

## Electromagnetism

### Volume 2

There are four forces in our universe. Two act only at the very smallest scales and one only at the very biggest. For everything inbetween, there is electromagnetism. The theory of electromagnetism is described by four gloriously simple and beautiful vector calculus equations known as the Maxwell equations. These are the first genuinely fundamental equations that we meet in our physics education and they survive, essentially unchanged, in our best modern theories of physics. They also serve as a blueprint for what subsequent laws of physics look like.

This textbook takes us on a tour of the Maxwell equations and their many solutions. It starts with the basics of electric and magnetic phenomena and explains how their unification results in waves that we call light. It then describes more advanced topics such as superconductors, monopoles, radiation, and electromagnetism in matter. The book concludes with a detailed review of the mathematics of vector calculus.

**David Tong** is a Professor of Theoretical Physics at the University of Cambridge and a Fellow of Trinity College. He is known for his contributions to quantum field theory and its application to diverse areas of physics, including particle physics, condensed matter, cosmology, and quantum gravity. His lecture notes on theoretical physics have gained a global following due to their clear explanations and easy-going, accessible style.

*Lectures on Theoretical Physics*

- Volume 1: Classical Mechanics
- Volume 2: Electromagnetism
- Volume 3: Quantum Mechanics
- Volume 4: Fluid Mechanics
- Volume 5: You may need to be patient

# Electromagnetism

## Lectures on Theoretical Physics, Volume 2

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DAVID TONG  
University of Cambridge





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*For Dad*

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