REAL-WORLD PASSIVE COMPONENTS

CHAPTER 1x

In the introductory first chapter of AoE3 we saw the basics of passive components – resistors, capacitors, inductors (and transformers), and diodes – and we've treated them as idealized components. In reality things are more complicated: for example, resistors have some "parasitic" capacitance and inductance; their resistance varies with temperature ("tempco"), with applied voltage ("voltage coefficient"), and with the passage of time ("aging"). Even plain old *wire* isn't a simple thing: wire has resistance and inductance, both dependent upon frequency; and it comes in a bewildering variety of sizes, configurations, and varieties of insulation.

Most of the time you can ignore these real-world deviations from the ideal. But a good circuit designer must know about them, and particularly which ones do matter in the design of any particular circuit. In this first "x-chapter" we peel away the ideal (and boring) façade of basic components, revealing their rich interior.

Review of Chapter 1 of AoE3

To bring the reader up to speed, we start this chapter with the end-of-chapter review from the main volume's Chapter 1:

¶A. Voltage and Current.

Electronic circuits consist of components connected together with wires. *Current* (I) is the rate of flow of charge through some point in these connections; it's measured in amperes (or milliamps, microamps, etc.). *Voltage* (V) between two points in a circuit can be viewed as an applied driving "force" that causes currents to flow between them; voltage is measured in volts (or kilovolts, millivolts, etc.); see §1.2.1. Voltages and currents can be steady (dc), or varying. The latter may be as simple as the sinusoidal alternating voltage (ac) from the wallplug, or as complex as a high-frequency modulated communications waveform, in which case it's usually called a *signal* (see ¶B below). The algebraic sum of currents at a point in a circuit (a *node*) is zero (Kirchhoff's current law, KCL, a consequence of conservation of charge), and the sum of voltage drops going around a closed loop in a circuit is zero (Kirchhoff's voltage law, KVL, a consequence of the conservative nature of the electrostatic field).

¶B. Signal Types and Amplitude.

See §1.3. In digital electronics we deal with *pulses*, which are signals that bounce around between two voltages (e.g., +5 V and ground); in the analog world it's *sinewaves* that win the popularity contest. In either case, a periodic signal is characterized by its frequency f (units of Hz, MHz, etc.) or, equivalently, period T (units of ms, μs, etc.). For sinewaves it's often more convenient to use *angular* frequency (radians/s), given by $\omega=2\pi f$.

Digital amplitudes are specified simply by the HIGH and LOW voltage levels. With sinewaves the situation is more complicated: the amplitude of a signal $V(t)=V_0 \sin \omega t$ can be given as (a) *peak* amplitude (or just "amplitude") V_0, (b) *root-mean-square* (rms) amplitude $V_{\text{rms}}=V_0/\sqrt{2}$, or (c) peak-to-peak amplitude $V_{\text{pp}}=2V_0$. If unstated, a sinewave amplitude is usually understood to be V_{rms}. A signal of rms amplitude V_{rms} delivers power $P=V_{\text{rms}}^2/R_{\text{load}}$ to a resistive load (regardless of the signal's waveform), which accounts for the popularity of rms amplitude measure.

Ratios of signal amplitude (or power) are commonly expressed in *decibels* (dB), defined as $\text{dB} = 10\log_{10}(P_2/P_1)$ or $20\log_{10}(V_2/V_1)$; see §1.3.2. An amplitude ratio of 10 (or power ratio of 100) is 20 dB; 3 dB is a doubling of power; 6 dB is a doubling of amplitude (or quadrupling of power). Decibel measure is also used to specify amplitude (or power) directly, by giving a reference level: for example, −30 dBm (dB relative to 1 mW) is 1 microwatt; +3 dBVrms is a signal of 1.4 V rms amplitude (2 Vpeak, 4 Vpp).

Other important waveforms are square waves, triangle waves, ramps, noise, and a host of *modulation* schemes by which a simple "carrier" wave is varied in order to convey information; some examples are AM and FM for analog communication, and PPM (pulse-position modulation) or QAM (quadrature-amplitude modulation) for digital communication.

¶C. The Relationship Between Current and Voltage.

Chapter 1 concentrated on the fundamental, essential, and ubiquitous *two-terminal linear devices*: resistors, capacitors, and inductors. (Subsequent chapters dealt with *transistors* – three-terminal devices in which a signal applied to one terminal controls the current flow through the other pair – and their many interesting applications. These include amplification, filtering, power conversion, switching, and the like.) The simplest linear device is the *resistor*, for which $I=V/R$ (Ohm's Law, see §1.2.2A). The term "linear" means that the response (e.g., current) to a combined sum of inputs (i.e., voltages) is equal to the sum of the responses that each input would produce: $I(V_1+V_2)=I(V_1)+I(V_2)$.

¶D. Resistors, Capacitors, and Inductors.

The resistor is clearly linear. But it is not the only linear two-terminal component, because linearity does not require $I \propto V$. The other two linear components are *capacitors* (§1.4.1) and *inductors* (§1.5.1), for which there is a time-dependent relationship between voltage and current: $I=C\,dV/dt$ and $V=L\,dI/dt$, respectively. These are the *time domain* descriptions. Thinking instead in the *frequency domain*, these components are described by their *impedances*, the ratio of voltage to current (as a function of frequency) when driven with a sinewave (§1.7). A linear device, when driven with a sinusoid, responds with a sinusoid of the same frequency, but with changed amplitude and phase. Impedances are therefore complex, with the real part representing the amplitude of the response that is in-phase, and the imaginary part representing the amplitude of the response that is in quadrature (90° out of phase). Alternatively, in the polar representation of complex impedance ($Z=|Z|e^{i\theta}$), the magnitude $|Z|$ is the ratio of magnitudes ($|Z|=|V|/|I|$) and the quantity θ is the phase shift between V and I. The impedances of the three linear 2-terminal components are $Z_R=R$, $Z_C=-j/\omega C$, and $Z_L=j\omega L$, where (as always) $\omega=2\pi f$; see §1.7.5. Sinewave current through a resistor is in phase with voltage, whereas for a capacitor it leads by 90°, and for an inductor it lags by 90°.

¶E. Series and Parallel.

The impedance of components connected in series is the sum of their impedances; thus $R_{\text{series}}=R_1+R_2+\cdots$, $L_{\text{series}}=L_1+L_2+\cdots$, and $1/C_{\text{series}}=1/C_1+1/C_2+\cdots$. When connected in parallel, on the other hand, it's the *admittances* (inverse of impedance) that add. Thus the formula for capacitors in parallel looks like the formula for resistors in series, $C_{\text{parallel}}=C_1+C_2+\cdots$; and vice versa for resistors and inductors, thus $1/R_{\text{parallel}}=1/R_1+1/R_2+\cdots$. For a pair of resistors in parallel this reduces to $R_{\text{parallel}}=(R_1R_2)/(R_1+R_2)$. For example, two resistors of value R have resistance $R/2$ when connected in parallel, or resistance $2R$ in series.

The power dissipated in a resistor R is $P=I^2R=V^2/R$. There is no dissipation in an ideal capacitor or inductor, because the voltage and current are 90° out of phase. See §1.7.6.

¶F. Basic Circuits with *R*, *L*, and *C*.

Resistors are everywhere. They can be used to set an operating current, as for example when powering an LED or biasing a zener diode (Fig. 1.16); in such applications the current is simply $I=(V_{\text{supply}}-V_{\text{load}})/R$. In other applications (e.g., as a transistor's load resistor in an amplifier, Fig. 3.29) it is the *current* that is known, and a resistor is used to convert it to a voltage. An important circuit fragment is the *voltage divider* (§1.2.3), whose unloaded output voltage (across R_2) is $V_{\text{out}}=V_{\text{in}}R_2/(R_1+R_2)$.

If one of the resistors in a voltage divider is replaced with a capacitor, you get a simple *filter*: lowpass if the lower leg is a capacitor, highpass if the upper leg is a capacitor (§§1.7.1 and 1.7.7). In either case the −3 dB transition frequency is at $f_{3\text{dB}}=1/2\pi RC$. The ultimate rolloff rate of such a "single-pole" lowpass filter is −6 dB/octave, or −20 dB/decade; i.e., the signal amplitude falls as $1/f$ well beyond $f_{3\text{dB}}$. More complex filters can be created by combining inductors with capacitors, described in Chapter 6. A capacitor in parallel with an inductor forms a *resonant circuit*; its impedance (for ideal components) goes to infinity at the resonant frequency $f=1/(2\pi\sqrt{LC})$. The impedance of a *series LC* goes to zero at that same resonant frequency. See §1.7.14.

Other important capacitor applications in Chapter 1 (§1.7.16) include (a) *bypassing*, in which a capacitor's low impedance at signal frequencies suppresses unwanted signals, e.g., on a dc supply rail; (b) *blocking* (§1.7.1C), in which a highpass filter blocks dc, but passes all frequencies of interest (i.e., the breakpoint is chosen below all signal frequencies); (c) *timing* (§1.4.2D), in which an *RC* circuit (or a constant current into a capacitor) generates a sloping waveform used to create an oscillation or a timing interval; and (d) *energy storage* (§1.7.16B), in which a capacitor's stored charge $Q=CV$ smooths out the ripples in a dc power supply.

Additional applications of capacitors include: (e) *peak detection* and *sample-and-hold* (§§4.5.1 and 4.5.2), which capture the voltage peak or transient value of a waveform, and (f) the *integrator* (§4.2.6), which performs a mathematical integration of an input signal.

¶G. Loading; Thévenin Equivalent Circuit.

Connecting a load (e.g., a resistor) to the output of a circuit (a "signal source") causes the unloaded output voltage to drop; the amount of such *loading* depends on the load resistance, and the signal source's ability to drive it. The latter is usually expressed as the *equivalent source impedance* (or *Thévenin impedance*) of the signal. That is, the signal source is modeled as a perfect voltage source V_{sig} in series with a resistor R_{sig}. The output of the resistive voltage divider driven from an input voltage V_{in}, for example, is modeled as a voltage source $V_{\text{sig}}=V_{\text{in}}R_2/(R_1+R_2)$ in series with a resistance $R_{\text{sig}}=R_1R_2/(R_1+R_2)$ (which is just $R_1\|R_2$). So the output of a 1kΩ–1kΩ voltage divider driven by a 10 V battery looks like 5 V in series with 500 Ω.

Any combination of voltage sources, current sources, and resistors can be modeled perfectly by a single voltage source in series with a single resistor (its "Thévenin equivalent circuit"), or by a single current source in parallel with a single resistor (its "Norton equivalent circuit"); see Appendix D. The Thévenin equivalent source and resistance values are found from the open-circuit voltage and short-circuit current as $V_{\text{Th}}=V_{\text{oc}}$, $R_{\text{Th}}=V_{\text{oc}}/I_{\text{sc}}$; and for the Norton equivalent they are $I_{\text{N}}=I_{\text{sc}}$, $R_{\text{N}}=V_{\text{oc}}/I_{\text{sc}}$.

Because a load impedance forms a voltage divider with the signal's source impedance, it's usually desirable for the latter to be small compared with any anticipated load impedance (§1.2.5A). However, there are two exceptions: (a) a *current source* has a high source impedance (ideally infinite), and should drive a load of much lower impedance; and (b) signals of *high frequency* (or fast risetime), traveling through a length of cable, suffer reflections unless the load impedance equals the so-called “characteristic impedance” Z_0 of the cable (commonly 50 Ω), see Appendix H.

¶H. The Diode, a Nonlinear Component.

There are important two-terminal devices that are not linear, notably the *diode* (or *rectifier*), see §1.6. The ideal diode conducts in one direction only; it is a “one-way valve.” The onset of conduction in real diodes is roughly at 0.5 V in the “forward” direction, and there is some small leakage current in the “reverse” direction, see Figure 1.55. Useful diode circuits include power-supply *rectification* (conversion of ac to dc, §1.6.2), signal rectification (§1.6.6A), *clamping* (signal limiting, §1.6.6C), and *gating* (§1.6.6B). Diodes are commonly used to prevent polarity reversal, as in Figure 1.84; and their exponential current versus applied voltage can be used to fashion circuits with logarithmic response (§1.6.6E).

Diodes specify a maximum safe reverse voltage, beyond which avalanche breakdown (an abrupt rise of current) occurs. You don’t go there! But you can (and should) with a *zener diode* (§1.2.6A), for which a reverse breakdown voltage (in steps, going from about 3.3 V to 100 V or more) is specified. Zeners are used to establish a voltage within a circuit (Fig. 1.16), or to limit a signal’s swing.

1x.1 Wire and Connectors

1x.1.1 Wire gauge: resistance, heating, and current-carrying capacity

Table 1x.1 shows wire sizes, going to extremes at both ends. An easy way to remember it all (if you've left your copy of this book at home) is to note that (a) each wire size is 1 dB (in cross-sectional area, or resistance), and (b) #10 wire is 1 mΩ/ft.[1] At high frequencies the *skin effect* (see §1x.1.6B, below) causes an effective increase in resistance (as does the *proximity effect*, for closely packed multiple wires).

We like #26 Kynar-insulated solid wire for point-to-point wiring on circuit boards, and #22–26 stranded wire (with irradiated PVC insulation) for other internal instrument wiring (as well as for multiwire signal cables), where currents are small (<1 A, say). For larger currents, choose wire sizes according to how much voltage drop and heating you can tolerate. When winding inductors or transformers (see below), the wire size is constrained by power dissipation, quality factor ("Q"), and available core dimensions. For power transformers a typical wire size guideline is 1000 "circular mils" (the square of the wire diameter in mils) per amp, e.g., #20 enamel insulated magnet wire for a 1 A (rms) load current. Figure 1x.1 is a rough guide to the current-carrying capability of a given wire size, based on temperature rise above ambient. But other factors – such as the enclosure or conduit, and the thermal path for heat removal – affect the real-world maximum current.

1x.1.2 Stranding, insulation, and tinning

Stranding

Stranded wire is more flexible and supple than solid wire, and is preferred for cables and for wiring that undergoes motion (e.g., power cords, mouse, keyboard, network patch cables, oscilloscope and voltmeter probes, and so on). But solid wire is often better when wiring between fixed points (such as on a circuit board, or for house wiring) because you don't have to worry about persuading every strand to behave.

Stranded wire comes with standard numbers of finer strands, often 7 or 19 (the number of coins that fit nicely on a flat surface), with finer stranding providing more flexibility. For *really* supple wiring you want "ropelay" stranding, in which a group of fine stranded wires are themselves stranded into a larger wire. For example, you can get #24 stranded wire as 7 strands of #32 ("7/32"), or 19 strands of #36; as a ropelay you can get 7 groups of 15/44, for a total of 105 strands. In very thick cables the numbers get really large: we bought some #0 "extra-flexible" ropelay cable, stranded as 7x7x86/36 (4214 strands of #36!); the stuff was as supple as clothesline.

There are some exotic forms of stranded wire; two in particular are called *bunched conductor*, and *litz wire*. A bunched conductor consists of a twisted bundle of insulated strands that are stripped and connected together at each end; litz wire also consists of a set of insulated strands, woven however in such a manner that each strand visits the inner and outer portions of the wire as it runs along the length. These unusual forms of stranded wire are used to circumvent skin effect and proximity effect, discussed below in §1x.1.6B.

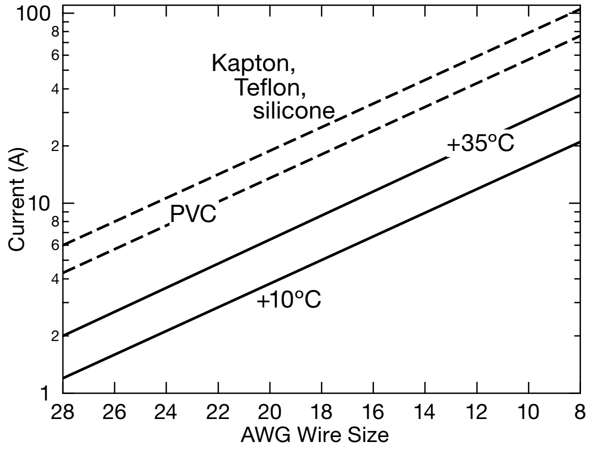

Figure 1x.1. Approximate current-carrying capability versus wire gauge, for 10°C and 35°C rise above ambient (solid lines), and for maximum insulation temperature at 30°C ambient for two insulation families (dashed lines). Derate these values for multiwire cables, by a factor of 0.8 (2–5 conductors), 0.7 (6–15 conductors), and 0.5 (16–30 conductors).

Insulation

The most common insulation is ***PVC*** (polyvinylchloride), which has respectable thermal and electrical characteristics. ***Polypropylene*** and ***polyethylene*** are also used, the latter particularly in coaxial cable. We favor ***irradiated PVC*** for wiring within instruments, because it is consid-

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[1] You can really impress your friends by knowing also that #10 wire is 0.1″ in diameter, or 5 mm^2 in area.

Table 1x.1: Copper Wire Table[a]

	Diameter[b]		Resistance[c]		Mass		Area[e]
AWG	mils[d]	mm	mΩ/ft	mΩ/m	lb/kft	kg/km	mm^2
0	325	8.26	0.098	0.32	320	476	53.5
2	258	6.55	0.156	0.51	201	299	33.6
4	204	5.18	0.249	0.82	126	187	21.1
6	162	4.11	0.395	1.30	79.5	118	13.3
8	129	3.28	0.628	2.06	50.0	74.4	8.36
10	102	2.59	0.999	3.28	31.4	46.7	5.26
12	80.8	2.05	1.59	5.22	19.8	29.5	3.31
14	64.1	1.63	2.53	8.30	12.4	18.5	2.08
16	50.8	1.29	4.02	13.2	7.82	11.6	1.31
18	40.3	1.02	6.39	21.0	4.92	7.32	0.82
20	32.0	0.813	10.2	33.5	3.09	4.60	0.52
22	25.4	0.645	16.1	52.8	1.95	2.90	0.33
24	20.1	0.511	25.7	84.3	1.22	1.82	0.20
26	15.9	0.404	40.8	134	0.769	1.14	0.13
28	12.6	0.320	64.9	213	0.484	0.720	0.084
30	10.0	0.254	103	338	0.304	0.452	0.053
32	7.95	0.202	164	538	0.191	0.284	0.034
34	6.31	0.160	261	856	0.120	0.179	0.021
36	5.00	0.127	415	1361	0.076	0.113	0.013
38	3.97	0.101	660	2164	0.048	0.071	0.0084
40	3.15	0.080	1050	3442	0.030	0.045	0.0053

(a) values at 25°C. (b) for solid conductor; stranded conductors are larger. (c) tempco = +0.4%/°C. (d) 1 mil = 0.001 inch = 0.0254 mm. (e) the area in "circular mils" is the square of the diameter in mils.

From very thick to very thin in the American Wire Gauge (AWG) sizes of copper wire. The resistance values have a positive temperature coefficient of 0.4%/°C. Sizes from #20–26 are typically used in signal cables and in instrument wiring; house wiring uses #14 and #12 for 15 A and 20 A circuits, respectively.

erably less susceptible to "melt-back" during soldering; it is also tougher, while retaining PVC's flexibility.[2] ***Teflon***® insulation is an expensive alternative, with superior thermal, electrical, and chemical properties: it is unaffected by soldering temperatures (no melt-back), it is chemically inert (unaffected by acids, alkalis, hydrocarbons, solvents, ozone, water, oil, and gasoline), and it retains its flexibility at low temperatures; it is rated for operation from −70°C to 200°C (or 260°C for TFE-type Teflon). However, owing to the absence of molecular cross-linking, Teflon is susceptible to "cold-flow" (also known as *creep*, or *compression set*): the Teflon insulation of a wire pulled tightly around a corner tends to cold-flow away from the contact zone.[3] "Magnet wire" consists of a bare solid copper conductor with a tough conformal insulating layer (or layers) (sometimes loosely called ***enamel)***, intended for inductors and transformers, with correspondingly high temperature ratings; for example, Belden's Beldsol® (nylon over polyurethane) is rated for operation to 270°C. Some other high-temperature insulation types (with excellent low-temperature properties as well) are ***silicone*** (which excels in flexibility), ***Tefzel***®, and ***Halar***®; these are rated to 150°C. For more detail see the literature from wire manufacturers such as Alpha and Belden.

Tinning

Nearly all electrical wiring is copper, which is often metal plated ("tinned"), both for compatibility with the insulating material, and for enhanced solderability (compared with bare copper). Tin (or tin alloy) plating is common for most plastic insulations (e.g., PVC), whereas Kynar- and Teflon-insulated wires are usually plated with silver (or nickel); enamel-insulated magnet wire (and litz wire) is ordinarily untinned, as are heavy wiring used in power distribution (e.g., "Romex" house wiring).

1x.1.3 Printed circuit wiring

Most circuit wiring that is not "on-chip" takes the form of PCB traces.[4] Except for the simplest circuits, contemporary PCBs are fabricated as multiple layers with plated-through holes, on a fiberglass-epoxy substrate known as FR-4 (formerly G-10). The standard thickness totals 0.062″ (1.6 mm), with a tough insulating *soldermask*[5] covering all but the exposed *pads*[6] that are to be soldered (to prevent solder bridges and also to protect the surface traces). An informational *silkscreen* legend is applied over the finished board (soldermask and all), indicating parts values and designations, and other generally useful stuff. Components are usually soldered on both sides, using robotic pick-and-place machinery to lay down the parts onto the pads (to which solder paste[7] has been applied), followed

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[2] We like the 19-strand (versus 7-strand) hookup wire, for greater flexibility; the Alpha part numbers are 7058/19–7054/19 (AWG even-numbered gauges 16 through 24). You can buy equivalents from wire dealers (such as Anixter), spooled from their bulk supplies in the lengths you want, generally at a considerable discount.

[3] This can happen also in a bundle of Teflon-insulated wiring tightly wrapped with cable ties or lacing. NASA has cautioned its spacecraft designers on this point, and suggests Tefzel and Kynar as alternative insulation materials.

[4] A misnomer, because the traces are not *printed*; rather, they are the remnants left after the unwanted copper has been chemically removed.

[5] Usually liquid photo-imageable solder mask over bare copper, "LPI-SMOBC."

[6] The connection points to the electrical components.

[7] An emulsion of solder particles and heat-activated flux.

by a scorching journey through a reflow oven during which the surface-mount components are soldered.[8]

1x.1.4 PCB traces

The wiring traces are copper, with the thickness specified in *ounces*, most commonly "1/2 ounce" or "1 ounce." These strange units refer to the weight of copper per square foot! You can figure it out from first principles, if you like, but the conversion factor is 1 ounce ↔ 0.00137″ (1.37 mils) ↔ 35 μm.[9] The *sheet resistance* of 1 oz copper is 0.5 mΩ/square (another strange unit, sometimes written Ω/□), and varies inversely with copper thickness. When used as a heatsink, a square inch of PCB copper is (very roughly) 50°C/W.

A. Resistance and current-carrying capacity

PCB traces are resistive, which causes a dc voltage drop IR proportional to current, and power dissipation I^2R proportional to the square of current. The dc resistance of a 1 ounce trace is $R = 0.47/w$ ohms/inch (or $0.19/w$ ohms/cm), where w is the trace width in mils (these values scale inversely in copper thickness). Because typical trace widths are in the range of 5–10 mils, the effects of their resistance (~ 0.05 to 0.1 Ω/inch) is generally insignificant, in terms of signal degradation, compared with the effects of capacitance and inductance (see below). However, trace resistance limits current carrying capacity, owing to I^2R heating; see Figure 1x.2.

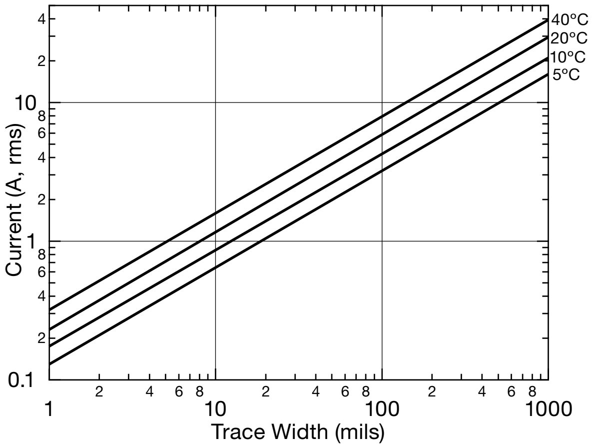

Figure 1x.2. Approximate PCB trace current limits, for 1 ounce (1.37 mil, or 35 μm) copper, as determined by I^2R heating, for the indicated values of temperature rise. For other copper thicknesses scale the x-axis values proportionally.

B. Capacitance and inductance

PCB traces, like all conductors, have capacitance and inductance. To a good approximation, these are proportional to trace length, and are a function of trace width, height above a conductive plane (power/ground planes, usually), and (for capacitance) the dielectric constant of the PCB substrate and soldermask. PCB layout software sometimes includes routines to calculate capacitance, inductance, transmission line impedance, and even propagation delays. It's useful to know the approximate range of values, though, which we've calculated and plotted in Figure 1x.3, for three values of substrate thickness: 10 mils, 30 mils, and 60 mils (corresponding to a typical multilayer board, a 0.032″ two-layer board, and a standard 0.062″ two-layer board). Note that the values depend primarily on the ratio of trace width to substrate thickness (w/h), and only weakly on the substrate thickness itself. Order of magnitude, traces have about a pF/cm of capacitance, and 5 nH/cm of inductance.

C. Transmission-line impedance and attenuation

Because a PCB trace of given width and spacing has some capacitance per unit length and some inductance per unit length (both dependant on geometry), it behaves like a *transmission line*, a subject treated in some detail in Appendix H. For now, the important facts are (a) for "fast" signals (those that change significantly in the time it takes a signal to travel to the end of a wire and back) you cannot treat a wire as a simple conductor with some single voltage on it – instead, signals travel along it as a wave, and in general will reflect off the far end and return to haunt you; (b) a transmission line has a *characteristic impedance* Z_0, which is the ratio of voltage to current in a traveling wave; (c) if you connect a resistor equal to the characteristic impedance (which is approximately a resistance, $R_0 \approx |Z_0|$, usually 50 Ω) across the far end, there will be no reflections (it will

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[8] If *through-hole* components are used, the board undergoes *wave soldering*, a terrifying process during which a fountain of molten solder is squirted forcefully against its underside. Lots of smoke! (And any surface-mount components on the bottom side will have been secured with a dot of adhesive, otherwise they will disappear in a cascade of molten solder.)

[9] The copper thickness you specify refers to the *finished* thickness – the process begins with PCB material with thinner cladding, which is then electroplated with copper (required to create plated-through holes and vias) to the final thickness. For example, a PCB with 1 ounce copper would begin life as 1/2 ounce clad board (or inner *core* layers, the thinner sheets that are stacked and laminated to make multilayer boards), plated up to 1 ounce copper thickness. In some PCB processes this might be covered with a thin tin plating.

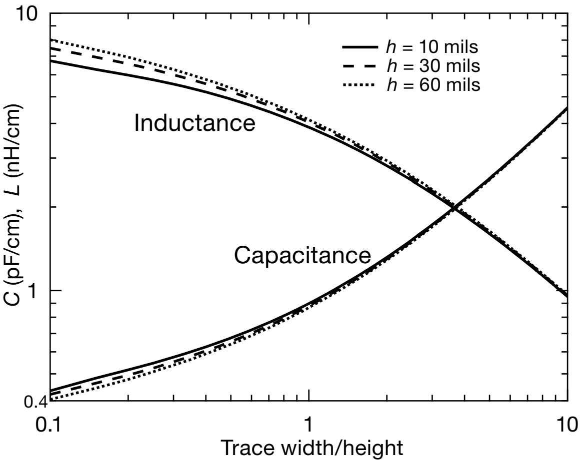

Figure 1x.3. Capacitance and inductance of printed circuit traces, as a function of the ratio of trace width to substrate thickness. These calculated values assume $\varepsilon=4.5$, typical of fiberglass-epoxy FR-4 PCB material.

swallow all signals); and (d) a transmission line so "terminated" at the far end looks purely resistive, with input impedance R_0 – its capacitance and inductance disappear entirely!

As seen in Appendix H, common transmission line configurations are "single-ended" coaxial cable (the familiar black RG-58 with BNC connectors that litter all laboratories), or "differential" twisted pair (the ubiquitous "cat-5" Ethernet used for computer networks). These are the standard forms of cables used to transport fast signals. But you sometimes have fast signals on printed circuit boards, for which you need to make a transmission line from PCB traces. There are two basic forms – *microstrip* (a top-layer trace or pair over an underlying ground plane, with a variant called *coplanar waveguide*), and *stripline* (an internal trace or pair, sandwiched between ground planes) – and either can be single-ended or differential; see Figure 1x.4.

The impedance of single-ended microstrip (a trace above a ground plane) is mostly a function of the ratio of trace width to height, and dielectric constant, and depends only weakly on the trace width itself (Fig. 1x.5); for the usual 50 Ω impedance on standard FR-4 circuit board, you want a trace width approximately 1.7 times the underlying insulation thickness. For a symmetrical stripline sandwich the trace is thinner, roughly 0.7 of the insulation thickness it sees on both sides.

For *differential* signals, which are popular for very fast signals, you use a pair of traces, either side-by-side (for microstrip, or "edge-coupled" stripline), or one above the other ("broadside-coupled" stripline). The impedance now depends on width, height, and conductor spacing. For differential transmission line the standard impedance is 100 Ω; some suitable PCB trace dimensions are given in Table 1x.2.

Contemporary digital electronics deals in fast signals (risetimes of a nanosecond or less) and wide bandwidths

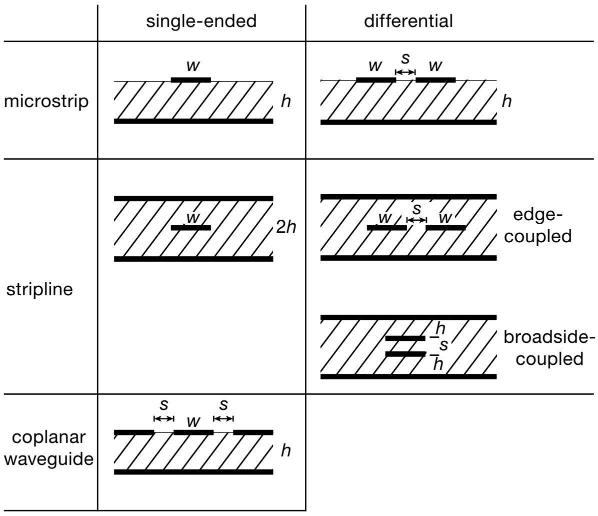

Figure 1x.4. PCB transmission line geometries, for both single-ended and differential signals. Microstrip traces are on an outer layer, whereas Stripline traces are buried.

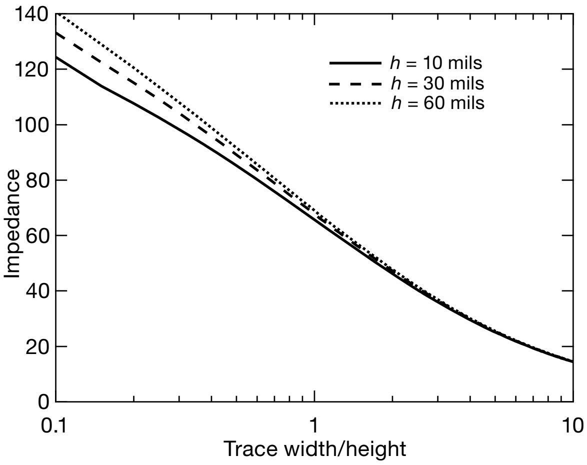

Figure 1x.5. Transmission line impedance of single-ended PCB microstrip, as a function of the ratio of trace width to substrate thickness. These calculated values assume $\varepsilon = 4.5$, typical of fiberglass-epoxy FR-4 PCB material. It's usually best to ask the board house which width to use for a 50 Ω (or 100 Ω pair, etc.) line, because FR-4 varies in dielectric constant, and they know what brand they're using.

Table 1x.2: Selected PCB Transmission Lines[a]

	h	*w*	*s*
	mils	mils	mils
50Ω single-ended			
microstrip	6	10	-
	8	14	-
	10	18	-
	30	55	-
stripline	5	3.5	-
	10	8	-
	15	13	-
100Ω differential			
microstrip	5	8	30
	5	7	11
	5	6	7
	5	5	5
stripline: edge-coupled	7	5	13
	10	6	8.5
	15	6	6
stripline: over/under	5	3.5	10
	7	5	15
	10	9	25

(a) FR-4, $\varepsilon = 4.3$

Dimensions for microstrip and stripline transmission lines, for the standard impedances of 50 Ω (single-ended) and 100 Ω (differential). See Fig. 1x.4 for symbols and geometries.

(to a gigahertz or more), for which even a short length of conductor on a circuit board must be treated as a transmission line, thus requiring proper termination (discussed extensively in Appendix H). It is also frequently the case that timing *skew* (the difference in propagation times) of multiple signals must be kept below very tight limits, as little as 25 picoseconds. Owing to the underlying dielectric, signals propagate along PCB traces at roughly half the speed of light, that is, 60 ps to 70 ps per centimeter. To keep skew less than 25 ps, then, the PCB trace lengths must be equalized to better than 4 mm. Even tighter constraints apply to the two traces of a clocking differential pair – for example, "DDR" memory recommends that the traces be matched in length to 0.5 mm![10] Figure 1x.6 shows an example: a set of single-ended data lines, plus a differential clock line pair, driving a memory chip.[11]

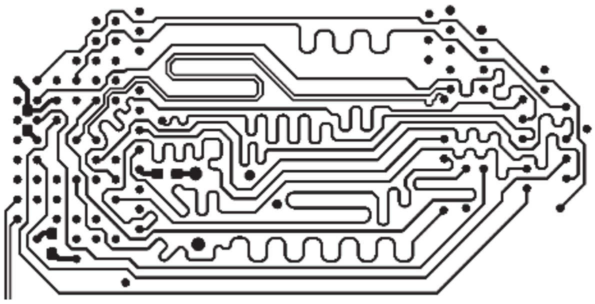

Figure 1x.6. The meandering traces on this portion of a circuit board from our lab are needed to equalize the propagation times of data (single traces) and clock (differential pair near top) going between an FPGA (left) and a DRAM (right).

1x.1.5 Cable configurations

If there's more than one wire, you call it a *cable*. There are many choices here: ***Coaxial*** cable ("coax") has an inner (usually stranded) conductor, with an outer shield; these are designed as transmission lines, with controlled impedance (see Appendix H on Transmission Lines), and usually designated with an "RG" type number.[12] At low frequencies you can think of coax simply as a shielded cable, with approximately 30 pF/ft (1 pF/cm) of capacitance. ***Multiwire*** cable (a variety of ***instrumentation cable***) is a bundle of strands, color-coded so you can untangle them at the other end; they may be individual wires, ***multipair*** (multiple pairs), or multiple triads. A multiwire cable may be ***unshielded***, or ***overall shielded*** (with a foil shield, a braided shield, or both). Multipair cables are also available with the pairs ***individually shielded***. Multiwire cables generally are not intended as transmission lines; however, certain multipair cables are designed for high-speed data transmission, with controlled impedance. The most common is "cat-5" (or "cat-5e" or "cat-6") 100 Ω (differential) impedance cable, with 4 twisted pairs, used universally for local-area networks (LANs); it's often called "UTP" (unshielded twisted pair) network cable (and shielded twisted pair cable is called "STP").[13]

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[10] See for example "Hardware Tips for Point-to-Point System Design: Termination, Layout, and Routing," Technical Note TN-46-14, Micron Technology, Inc.

[11] PCB propagation is faster on surface traces than on inside layers, because the effective "index of refraction" is $\sqrt{(1+\varepsilon)/2}$ (∼1.68), versus $\sqrt{\varepsilon}$ (∼2.1).

[12] For example, RG-58/U is the ubiquitous 50 Ω cable used for BNC patch cables, and RG-174 and RG-316 are skinnier variants, the latter with teflon insulation. Although 50 Ω is the by far the dominant cable impedance, the *video* world has chosen to be different, and uses 75 Ω cable, usually RG-59 or RG-6.

[13] Other popular high-speed data cables are SATA and Firewire (2 individually shielded twisted pairs, 100 Ω and 110 Ω differential impedance, respectively), and USB 2.0 (shielded pair, 90 Ω).

Flat cable (also called ***ribbon cable***) comes in the familiar unshielded ribbon with 50 mil pitch; variants include ribbon with an integral flexible ***ground plane*** on one side, or as ***fully shielded*** flat ribbon. For better signal integrity with differential signals you can get a ribbon of twisted pairs, flattened for 5 cm every 50 cm to allow for the same mass-termination insulation-displacement ("IDC") connectors used for conventional ribbon cable. For single-ended signals you can get a flat ribbon of individual coaxial cables, or a less expensive "near-coax" approximation in which the continuous outer shield does not completely surround the individual strands.

1x.1.6 Inductance and skin effect

A. Inductance

In addition to resistance, *any length of wire has inductance*. That must be so because a current creates a surrounding magnetic field, with its associated field energy; equating that energy with $\frac{1}{2}LI^2$ is often the easiest way to calculate wire inductance. The result is that the inductance is approximately proportional to wire length, and further depends somewhat on spacing to nearby conductors, on wire diameter, and on frequency (via skin effect). For an isolated straight wire of diameter d and length l, the low-frequency inductance is given approximately by

$$L/l \approx 2\left(\log_e \frac{4l}{d} - 0.75 + \frac{d}{l}\right) \quad \text{nH/cm}.$$

So, for example, a 10 cm length of 20-gauge wire ($d = 0.081$ cm) has an inductance of approximately 10.9 nH/cm, when distant from other conductors. The inductance is less for a conductor in proximity to a conductive surface (a "ground plane"), or to other wires; the relevant expressions are

$$L/l \approx 2\left(\log_e \frac{4h}{d}\right) \quad \text{nH/cm} \quad \text{(wire over plane)}$$

$$L/l \approx 2\left(\log_e \frac{2\pi h}{w}\right) \quad \text{nH/cm} \quad \text{(flat strip over plane)}$$

$$L/l \approx 4\left(\log_e \frac{2D}{d} - \frac{D}{l}\right) \quad \text{nH/cm} \quad \text{(wire pair)},$$

where h is the height of the conductor's center above the conducting plane (reasonably accurate for $h > 1.5d$ or $h > w$), D is the center-to-center wire spacing, and w is the width of a flat conductor (such as a PCB trace).[14]

A twisted wire pair has slightly less inductance than a parallel pair[15] (and it has considerably less susceptibility to signal pickup via magnetic coupling); you can get a much larger reduction in inductance by creating a twisted pair from a twisted four-wire cable, with diagonal wires tied together.[16] Figure 1x.7 shows inductances for several configurations.

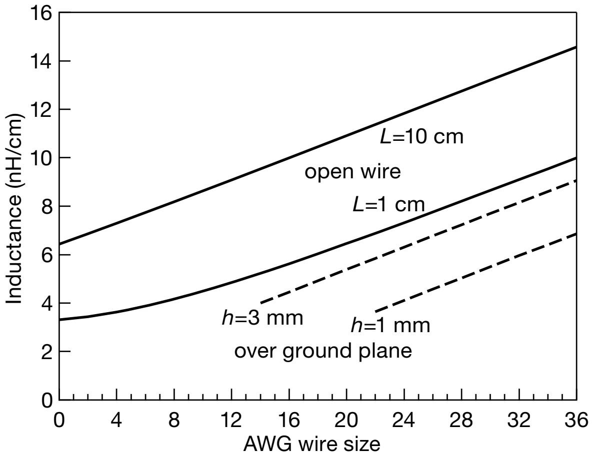

Figure 1x.7. Inductance per centimeter versus wire size for a wire over a ground plane (1 mm and 3 mm heights), and for a wire in isolation (1 cm and 10 cm lengths).

B. Skin effect

When alternating current flows through a conductor, the current is not uniform throughout the bulk – it is concentrated in an outer layer[17] (called the *skin depth*) of thickness $\delta \approx (\pi\mu\sigma f)^{-\frac{1}{2}}$, where σ is the conductivity, μ is the relative magnetic permeability (=1 for non-magnetic materials), and f is the frequency.[18] From a circuit point of view, skin effect causes an increase in the effective resistance (and loss), and a decrease of inductance (because the

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[14] There are excellent references on wire inductance and its effects, notably the "Black Magic" volumes by Johnson and Graham, Grover's *Inductance Calculations*, Terman's *Radio Engineers' Handbook*, and Ott's *Noise Reduction Techniques in Electronic Systems*.

[15] The D/l term is omitted in the equation above.

[16] We used this trick in the "stopped-light" experiments, where we had to limit the inductance of an 875 A cable to quickly kill the current, allowing trapped sodium ions to fall under gravity, watching the spread to measure their temperature.

[17] To be precise, the current density decreases exponentially, falling to $1/e$ (37%) of its surface value at a depth equal to δ.

[18] If you are interested in the physical origin of skin effect, you can think of it in either of two ways: (i) as the penetration depth of an electromagnetic wave incident on the finite-conductivity metal; or (ii) as resulting from the outward force on the moving charges caused by their motion through the alternating magnetic field they produce.