

Contents

Preface	xi
Acknowledgements	xv
Notation	xvii
List of Illustrations	xxi
PART ONE ORDINARY DIFFERENTIAL EQUATIONS	
1 First- and Second-Order ODE	
1.1 Introduction	3
1.2 Some Applications	8
1.2.1 The Envelope of a Family of Curves	9
1.2.2 Orthogonal and Isogonal Trajectories	11
1.3 Exercises	12
1.4 Solutions	20
2 Linear Systems	
2.1 Introduction	47
2.2 Some Applications	52
2.2.1 Kinematics	52
2.2.2 Controllability and Observability of Linear Systems	53
2.3 Exercises	58
2.4 Solutions	62
3 Series Solutions: Frobenius Theory	
3.1 Introduction	85
3.2 Real Analytic Functions	86
3.3 Equations with Analytic Coefficients	88
3.4 Regular Singular Points	91
3.4.1 Equations with Regular Singular Points	92
3.4.2 Singularity at Infinity	98
3.5 Exercises	100
3.6 Solutions	102
4 Regular Sturm–Liouville Theory and Boundary Value Problems	
4.1 Introduction	111
4.1.1 Boundary Value Problems	113
4.1.2 Existence and Uniqueness of Green’s Function	113
4.1.3 Green’s Function for the Adjoint Equation (Symmetry)	116
4.2 Prüfer Substitution	118

viii | Contents

4.3	Exercises	120
4.4	Solutions	122
5 Qualitative Theory		141
5.1	Introduction	141
5.1.1	Liapunov Stability, Liapunov Function	143
5.1.2	Linearization	144
5.1.3	Liapunov Function and Stability	145
5.1.4	Hamiltonian System	147
5.1.5	Two-Dimensional Potential Flows	149
5.2	Exercises	150
5.3	Solutions	154
PART TWO PARTIAL DIFFERENTIAL EQUATIONS		183
6 First-Order Partial Differential Equations		185
6.1	Method of Characteristics	185
6.2	Hamilton–Jacobi Equation	188
6.3	Conservation Laws	190
6.4	Exercises	193
6.5	Solutions	201
7 Classification of Partial Differential Equations		235
7.1	Introduction	235
7.2	Classification	237
7.3	Second-Order Linear Equations in Two Variables	240
7.3.1	Reduction to Canonical Form	240
7.4	Higher-Order Equations	242
7.5	Exercises	242
7.6	Solutions	243
8 Laplace and Poisson Equations		247
8.1	Introduction	247
8.2	Exercises	255
8.3	Solutions	262
9 Heat Equation		285
9.1	Homogeneous Equation: Fourier–Poisson Formula	285
9.1.1	Inhomogeneous Equation: Duhamel’s Principle	286
9.1.2	Heat Equation in a Finite Interval: Fourier Method	286
9.2	Exercises	287
9.3	Solutions	292
10 One-Dimensional Wave Equation		307
10.1	Homogeneous Equation: D’Alembert’s Formula	307
10.2	Domain of Dependence and Other Concepts	308
10.3	Discontinuous Initial Data: Propagation of Singularities	309
10.4	Inhomogeneous Equation: Duhamel’s Principle	310
10.5	Equation with Two Distinct Speeds	311

Contents | **ix**

10.6	Characteristic Parallelogram Property	312
10.7	Telegraph Equation	314
10.8	Exercises	314
10.9	Solutions	317
11	Wave Equation in Higher Dimensions	329
11.1	Introduction	329
11.1.1	Two-Dimensional Wave Equation: Method of Descent	331
11.1.2	Telegraph Equation	332
11.2	Exercises	336
11.3	Solutions	339
References		353
Index		357