The Conway–Maxwell–Poisson Distribution

While the Poisson distribution is a classical statistical model for count data, the distributional model hinges on the constraining property that its mean equals its variance. This text instead introduces the Conway–Maxwell–Poisson distribution and motivates its use in developing flexible statistical methods based on its distributional form.

This two-parameter model not only contains the Poisson distribution as a special case but, in its ability to account for data over- or under-dispersion, encompasses both the geometric and Bernoulli distributions. The resulting statistical methods serve in a multitude of ways, from an exploratory data analysis tool to a flexible modeling impetus for varied statistical methods involving count data.

The first comprehensive reference on the subject, this text contains numerous illustrative examples demonstrating R code and output. It is essential reading for academics in statistics and data science, as well as quantitative researchers and data analysts in economics, biostatistics and other applied disciplines.

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To those that "count" most in my life:

My family, especially my son

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Preface

Welcome to *The Conway–Maxwell–Poisson Distribution* – the first coherent introduction to the Conway–Maxwell–Poisson distribution and its contributions with regard to statistical theory and methods. This two-parameter model not only serves as a flexible distribution containing the Poisson distribution as a special case but also, in its ability to capture either data over- or under-dispersion, it contains (in particular) two other classical distributions. The Conway–Maxwell–Poisson distribution thereby can effectively model a range of count data distributions that contain data over- or under-dispersion, simply through the addition of one parameter. This distribution's flexibility offers numerous opportunities with regard to statistical methods development. To date, such efforts involve work in univariate and multivariate distributional theory, regression analysis (including spatial and/or temporal models, and cure rate models), control chart theory, and count processes. Accordingly, the statistical methods described in this reference can effectively serve in a multitude of ways, from an exploratory data analysis tool to an appropriate, flexible count data modeling impetus for a variety of statistical methods involving count data.

The Conway–Maxwell–Poisson Distribution can benefit a broad statistical audience. This book combines theoretical and applied data developments and discussions regarding the Conway–Maxwell–Poisson distribution and its significant flexibility in modeling count data, where this reference adopts the convention that the counting numbers are the natural numbers including zero, i.e. $\mathbb{N} = \{0, 1, 2, \ldots\}$. Count data modeling research is a topic of interest to the academic audience, ranging from upper-level undergraduates to graduate students and faculty in statistics (and, more broadly, data science). Meanwhile, the compelling nature of this topic and the writing format of the reference intend to draw quantitative researchers and data analysts in applied disciplines, including business and economics, medicine and public health, engineering, psychology, and sociology – broadly anyone interested in its supporting computational discussions and examples using R. This reference seeks to assume minimal prerequisite statistics coursework/knowledge (e.g. calculus and a calculus-based introduction to probability and statistics that includes maximum likelihood estimation) throughout the book. More advanced readers, however, will benefit from additional knowledge of other subject areas in some chapters, for example, linear algebra or Bayesian computation.

Along with this reference's discussion of flexible statistical methods for count data comes an accounting of available computation packages in R to conduct analyses. Accordingly, preliminary R knowledge will also prove handy as this reference brings to light the various packages that exist for modeling count data via the Conway–Maxwell–Poisson distribution through the relevant statistical methods. The Comprehensive R Archive Network (CRAN) regularly updates its system. In the event that any package discussed in this reference is subsequently no longer directly accessible through the CRAN, note that it is archived and thus still accessible for download and use by analysts.

Acknowledgments

This book is not only the culmination of my years of work and research developing this field but also represents the vastness of contributions in statistical methods and computation by the many authors that are cited in this reference. I am particularly thankful to Dr. Galit Shmueli, the first to introduce me to the Conway–Maxwell–Poisson distribution. That initial collaboration sparked my great interest in this topic where I could easily recognize the broad and diverse areas for potential research, some of which has since come to fruition by myself and/or others. Accordingly, I further thank my colleague and student collaborators with whom we have accomplished a great deal. Thanks as well are extended to other researchers in the larger field of count data modeling with whom I have not only established a camaraderie, but some of whom have also provided discussions and/or analysis associated with their respective models that do not provide an `R` package for direct computation, including Dr. N. Balakrishnan regarding various works on cure-rate models, Dr. Alan Huang regarding the mean-parametrized Conway–Maxwell–Poisson distribution and the `mpcmp` package, Dr. Felix Famoye regarding the bivariate generalized Poisson, Mr. Tong Li for the trivariate-reduced bivariate COM–Poisson, and Dr. S. H. Ong for both Sarmanov-formulated bivariate COM–Poisson models.

Words cannot fully describe how much I thank and appreciate Sir David Cox for his vision and encouragement for me to author this reference; I am only sorry that he was not able to see it come to fruition. I also am forever thankful and appreciative to Dr. Darcy Steeg Morris for our countless, fruitful conversations as she aided with editing the content in this reference that not only helped to shape its vision, but further ensure its accuracy, understanding, and comprehension.

Completing such a project does not happen in a vacuum. I thank family and friends who, through word and deed, kept me encouraged, motivated, and focused to see this project to and through the finish line. I particularly thank Dr. Kris Marsh for our many hours spent together in our self-made writing group. I am also appreciative to Georgetown University for their financial support through avenues including the Senior Faculty Research Fellowship which aided the preparation of this reference.