Cambridge University Press & Assessment
978-1-009-66307-6 — Philosophy of Mathematics from Descartes to Kant
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Excerpt
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1 Introduction

The mathematical method and the nature of mathematical knowledge were subjects of intense philosophical discussion in the seventeenth and eighteenth centuries. The significance of this debate for philosophy generally is evident from the Prize Essay question posed in 1761 by the Prussian Royal Academy: “One wishes to know whether the metaphysical truths in general, and the first principles of *Theologiae naturalis* and morality in particular, admit of distinct proofs to the same degree as geometrical truths.”[1] The question reflects the philosophical debate over whether the mathematical method can or should be employed in philosophy. Implicit in the Prize Essay question is the conviction that geometrical truths *do* admit of distinct proof and so are known with complete certainty. Euclidean geometry was widely accepted as a paradigm of certainty, but the comparison between geometry and philosophy requires a proper understanding of how Euclidean demonstration ensured that certainty. It was in this spirit that philosophers like Locke, Leibniz, Wolff, Lambert, Mendelssohn, and Kant undertook the philosophical investigations of the mathematical method that will be presented here. Accordingly, the emphasis in this *Element* is on metaphysical and epistemological questions about geometrical demonstration in the seventeenth and eighteenth centuries as they were treated by philosophers of the time. While some of the philosophers we will consider were also practising mathematicians, this is not a work on the history of mathematics or of mathematical practice.

At the heart of our discussion here is the role of the diagram or construction in Euclidean demonstration, so let’s begin with a simple example of a Euclidean demonstration (Figure 1). Book 1, Proposition 1, which takes up the problem of constructing an equilateral triangle on a given finite straight line. Begin with the line AB. By Euclid’s third postulate, we may draw a circle BCD with centre A and radius AB, and another circle ACE with centre B and radius AB. Euclid’s first postulate allows us to draw lines AC and BC. Thus we have a triangle ABC.

Because A is the centre of BCD, by Euclid’s Definition 32, AB = AC; because B is the centre of ACE, again by Definition 32, BC = BA. By Euclid’s axiom 1 (things which are equal to the same are equal to one another), AB = AC = BC. So all three sides of triangle ABC are equal, and ABC is the requested equilateral triangle (by Definition 21). It’s not hard to see the kinds of epistemological questions philosophers might have about this kind of demonstration, the evidential role of the diagram, and the purported certainty it provides.[2]

[1] Cited in Walford and Meerbote (1992), p. lxii. Compare also the *quaestio de Certitudine Mathematicarum;* see Mancosu (1993), Chapter 1.

[2] For a detailed contemporary defence of the epistemic success and continued relevance of diagram-based geometrical reasoning, see Manders (2008). See also Arana (2016).

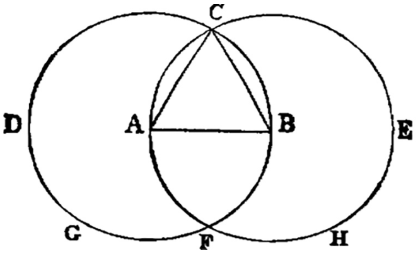

Figure 1 Euclid's construction of an equilateral triangle on a given finite straight line

These questions are raised in rather stark terms by Leibniz in his *New Essays*. Leibniz contrasted his own account of geometrical demonstration with those, like Locke's, that appeal to "what images tell us" (*NE* IV.xii.12). Leibniz takes geometrical demonstration to provide us with a glimpse of the true source of eternal truths and how we come to understand necessity in a way that the "confused ideas of the senses" – images – don't allow (*NE* IV.xii.6). Science should not be founded on the imagination, drawn from sense experience (*NE* IV.xii.4). Rigorous demonstration instead consists in "the connecting of definitions by means of axiomatic identities" (*NE* IV.xii.4):

> The force of the demonstration is independent of the drawn figure (*la figure tracée*), whose only role is to make it easier to understand what is meant and to fix one's attention. It is universal propositions – definitions, axioms and theorems which have already been demonstrated – that make up the reasoning, and *they would support it even if there were no diagram* (*NE* IV.i.9).

This contrast underlies the traditional way of distinguishing rationalists like Leibniz and Wolff from empiricists like Locke. As we'll see, this story, not surprisingly, is not so straightforward. Recent scholarship has seen a reassessment of Wolffian rationalism by showing that his theory of science – including mathematics – admits appeals to empirical justification, to experience. In fact, Dunlop has argued that even Leibniz recognized an essential role for sensibility in mathematical cognition.[3] What follows is a survey of various early modern figures on the issue of demonstrative certainty in geometry and the role therein for sensibility. I hope to show that there's a common challenge that these figures are struggling with, one that Kant eventually brings to the fore and explicitly addresses.

It seems appropriate that a discussion of certainty in the early modern period should begin with Descartes.

[3] Dunlop (2014).

2 Descartes: From the *Rules* to the *Meditations*

Descartes was a practising mathematician, natural philosopher, and philosopher. He is known for, among many other things, developing the techniques that made analytic geometry possible and for developing a comprehensive new physics. Our interest here, however, is in Descartes' conviction that there was a method which, if followed correctly, would lead us to the certain knowledge within our cognitive reach. The model for that method was mathematics.

In his early unfinished work, *Rules for the Direction of the Mind*, Descartes takes arithmetic and geometry as certain and indubitable, but also sees therein an essential role for the imagination, and therefore the senses. On the other hand, the later *Meditations* is often taken to be a quintessentially rationalist work: after all, he believes that the "greatest benefit" of the "extensive doubt" of the *Meditations* is to provide "the easiest route by which the mind may be led away from the senses" (*Rules*, *Synopsis*). This tension between the earlier and later accounts foreshadows the themes we'll trace in the following sections.

2.1 Rules for the Direction of the Mind

The stated aim of Descartes' *Rules for the Direction of the Mind* is to form "true and sound judgements about whatever comes before it" (*Rules* 359).[4] Descartes tells us that "as yet" the only certain and indubitable cognition we have is of arithmetic and geometry (*Rules* 364). This is because their objects are "so pure and uncomplicated that they need make no assumptions at all that experience renders uncertain"; they consist rather in the "rational deduction of consequences" that "cannot be erroneous when performed by an understanding that is in the least degree rational." Because the goal of Descartes' study is certain knowledge, and because the only intellectual activities by which we can acquire certain knowledge of things are intuition and deduction, the study is concerned only with what we can "clearly and evidently intuit or deduce with certainty" (*Rules* 366). We'll see that mathematics is not just the model for certain knowledge of the physical world, but that all such knowledge *is* mathematical knowledge.

By "intuition," Descartes says, he doesn't mean "the fluctuating testimony of the senses, or the deceptive judgement of the imagination as it botches things together, but 'the indubitable conception of a clear and attentive mind, which proceeds solely from the light of reason'" (*Rules* 369). The self-evidence and certainty of intuition are required not just for apprehending single propositions, but also for reasoning, for deducing propositions: for example, the inference

[4] Page references to the *Rules* are to the AT pagination in the margins.

that $2 + 2 = 3 + 1$ requires that we intuitively perceive that $2 + 2 = 4$, that $3 + 1 = 4$ and that we intuitively perceive that $2 + 2 = 3 + 1$ follows necessarily from the latter two propositions. So a deduction is a succession of intuitions. In Rule 3, Descartes specifies the objects of certain knowledge as "pure and simple natures," of which there are very few, which we can "intuit straight off and *per se* (independently of any others) either in our sensory experience or by means of a light innate within us" (*Rules* 383). Although "only the intellect is capable of *perceiving* truth," in order to intuit simple propositions distinctly "it has to be assisted by imagination, sense-perception and memory." The question I want to focus on is how the imagination assists the intellect in intuiting simple propositions distinctly and thereby perceiving truth. This becomes clearer in Rule 12, where the very general picture of our cognitive activities is filled out. An account of knowledge must consider only two factors: the knowing subject and the objects of knowledge. Accordingly, Descartes provides a hypothetical mechanistic account of our "instruments of knowing" (*Rules* 412–415), followed by an account of their objects.

First, consider the instruments of knowing. Sense-perception is merely passive: by means of contact with an external object, the external shape of the sense organ is really changed, just as the shape of the surface of wax is altered by a seal. The image received from the senses is simultaneously transmitted to the common sense, which is "another part of the body." The common sense in turn impresses the images or ideas on the imagination, again as if on wax. The imagination is also a genuine part of the body, "large enough to allow different parts of it to take on many different figures." All of this is a function of the body, without the aid of reason.

The cognitive power in the strict sense, by contrast, is a "purely spiritual" single power, distinct from the body. According to its different functions, it is variously called pure intellect, imagination, memory, or sense-perception; it is nonetheless a single power. It can be passive or active. When it applies itself (together with the imagination) to the common sense, we say it sees or touches; when it applies itself to figures already in the imagination, we say it remembers; when it applies itself to the imagination to form new figures, it imagines or conceives. Finally, when it acts on its own, it is said to understand.

Descartes next considers the objects of knowledge, only insofar as they are perceived by the intellect. Simples are things so clearly and distinctly known that we can't divide them into others more distinctly known. We conceive of everything else as "in a sense composed of these" simples. It's in the composition that error can creep in.

These *simples* are either purely intellectual, purely material or common to both. The intellectual simples are those the understanding recognizes by an innate light, without the aid of any corporeal images: Descartes' examples are doubt, ignorance, or volition. The material simple natures are present only in bodies: for example, shape, extension, and motion. The common simples are those we predicate of either corporeal or non-corporeal things: for example, existence, unity, and duration. These simple natures are all self-evident and never contain any falsity. According to Descartes, we can never understand anything beyond those simple natures and a certain compounding of one with another.

Composite natures are known by us either from experience or because we put them together. We can go wrong only in the latter case, where we ourselves compose the objects of our belief. The only way of combining simple natures that enables us to be certain of their truth is by deduction. Deduction is in effect a mental intuition of the necessary connections between simple natures: we should only conjoin things when we intuit that the conjunction of one with the other is wholly necessary, as for example when we "deduce that nothing that lacks extension can have a shape, on the grounds that there is a necessary connection between shape and extension" (*Rules* 425). So "mental intuition extends to all these simple natures and to our knowledge of the necessary connections between them" (*Rules* 425). The whole of human knowledge consists in "our achieving a distinct perception of how all these simple natures contribute to the composition of other things" (*Rules* 427).

In the next twelve Rules, Descartes turns his attention specifically to knowledge of corporeal things, such as, for example, the nature of a magnet and the sounds emitted from strings of different thickness and tensions. This is where we see clearly how physical problems should be reduced to problems that can be expressed in arithmetic or geometry. Rule 13 enjoins us when considering such things to "abstract from . . . every irrelevant conception and reduce [the problem] to such a form that we are no longer aware of dealing with this or that subject-matter, but only with certain magnitudes in general and the comparison between them" (*Rules* 430).

Why does the investigation of material things reduce to a comparison of magnitudes in general? As we've seen, all knowledge, apart from that obtained from "simple and pure intuition," results from deduction, which in Rule 14 Descartes describes as "a comparison between two or more things" (*Rules 440*). The goal of deduction is to reduce the terms compared to equality. But "nothing can be reduced to equality except what admits of difference of degree, and everything covered by the term 'magnitude'." Thus "all we have to deal with here are magnitudes in general" (*Rules 440*). Of course, one thing can be more or less white than another, one sound more or less sharp than another, "but we cannot determine exactly whether the greater exceeds the lesser by a ratio of 2 to 1 or 3 to 1 unless we have recourse to a certain analogy with the extension of a body that has shape" (*Rules* 441). What's needed is a way of representing measurable features of material things in such a way that they can be compared.

According to Rule 14, such problems should be "re-expressed in terms of the real extension of bodies and should be pictured in our imagination entirely by means of bare figures" (*Rules* 438); this will allow them to "be perceived much more distinctly by our intellect." Why? Here is where the mechanistic account of our cognitive faculties comes in. Descartes explicitly refers back to Rule 12: if we conceive (as we did there) of the imagination and its ideas as "nothing but a real body with real extension and shape," we can see that it can "display more distinctly all the various differences in proportions" (*Rules* 441). The mechanistic account of the senses and imagination seems to ground the account of certain knowledge of corporeal things, and, as we'll see, the certainty of mathematical knowledge: "the Rules I'm about to expound are much more readily employed in the study of [arithmetic and geometry]"; in fact, they're *all that are needed* in arithmetic and geometry (*Rules* 442).

At this point, "we are assuming that every problem has been reduced to the point where our sole concern is to discover a certain extension on the basis of a comparison with some other extension which we already know" (*Rules* 447). Since "unity is the common nature which . . . all the things which we are comparing must participate in equally," we begin with a unit, which can be "either one of the magnitudes already given or any other magnitude, and this will be the common measure of all the others" (*Rules* 449). The only figures that we need for the purpose of expressing differences in relation are "rectilinear and rectangular surfaces, or straight lines" (*Rules* 452).

In Rule 15, Descartes explains that we can form more distinct images of these figures in our imagination if we draw them, thereby visually displaying them before our external senses. For example, we shall depict the unit in three ways, *viz.* by means of a square, □,if, we think of it only as having length and breadth; by a line, ——, if we regard it as having just length; or, lastly, by a point, .,if we view it as the element which goes to make up a set.[5]

In Rule 18, Descartes describes "all the operations needed for working out a problem, i.e. for deducing some magnitudes from others," that is, addition, subtraction, multiplication, and division (*Rules* 461). For example, "when we come to know one magnitude on the basis of our prior knowledge of the parts

5 Rule 16 describes the use of "very concise symbols" in place of the complete figure when the figure "does not require the immediate attention of the mind" (*Rules* 454).