.

Immediately after the publication of the seminal papers by Heisenberg and Schrödinger, which laid the foundations for modern QM, a number of scientists, mainly physicists and mathematicians – including the founders themselves – began to test the basic equations and apply them to the simplest problems at hand. The application of algebraic methods developed by mathematicians furthered the systematic development of matrix mechanics, and methods of solving partial differential equations applied to physics were developed to deal with the Schrödinger equation. Since only a handful of simple problems were found to be amenable to an exact solution, a variety of approximation methods were developed to deal with problems that did not admit of an analytical solution or whose solution was too complex for practical application, perturbation theory being the most extended. Methods for dealing with composite or many-body systems, or systems in a highly excited state, or problems involving a time-dependent interaction, or statistical methods for dealing with an ensemble of similar quantum systems … are now widely used. Most of these tools and methods, which form the core of the present text, are presented at an appropriate level in many QM textbooks, which the student is advised to consult regularly.

At the same time, new experimental techniques were developed to confirm the predictions of the nascent theory and to discover additional features of quantum systems that were promptly taken up by theorists for further elaboration and interpretation.

The result was a quantum effervescence around 1900 to 1927 – to define a period – that had a major impact on physics at the time and has not abated over the years. New instruments based on quantum phenomena, such as microscopes using the wave properties of electrons, quantum tunneling or atomic force microscopes, lasers based on the stimulated emission of radiation, optical tweezers based on nonlinear quantum optics, diagnostic tools based on NMR (nuclear magnetic resonance), the GPS (global positioning system) based on the astonishing precision of atomic clocks, related paraphernalia based on solid-state physics, and so on, have become the daily bread of modern laboratories and common technological and medical applications. In various branches of quantum physics, data-intensive calculations and computer simulations have become an important complementary tool.

1.1.2 Scope and purpose of this book

In principle, each of the preceding topics is a subject of study for QM. In practice, however, an undergraduate textbook is largely confined to the development of the basic theory, the exposition of quantum formalism, and its applications to the study of the lightest atoms and molecules, electrons bound or scattered by simple potentials, and elements of condensed matter, including crystals and metals. Depending on the authors' preferences or areas of research, some texts (the majority) focus on the formal aspects of the theory, others on the more fundamental and conceptual aspects, or on topics of current practical interest. Given the wide range of physics that QM touches on, it would be illusory to attempt to produce a truly comprehensive textbook, or to pretend that this one is. The present book is conventional in the sense that it introduces the student to the main formal and practical aspects of QM, but at the same time it is unconventional in the sense that it systematically strives to provide physical explanations that are as clear as possible and, where feasible, novel elements aimed at enabling the student to acquire a coherent picture of the physics underlying quantum formalism. In short, this book discusses standard QM, but from a nonstandard constructive perspective.

The student may have encountered the favored slogan "shut up and calculate"[2] or been told that their questions are "illegitimate questions." In science there are no illegitimate questions, there are only illegitimate answers. The student soon discovers that *understanding* QM is much more complex than *learning* QM; that he or she has learned to calculate without a true understanding of the underlying physics. Indeed, modern QM is in some ways cryptic and enigmatic, in stark contrast to the clarity and elegance of its formal apparatus. In preparing this textbook, the authors have been mindful of these difficulties and, rather than avoiding them, have tried to pave the way, as far as possible, to an understanding of the physics behind the mathematics. This aim is reflected both in the content and in the realistic and objective argumentation throughout the book.

[2] Concerned about attitudes toward QM and feeling uncomfortable with the Copenhagen interpretation (see Section 1.3.1), David Mermin (1989) wrote: "If I were forced to sum up in one sentence what the Copenhagen interpretation says to me, it would be 'Shut up and calculate!' " This meme has become part of modern quantum folklore because of its telling power.

1.2 Is Quantum Mechanics a Complete Theory?

As some physicists and philosophers of science have pointed out, despite its successes, current QM lacks adequate explanations for important observed atomic properties. Among those that have attracted the most attention are quantum fluctuations, the stability of atoms, atomic transitions, and even quantization itself. Let us briefly review these topics.

In the years 1924–1926, the present-day QM was formulated independently by L. de Broglie,[3] W. Heisenberg,[4] and E. Schrödinger,[5] who were separated by several hundred kilometers and by opposing points of view. At the time, Heisenberg, who had just given birth to his highly successful theory that would soon become known as *matrix mechanics*, was still tinkering with his strange (at least to physicists of the time) but successful algebra. Heisenberg's observations were highly significant: he realized that, in contrast to the classical case, his quantum description in terms of matrices – a form of operators, as we will see in Chapter 3 – for the dynamical variables of atomic electrons implied inevitable fluctuations. In the absence of an alternative explanation for such behavior, he decided to accept the fluctuations as inherent to quantum particles in general, since they were all described by the same kind of matrices. Thus, a few months after his careful investigation, he added the notion of *quantum fluctuations* to his theory as a *postulate*, namely that the quantum particles possess an *intrinsic* (*irreducible*) and *permanent* fluctuating motion. He then developed the subject further and arrived at a universal relationship between the position and momentum fluctuations of a quantum particle. This relationship became known as the *uncertainty principle* or *indeterminacy principle* (depending on personal preferences)[6] and was expressed as the "uncertainty relation" or "indeterminacy relation" – both popular names for the Heisenberg inequality. The immediate and widespread success of this work made Heisenberg an eminent public figure, and his uncertainties or indeterminacies have been immoderately extrapolated to other areas of knowledge, acquiring various scientific or philosophical, even mystical, connotations.

The general acceptance of quantum fluctuations as a basic *postulate* of QM has several major implications. At the time of its introduction, there was no evidence for a mechanism that could explain random motions in quantum systems (and this remained so for decades). Fluctuations of the quantum variables were observed, they were a fact; but the postulate that they were intrinsic – or innate, with no known explanation – turned QM into a noncausal theory, a theory that denied the existence of a possible cause for the fluctuations. Moreover, the theory under construction was

[3] Louis Victor Pierre Ramon de Broglie (1892–1987) was an eminent French theoretical physicist who was awarded the Nobel Prize in 1929 for his discovery of the wave properties of matter.

[4] Werner Karl Heisenberg (1901–1976) published his groundbreaking paper on quantum theory in 1925 (at the age of 24), for which he won the Nobel Prize in 1932. His scientific career was exceptional, although it was marred by his work for the German regime during the Second World War (1939–1945).

[5] Erwin Rudolf Josef Alexander Schrödinger (1887–1961) was an eminent Austrian theoretical physicist and philosopher, one of the creators of QM, for which he shared the 1933 Nobel Prize with P. A. M. Dirac.

[6] It should be noted that the terms "uncertainty principle" and "indeterminacy principle" have different connotations: the former refers to *our* impressions or to *our* knowledge and the latter implies an actual lack of determinism in the theory.

indeterministic because of the postulate of irreducible, uncertain, and unpredictable random motions. In short, Heisenberg's matrix mechanics led to a noncausal and indeterministic theory. However, this radical departure from central tenets of classical science came about through a postulate, not as the inevitable result of direct experimentation.

Let us look at the question from the atomic perspective. For simplicity, we consider the hydrogen atom. Its quantum mechanical description can be carried out in detail and with exact analytical procedures using the Schrödinger equation, as will be seen in Chapter 12. This equation contains the Coulomb force between the nucleus (a single proton) and the electron that make up the atom; no other force appears explicitly. Although this force alone cannot be held responsible for the quantum fluctuations, the fact remains that QM *correctly* predicts the energy spectrum of the hydrogen atom and several other of its important atomic properties. So the cause, the reason, the explanation of quantum fluctuations is hidden in the quantum formalism.

Next we consider the issue of atomic stability. Atomic electrons, being charged particles undergoing complicated motions, are expected to radiate as any charged particle does when accelerated. This means that the atom should lose energy and collapse after a short time. But atoms are known to be stable. Returning to the Schrödinger equation, we find that the Coulomb force, the only force that appears in the description of the atom, is also unable to explain the source of the extra energy that is constantly and continuously required to stabilize the system. Again, something is missing – or hidden in the formalism.

Related to the preceding is the question of atomic quantization. Formally, there is no problem: the quantum-mechanical formalism leads to the existence of quantized states. But *physically* it is a puzzle: what is the mechanism that allows some energy states but forbids others? There must be some (hidden) balancing process behind the well-defined orbits corresponding to the stationary quantized solutions so accurately predicted by Schrödinger's theory. Formal explanations along the lines of "each quantum state corresponds to a local minimum of the energy of the distribution of particles," may be interesting and correct, but they do not solve the puzzle. You will see that textbooks on QM usually consider the matter to be solved by mathematics, and in fact the mathematical explanation is the best one you can get – indeed the only one – as long as you give up trying to understand the phenomenon.

To this we can add the question of "spontaneous" transitions between atomic states, which QM does not account for; one has to resort to quantum electrodynamics (QED) to find an explanation. The fact is that these transitions take on a spontaneous appearance when an approximation is made to obtain QM from QED, which means that something important (the cause) contained in QED is lost in the process.

We conclude that although QM is correct in terms of its formal apparatus – and we may add its extended predictive capacity – it is a theory that has never failed – it is an *incomplete* theory in that it lacks one (or more?) crucial element(s) for its full understanding. In other words, its *ontology* – the collection of entities that make up the quantum world and their relationships – has yet to be fully deciphered.

1.2.1 On the seat of quantum phenomena

An important aspect that makes QM difficult to understand is that it expresses all its results as if they occurred in an abstract Hilbert space,[7] rather than in the spacetime of our everyday experience. Matrix mechanics was born as a particularly obscure theory, so much so that it was immediately qualified as not visualizable; H. A. Lorentz[8] spoke out against this obscurity at the Fifth Solvay Conference in 1927 – where QM as we know it today took root – and demanded a clear and visualizable description in our spacetime: "I should like to preserve this ideal of the past, to describe everything that happens in the world with distinct images. I am ready to accept other theories, on condition that one is able to re-express them in terms of clear and distinct images."[9] It is instructive to compare this demand with the following comment by W. Pauli[10] concerning the electron spin: "The lack of a concrete picture is the most satisfactory state of affairs."[11] These contrasting remarks by two of the most eminent theoretical Nobel-Prize-winning physicists illustrate well the two poles of theoretical thinking in Western philosophy under the influence of a realistic versus an idealistic outlook.

Somehow, a search along Lorentz's direction was proposed by H. Groenewold[12] and simultaneously and independently by J. E. Moyal,[13] each building on earlier (1934) ideas by E. Wigner.[14] At present, however, there is no satisfactory statistical phase-space description of QM, as there is, for example, of classical statistical mechanics; no quantum theory describes in detail in real spacetime what is happening, for example, inside an atom, what an electron is doing right now. This makes it difficult to think of QM today as a well-developed, finished statistical theory. Chapter 3 introduces the reader to Heisenberg's matrix mechanics, and Chapter 10 contains the explanation provided by stochastic electrodynamics (SED) for the emergence and physical meaning of the quantum operators underlying the Hilbert-space formalism. This, in turn, will explain the reason for the inability of current QM to provide a (visualizable) phase-space description.

[7] David Hilbert (1862–1943) was a highly influential German mathematician.
[8] Hendrik Antoon Lorentz (1853–1928) was an influential Dutch theoretical physicist, who shared the 1902 Nobel Prize with Pieter Zeeman for the Zeeman effect.
[9] Quotation taken from Bacciagaluppi and Valentini (2009).
[10] Wolfgang Ernst Pauli (1900–1958) was an eminent Austrian theoretical physicist and mathematician, a member of the group of young physicists – the *wunderkinder* – who developed quantum theory in its Copenhagen version, including quantum field theory and QED. He was awarded the Nobel Prize in 1945 for his contributions, in particular the *exclusion principle.*
[11] Quoted by B. L. van der Waerden (1960), p. 202.
[12] Hilbrand Johannes Groenewold (1910–1996) was a Dutch physicist, who in his PhD thesis in 1946 sought a phase-space description of QM.
[13] José Enrique Moyal (1910–1998) was an Australian mathematician and physicist.
[14] Eugene Paul Wigner (1902–1995) was an important Hungarian–US theoretical physicist, who was awarded the Nobel Prize in 1963 for his contributions to the development of quantum theory.

1.3 Main Issues in the Interpretation of Quantum Mechanics

The current QM is not only incomplete, but an unfinished theory. Any physical theory consists of at least two parts, one being its formal (logical) apparatus and the other being the set of specific meanings attributed to its nonlogical elements (the semantics). This is the process that gives the formalism a physical meaning; in short, that links mathematics and nature, identifying, for example, its symbols m, f, a, E ... with mass, force, acceleration, electric field, and so on. The formal apparatus of a theory is well defined, but such identifications (correspondences) are freely assigned, which opens the door to more than one specific reading and allows it to change according to personal preferences or the trends of the time.

In a mature theory all identifications are well defined and consolidated by practice, so there is a single and universal interpretation, a fixed semantics whose meanings cannot be changed without changing the theory itself. Nobody pretends to change the identifications of Newtonian mechanics or Maxwellian electromagnetism. But this is not the case with QM, which is plagued by around two dozen interpretations.

Quantum phenomena normally take place on a scale of length and time that does not allow them to be observed directly: we cannot follow the trajectory of an atomic electron, for example, or its transition from one state to another, or the paths of individual particles in a double-slit array. The quantum formalism has been built on indirect information, extracted, for example, from atomic spectra, or the interference pattern registered on the detection screen, or the various physical properties of bulk matter. The lack of direct contact with the individual quantum objects thus leaves considerable room for interpretation.

And indeed, since its inception, a number of different interpretations of QM have emerged; some are more popular than others, but none is universally accepted. Philosophical precepts have an important influence on the overall interpretation of the quantum formalism and on the meaning assigned to the elements of the description. This makes the subject extremely rich and complex, so much so that a serious analysis of it would require at least one specialized volume. Nevertheless, considering the importance of the subject, we shall briefly discuss some of the salient points on which the various interpretations differ. The interested reader is invited to consult the relevant bibliography at the end of the chapter; in particular, Wikipedia (2025) contains a detailed discussion of some of the interpretations.

1.3.1 Realism and idealism

At the risk of unwarranted simplification – a risk that we are forced to take for the sake of brevity and simplicity – we can say that usually either a realistic or an idealistic (even mystical) position is at stake. Take, for example, the wave function ψ, which is used in QM to describe the state of a system. According to the realistic interpretation adopted throughout this book, ψ refers to a physical system – be it a particle, an atom or any other object – that has an existence of its own. By contrast, the view expressed by Pauli (1960) that "the concept of a material object, of a constitution and nature independent of the observer, is foreign to modern physics, which, forced by the facts, has had to renounce this abstraction," is idealistic, by implying that the object vanishes to be replaced by the world of ideas that appear in the mind of the researcher. Another example of the idealistic interpretation is the statement by Heisenberg (1958) that an elementary particle "is not a material formation in space and time, but in some way a symbol, adopted by which the laws of nature acquire a particularly simple form."

Furthermore, according to an objective view, ψ describes a system's physical state that follows its own laws, governed by an equation (the Schrödinger equation) without our intervention, that is, independent of us, our sensory experience, or our knowledge or ignorance of it. By contrast, according to the subjective view, ψ refers to our knowledge of the system. Thus it has been said that "quantum theory does not refer to nature, but to our knowledge of nature" (Heisenberg, 1951). Similarly, probabilistic statements in quantum theory are often interpreted as subjective judgments, as a measure of our ignorance of the system, rather than, for example, as statistical assessments of the fraction of favorable cases that will occur in the given situation, independent of any observer, who may or may not exist.

A scientist, especially a classical physicist, tends spontaneously to take a realistic and objective view when investigating nature. However, this view is neither the only possible nor the most common. In the mainstream empiricist interpretation, ψ is considered to refer only to properties of the observational data that relate to the system (which may or may not have its own independent existence, depending on the school of thought). In such a view, since the data can be influenced by our observations, it is the latter that define the state: without observations (without an observer), there is no definite state. The assumption that the observed system and the observer are logically inseparable at the quantum level is a central feature of the *orthodox interpretation* of QM, conventionally called the *Copenhagen interpretation*, with N. Bohr[15] (who lived and worked in Copenhagen) as one of its main founders, and Heisenberg, M. Born,[16] and P. A. M. Dirac[17] as its initial supporters. However, the term "orthodox" should not be taken to imply unanimity of opinion, but only the existence of certain features common to the various strands of this hegemonic school.

1.3.2 Individual vs statistical meaning of ψ

Another major issue is whether ψ refers to the state of a single system, or to that of a representative member of an ensemble of equally prepared systems. The latter is the

[15] Niels Henrik David Bohr (1885–1962) was a Danish physicist who made fundamental contributions to the understanding of quantum atomic structure and, more generally, to quantum theory; in particular, he proposed the first model of an atom with discrete energy levels, for which he was awarded the 1922 Nobel Prize in Physics. As a major promoter of scientific research, he succeeded in mainstreaming the Copenhagen version of QM.

[16] Max Born (1882–1970) was a German theoretical physicist and mathematician, a distinguished mentor, and key figure in the development of QM, for which he was awarded the Nobel Prize in 1954. During the early development of quantum theory, Born was a central figure in Göttingen, the *alma mater* of famous mathematicians and theoretical physicists.

[17] Paul Adrien Maurice Dirac FRS (1902–1984) was an English engineer, mathematician, and theoretical physicist, considered one of the most important scientists of the twentieth century. He made notable contributions to QM and quantum field theory, for which he shared the 1933 Nobel Prize with Schrödinger. It is commonly considered that his was the first thesis on quantum mechanics.

statistical or *ensemble interpretation* of QM, which was first proposed by J. C. Slater[18] in 1929 and has been supported over time by a range of physicists, including highly relevant figures such as A. Einstein,[19] Schrödinger, K. Popper,[20] D. I. Blokhintsev,[21] and others – but still enjoys limited popularity.

Much more widespread is the orthodox interpretation, according to which ψ describes the state of an *individual* system. This fundamental discrepancy gave rise to the famous polemic between Einstein, on the one hand, and Bohr and Born, on the other, which lasted for several decades, until Einstein's death in 1955, and is documented – albeit only partially – in several very instructive compilations (see the "Bibliographical Notes" section at the end of the chapter). For example, in his 1953 essay in honor of Born, Einstein showed that the classical limit of the wave function for particles bound to a potential well describes an ensemble of particles, not a single particle, and argued that since the character of the description cannot change by passing to the limit, one must conclude that in general ψ describes an ensemble. Similar reasoning has led to a number of results that are natural if one adopts the ensemble interpretation, but paradoxical if one adopts the orthodox one; we will encounter some of them throughout this book. The so-called *wave-function collapse* or *reduction* introduced by J. von Neumann,[22] which is discussed in Sections 14.5.3 and 15.5.7, is a notable example.

Within the ensemble interpretation of QM it is possible to overcome some obscure conceptual or philosophical points, and even others of a more straightforward physical nature. This is ultimately the reason for our choice. It is important to note, however, that this choice is not exempt from difficulties. An example that sums up the problem well is that although the quantum description is essentially statistical, it is generally incapable of providing a coherent statistical description in phase space, as was mentioned earlier.

Another question that has raised serious difficulties of interpretation relates to the nature of the quantum corpuscle: is it a wave, or a particle, or both in a complementary way, or a *sui generis* entity, neither wave nor particle, but possessing the properties of both? This topic will be discussed in detail in Section 2.8, where it will be argued that in the experiment of electron diffraction, the electrons *are and remain particles* all the time; they are simply distributed in space according to the structure of a diffraction pattern; in short the "wave" pattern is a multi-particle drawing.

The example just discussed shows that ψ does not fully describe the behavior of a single particle, that is, it can only give an incomplete description. To claim that the

[18] John Clarke Slater (1900–1976) was a US-born physicist, best known for developing the theory of the electronic structure of atoms, molecules and solids and for his contributions to microwave electronics.

[19] Albert Einstein (1879–1955). Undoubtedly the most famous scientist of the twentieth century, he was inter alia the creator of special and general relativity. Einstein was a major contributor to the foundation of quantum theory and a severe critic of the direction the theory took under the influence of the Copenhagen school.

[20] Karl Raimund Popper (1902–1994), born in Austria, was one of the most influential philosophers of science of the twentieth century, best known for his theory of empirical falsification.

[21] Dimitri Ivanovich Blokhintsev (1908–1979) was a prominent Soviet theoretical physicist, founder and first director of the Joint Institute for Nuclear Research in Dubna.

[22] John von Neumann (1903–1957) was a Hungarian–US mathematician, physicist, computer scientist, engineer and polymath. Among many other contributions, he pioneered the application of operator theory to quantum mechanics, which is contained in his influential work *Mathematical Foundations of Quantum Mechanics*, published in 1932.