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### Syntax and Semantics of Petri Nets

Petri nets are one of the most popular tools for modeling distributed systems. This book provides a modern look at the theory behind them, by studying three classes of nets that model (i) sequential systems, (ii) non-communicating parallel systems and (iii) communicating parallel systems. A decidable and causality-respecting behavioral equivalence is presented for each class, followed by a modal logic characterization for each equivalence. The author then introduces a suitable process algebra for the corresponding class of nets and proves that the behavioral equivalence proposed for each class is a congruence for the operator of the corresponding process algebra. Finally, an axiomatization of the behavioral congruence is proposed. The theory is introduced step by step, with clear explanations and examples provided throughout, to remain accessible to readers without specialized training in concurrency theory or formal logic. Exercises with solutions solidify understanding, and the final chapter hints at extensions of the theory.

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# Syntax and Semantics of Petri Nets

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## Preface

The aim of the book is to contribute to the theory of Petri nets, a popular tool for modeling distributed systems, by studying three classes of nets that model (i) sequential systems, (ii) non-communicating parallel systems and (iii) communicating parallel systems.

For each class of nets, a suitable behavioral equivalence is presented that satisfies the following criteria: (a) resource sensitive (meaning that it relates global states of the same size only); (b) causality preserving (meaning that it captures correctly the causal dependencies among the performed events); (c) decidable (i.e., the equivalence checking problem can be solved algorithmically). Moreover, for each behavioral equivalence a suitable modal logic characterization is presented: Two nets are behaviorally equivalent if and only if they satisfy the same logical formulae.

Each of the three net classes is “alphabetized” by providing a simple process algebra that represents all (and only) those nets in that class, up to isomorphism. The behavioral equivalence proposed for each class of nets is proved to be a congruence for the operators of the corresponding process algebra. By combining these two results, a *causality-preserving compositional semantics* for each of the three net classes is achieved, a long-awaited result in concurrency theory. Finally, a suitable axiomatization is presented for each behavioral congruence that is sound and complete for the first two classes, and sound but not complete for the last one.

The theory in this book is presented in as simple a style as possible, even though the results described are rather profound and have been considered extremely difficult to achieve. Accordingly, the book can be used for a semester course on concurrency theory at graduate level and is also suitable for self-study, with many exercises supplied and solutions outlined in the Appendix.

## Acknowledgments

The goals of this book were inspired by E.-R. Olderog's seminal book *Nets, Terms and Formulas* [88], where, for the first time, a three-level view of distributed systems was proposed. At a very concrete level, Petri nets were used to “describe processes as concurrent and interacting machines” [88]. At a higher level, the terms of a suitable process algebra were used as “an abstract concurrent language” [88], the semantics being given in terms of Petri nets, which offered the means for describing and analyzing systems in a *compositional* manner. At a more abstract level, logical formulae could be used as “a specification language for processes” [88]. The main differences with respect to Olderog's book are that:

- (a) he considered only *safe* Petri nets (i.e., nets where each place can contain one token at most);
- (b) his proposed process algebra did not alphabetize the class of finite safe Petri nets (i.e., it is not possible to describe all the finite safe Petri nets with terms of his algebra);
- (c) the behavioral relation proposed by him was not an equivalence [3], and so not even a congruence, so that the goal of compositionality was not really achieved;
- (d) his proposed logic is a first-order predicate logic, from which he could automatically derive process terms, while my goal is less ambitious, as the logics I introduce are modal logics characterizing the behavioral equivalences of interest.

The work by Philippe Schnoebelen and co-workers about *place bisimulation* [3, 4] has inspired some parts of this book. In them I develop this idea, which unfortunately those authors soon abandoned, by adding decidability and compositionality results.



### *Acknowledgments*

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The work by Rob van Glabbeek in [39] was important for this book, as his *structure-preserving bisimulation* is a central issue in Chapter 4 and, moreover, it played a pivotal role in proving that the bisimulation-based behavioral equivalences I describe are all causality respecting.

I have to thank Ugo Montanari and Pierpaolo Degano, who introduced me to Petri nets. Their pioneering work (done jointly with Rocco De Nicola), relating Petri nets and process algebras, was inspirational for my earlier work [46], some of which is reused, in a revised and simplified form, in this book.

My wife Nicoletta and my daughters Lucia, Emma and Caterina are warmly thanked for their support during the writing of this book. Last, but not least, I would like to thank David Tranah and Anna Scriven for their constant support and valued assistance in publishing this book.