978-1-009-57489-1 — Philosophy of Mathematics after Wittgenstein
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Excerpt
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Introduction
A Disappearing Act

In *Philosophical Investigations* §133, Wittgenstein writes that his aim is to make philosophical problems "*completely* disappear." Although mathematics is not explicitly mentioned in this remark, he very likely had in mind the philosophical problems about it too. Under this assumption,[1] my primary goal here is to probe the extent to which this aim is attainable. More precisely, my central task is to mount a qualified defense of his contention that it is possible to *dissolve*, and thereby *eliminate*, the *traditional* philosophical problems about mathematics.

This book differs from other reconstructions of Wittgenstein's views on mathematics by having a distinct agenda. While other available accounts typically aim to be comprehensive,[2] this work doesn't. It focuses on his later period and especially on the line of thought that substantiates a radical thesis: that philosophy of mathematics *after* Wittgenstein applied his methods to it appears devoid of its time-honored problems.

Obviously, such a claim may sound alarming to many contemporary philosophers of mathematics. It makes Wittgenstein look like a threat, especially since he is genuinely committed to this kind of 'destructive' or 'eliminativist' project. (The theme of dissolution is not a sporadic occurrence in his thinking, but a central motif: it resurfaces in the post-tractarian

[1] This is plausible given his lifelong interest in mathematics, and also in light of von Wright's (1982a, 132) recollection that "Wittgenstein in 1945 [was] contemplating a second volume dealing with the philosophy of logic and mathematics . . . This volume would presumably have embodied a new version of the second half of the early version of the *Investigations*, with additions from his many writings on those topics during the period 1937–44." (See also Pichler 2023, 48 and McGuinness 2002, 283–286.) Indeed, in the 1945 *Preface* to PI (Ts-243) Wittgenstein mentions the foundations of mathematics as one of the book's topics.

[2] Book-length treatments include Kielkopf (1970), Klenk (1976), Wright (1980), Shanker (1987), Frascolla (1994), Marion (1998), Berts and Solin (2010), Clark (2017), Schroeder (2021), Floyd (2021), and Scheppers (2023). Other helpful discussions have appeared in articles and book chapters (see the References).

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period,[3] after playing a prominent role earlier in the *Tractatus*.)[4] In fact, a professional philosopher of mathematics may even hope that such a project will *not* succeed. Who would be willing to entertain the thought, let alone accept it, that they are "under a spell" (Ms-158, 37 r) when trying to solve these problems? That these conundrums, on whose discussion one builds one's entire career, are illusory?[5]

Yet note that his very contention, admittedly extreme and apparently 'antiphilosophical,' is, after all, a *philosophical* one. As such, it is bound to be open to *further* philosophical examination – something that Wittgenstein himself actually envisaged.[6] Hence, even if one realizes the magnitude of the threat and becomes worried, one can rest assured that the game is far from over, so to speak. Many challenging questions *can* and *should* be asked about his ambitious 'dissolution solution' of these problems. Moreover, mathematics is a dynamic discipline, and new philosophical puzzles (about its concepts and proofs) will always appear on the horizon.[7]

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As I noted, several reconstructions of Wittgenstein's thinking on mathematics have been carried out in the past. But I should stress that although the present attempt has benefited from these enlightening works, it does not pursue the same objectives. Most importantly, I relinquished any ambition to provide a comprehensive account of his (early and late) thinking on mathematics. Virtually every analysis in the book is in the service of my primary goal, to support the eliminativist thesis stated at the outset. The aim was simply to understand *why* the central problems in this field vanish: that is, to *show* the readers *how* – rather than merely *tell* them *that* – these

[3] In PG (§9, p. 47) he notes that "while thinking philosophically we see problems in places where there are none. It is for philosophy to show that there are no problems."
[4] See propositions 4.003(3), "And it is not surprising that the deepest problems are in fact not problems at all," and 6.5(2), "The riddle does not exist." Note, however, that the dissolution idea in the later works is an echo of, but not identical to, what we find in 1921–1922 in TLP. Unfortunately, I don't have space to explore the differences between them, and I will focus on the later work.
[5] Wittgenstein's scorn for 'academic' philosophers is (in)famous; ironically, this category of people (including the present author) keeps his legacy alive.
[6] Rhees reports Wittgenstein's confession that "In my book I say that I am able to leave off with a problem when I want to. But that is a lie; I can't." In Hallett (1977, 230).
[7] Examples of such puzzles include the status of computer proofs (Tymoczko 1979, Avigad 2008), and of visual, nondeductive, and probabilistic proofs (Giaquinto 2007, Baker 2009, Paseau 2015). One can also mention studies of mathematical practices and issues about mathematical explanation (e.g., Mancosu 2008; Mancosu, Poggiolesi, and Pincock 2023), as well as the topic of mathematical cognition (e.g., Bangu 2018b, Schlimm and Vold 2020). Unfortunately, I will not have space to discuss any of these here.

puzzles (should) dissolve. (This is of course a nod to the famous advice 'Show, don't tell' meant to describe a writing technique, often attributed to Chekhov and Hemingway. It also registers my hope that the secondary literature will begin to take Wittgenstein's eliminativist idea more seriously – in general, not only about mathematics.)[8]

However, demonstrating that this eliminativism is a viable philosophical option depends on first being able to extract it, and articulate it, from his often-opaque remarks. So, the question to ask right at the beginning is whether there is enough substance in his later works to fuel this project. In this regard, reservations have been expressed in the past. For instance, the central work we'll discuss here, the posthumously published RFM collection, has been described by the eminent logician Georg Kreisel as a "surprisingly insignificant product of a sparkling mind" (1958, 158).

I must admit that such harsh judgments are understandable at first glance. Even expert interpreters of Wittgenstein's other works often wander astray in the labyrinth of his reflections on mathematics. Quite frequently we are not told what to believe, only what we should *not* believe. Attempts to identify clearly delineated arguments running through the remarks are generally inconclusive. Out of the many questions the reader is presented with, only a few are answered. Just as in the PI, there are sometimes distinctive characters ('voices') speaking, and it is a difficult task to tell which one to listen to (if any). In addition, another possible issue is the different status of the works he left behind – some are writings (either in raw manuscript form, or as typescripts), others are notes taken by people in his audiences. Wittgenstein's literary executors – G. E. M. Anscombe, R. Rhees, and G. von Wright – tell us that only two parts out of the seven that they put together as RFM (i.e., parts I and VI) are "complete reproductions of texts of Wittgenstein's" (RFM, "Editors' Preface to the Revised Edition," p. 32),[9] and consequently should be regarded as more authoritative than others.

The main worry, however, is about the very substance of what we inherited, regardless of how trustworthy we take the source to be. Such

[8] Commentators typically refrain from doing so. To single out a prominent one, Hacker's comprehensive *Insight and Illusion* only *tells* the reader what Wittgenstein envisaged – e.g., "that these pseudo-theories are quasi-mythological constructions . . . do not stand in need of either refutation or confirmation, but of dissolution" (1986, 175). As far as I can see, this remarkable book does not in general engage in a systematic and detailed attempt to *show* how this dissolution proceeds. More recently, however, Schroeder (2006, 154–168) took steps in this direction.

[9] RFM I is mainly based on materials in Ms-117–119, and Ms-115 and has a special status, since Wittgenstein made typescripts of them (Ts-221a, 222–224). As noted, it was meant to be the second half of an earlier version of the PI. RFM VI is based on Ms-164.

a concern is, I suspect, shared by many. It has been sharply summarized by Michael Potter:

> We know what he was against: Platonism, logicism, intuitionism, and Hilbertian formalism all at various times come in for his criticism. But what, by contrast, was he for?
>
> . . . [I]t is hard to see what he had to offer instead. Indeed, one struggles to present, even in outline, a positive account of mathematics that can reasonably be called Wittgensteinean. Too often Wittgenstein seems more concerned with offering meta-level advice about how to go about finding a correct account, rather than with developing the account itself. (2011, 136)

These are real difficulties – and my *show, don't tell* approach is meant to tackle them head-on. I believe that we should not give up yet; we can be optimistic about answering Potter's question. As I hope to demonstrate, Wittgenstein's later works on mathematics, when read charitably, do contain an important philosophical fixed point, that is, a reasonably coherent eliminativist vision.

My confidence is also bolstered by the fact that Potter himself lists two elements to be incorporated into such a reading – "that numbers are not objects, and that arithmetical equations are not tautologies" (2011, 136) – ideas that Wittgenstein retained from his early and middle periods. These views (which, indeed, are stated in the 'negative' mode) will find a place in my reconstruction. But the reader should be warned, once again, that his contribution will *not* appear in the form of a comprehensive theory, as is typical in the Western philosophical tradition.

So, *what was Wittgenstein for*? A full answer will hopefully emerge in due course; a condensed one is as suggested: he designed, though not always explicitly pursued, a series of strategies to dissolve, and thus eliminate, the *traditional* problems in the philosophy of mathematics.[10] He aimed to expose them as pseudoproblems, as ill-posed questions, which appear as genuine difficulties only because the traditional schools uncritically accepted certain presuppositions regarding the working of language. The examination of these deeply entrenched assumptions consists of doing work of a descriptive, and often humdrum, nature. This enterprise, which he calls a 'grammatical' investigation (PI §90), amounts to 'looking'

[10] My approach is complementary to that of Floyd and Mühlhölzer (2020, x). They "are especially concerned to deny that [Wittgenstein] was aiming to put an end to philosophy as such, dogmatically, on his own." I agree that putting an end to *traditional* philosophy was not to be done dogmatically; as I detail here, an entire argumentative project can be reconstructed to this effect. On the other hand, I do believe that he envisaged such an end.

(at language) and 'seeing' (how it works) (PI §66). This is a process through which we are reminded of all-too-familiar linguistic facts and conceptual distinctions.

When it comes to applying this kind of investigation to mathematics, the result is a unique perspective on the relation between these venerable intellectual fields: philosophy no longer appears as a provider of explanatory theories aimed at solving philosophical problems raised by mathematics, but is a 'grammatical' endeavor aimed at achieving "*complete* clarity" (PI §133) about what generates these problems in the first place. And, once this clarity is obtained, the result is the dissolution of these problems. We are liberated from them upon the realization that they are somehow unreal: "You are under the misapprehension that the philosophical problem is *difficult*, whereas it's hopeless" (Ms-158, 37 r; in English in original). The 'therapeutical' intention of this exercise is obvious, and I shall emphasize it throughout the book. Just as the goal of the doctor is not to be needed, for Wittgenstein the goal of philosophizing is reached when these problems are eliminated: "Thoughts that are at peace. That's what someone who philosophizes yearns for" (CV 43^e (1944)).[11]

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Let me acknowledge five methodological commitments that guided the work on this book. First of all, what follows is an attempt to articulate Wittgenstein's position on mathematics post-1936,[12] and to do so by relying on substantial textual support. Yet I will sometimes go beyond the letter of the text. I will not suppress my opinions about what it seems to me that Wittgenstein *would* have said, or, more contentiously, about what he *should* have said. These elaborations are not motivated by the (laughably arrogant) intention to somehow 'improve' his work; I will only assume that some of his terse, often cryptic, remarks become clearer when rephrased. Hence, I will labor under the presumption that he *could* have said what he (seems to me to have) said in a more transparent – though admittedly less memorable – manner. In addition, I believe that his insights can be better

[11] This remark was written in 1944; but, as I have already noted, similar pronouncements are ubiquitous in his work, spanning his entire career. In addition to what he says in TLP, one more sample is in PG (from the early 1930s), where he notes that "philosophical problems are misunderstandings which must be removed by clarification of the rules according to which we are inclined to use words" (PG §32, p. 68).

[12] The year 1936 is typically considered the beginning of the later, post-transitional period, coinciding with Wittgenstein's stay in Skjolden, Norway; see Conant (2011). Pichler (2018) recently discusses the standard tripartite periodization (early, middle/transitional, late); see, however, Gerrard (2002) for arguments against it.

conveyed when reshaped into a more linear train of thought, and when illustrated by more examples from the history and practice of mathematics.

Nevertheless, staying close to the texts was very important, and I will often expound on individual remarks, or parts of them. However, what follows is not primarily an exegetical exercise, nor a detailed, line-by-line, commentary of any portion of his work (on mathematics).[13] Even if I had tried to do this, I still could not offer any guarantee that Wittgenstein *really* held the views I attribute to him here – so, in this book 'Wittgenstein' always stands for 'Wittgenstein as I understand him.' But I do entertain the hope that my comprehension of the texts is correct[14] and, as far as I can make sense of his key ideas, they seem to me essentially right. Moreover, I believe that these insights are highly original and constitute a major achievement in the history of thought.[15] Yet, once again, I take my praising of his work to be compatible with occasionally voicing frustration with the "cryptic, fragmentary, and unfinished" (Anderson 1958, 446) presentation of his ideas. So, they need to be reconstructed properly – and, once this is done, I submit that they pose a formidable challenge to the conceptions of mathematics advanced by the traditional philosophical schools. This is a challenge that, I'm afraid, has barely begun to be recognized now, seventy-five years after his death. With very few (but notable) exceptions, contemporary philosophers of mathematics proceed as if Wittgenstein never addressed the subject.

Second, I will be very discerning about which remarks I include in my discussion. This is so not only because the argument of the book is meant to hit a well-circumscribed target, but also because I generally agree with the commentators who worry that the materials Wittgenstein left behind are of variable quality. In fact, he himself noted that not all remarks he jotted down are equally important – in a suggestive simile: "Of the sentences that I write down here, only the occasional one represents a step forward; the others are like the snip of the barber's scissors, which must be kept in motion so as to make a cut with them at the right moment" (CV 66[e] (1948)).

Indeed, it is quite clear that many of the entries in the manuscripts are such 'snips.' Hence, my strategy was to identify the 'cuts' and reconstruct

13 See Mühlhölzer (2010) for a commentary on RFM III.
14 It is of course possible that I misattribute views to him. Then I will have to accept the ownership of these views and redescribe the book as an attempt to articulate a Wittgenstein-inspired position.
15 This is to agree with his self-assessment that his "*chief* contribution was in the philosophy of mathematics," reported in Monk (1990, 466).

his eliminativism from them.[16] Thus, not everything one finds in his later works will be, or even *can* be, accommodated by the present account. I must confess that I sometimes encountered lines whose role in the eliminativist project escaped me. More generally, in some remarks, to put it bluntly, I had no idea what he was talking about; so, I left them out of discussion altogether.

A third feature of my approach is that I read Wittgenstein's oeuvre with the mindset of a contemporary philosopher of mathematics, that is, someone still struggling with these problems, and willing to understand how he addressed them (if at all). This is in direct contrast with other approaches, for example Fogelin's, who admits that "one thing I have *not* tried to show is that the insights found in Wittgenstein's later philosophy can be integrated with some of the standard forms of philosophizing that flourish on the contemporary scene" (2009, 167).

A fourth methodological maxim is that I will avoid tackling questions about the trajectory of Wittgenstein's thinking over time – with one exception. Understanding how his early, middle, and late 'phases' relate to each other is an important exegetical task which, fortunately, is already well accomplished.[17] I won't have much to say on this here, the only exception being Wittgenstein's attitude toward Cantorian transfinite set theory. Based mostly on my perusal of the *Nachlass*, I shall suggest that his attitude changed from the PR and PG of the early 1930s to the end of that decade, when he delivers LFM in Cambridge in 1939, change that continued well into the 1940s. As we'll see in Chapters 6 and 7, he begins as an unabashed revisionist, that is, as someone who would like to 'correct,' or revise mathematics, but ends up as a nonrevisionist (of sorts). In the early 1930s he makes harsh pronouncements against set theory; he claims, for instance, that it "builds on . . . nonsense" (PR §174), and that the textbook proposition "'between the everywhere dense rational points, there is still room for the irrationals'" is "balderdash" (PG §40, p. 460; see also §41–43). Yet, as time passes, it seems to me that he gradually moves away from this combative stance toward a more nuanced position, reaching a point where he becomes what I shall call a *reluctant* nonrevisionist. He is

16 Another useful distinction, due to Pichler, is between thoughts while 'in chemical reaction' versus thoughts present in the 'precipitate.' The first are found only in the remarks in handwritten form, whereas the second category have often also been typewritten. This must not be understood to mean that everything in the typescripts was to be kept (they are sometimes heavily edited too!), or that some of the handwritten remarks couldn't also be 'precipitate.' Pichler's distinction relates roughly to the distinction between 'writings' and 'works' in Wittgenstein. (See Pichler 2023.)

17 See, e.g., Gerrard (1996), and most recently Schroeder (2021) and Floyd (2021).

a nonrevisionist insofar as he explicitly bars himself from "interfer[ing] with the mathematicians" (LFM 13) and from attempting to "prove that calculations are wrong" (RFM II-62). Nevertheless, on closer inspection, it turns out that his new stance still leaves room for *some* interference to happen (as RFM II, written in 1937–1938, reveals), albeit in an *indirect* way. In my reading, he ultimately embraces nonrevisionism, but half-heartedly.[18]

Finally, the fifth methodological constraint here is related to the previous point and to the first constraint. While, as I said, my discussion relies for the most part on works that Wittgenstein produced after 1936, this investigation of his (non)revisionism will require considering remarks composed from 1929 to circa 1935, or in what has become known as his 'middle' or 'transitional' period (such as the remarks collected in above-mentioned PG and PR). However, among the remarks from this period I plan to invoke only those relevant to his post-1936 conceptions. When exegetical disputes on this relevance arise, I will take sides in them – but only if I judge that this helps me reach the goals of the book: to understand why Wittgenstein believed that he could dissolve the traditional problems in the philosophy of mathematics, and also to show that there are good reasons to think that he was by and large right to believe so.

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To paraphrase PG §141 ("Philosophy is philosophical problems"), philosophy of mathematics is philosophical problems about mathematics. It is the existence of a battery of such venerable puzzles that led to the emergence and development of various philosophies of mathematics: platonism,[19] (anti)realism, logicism, empiricism, conventionalism, intuitionism, formalism-finitism, structuralism, and so on. Consequently, an important issue to address in the book is the relation between these traditional doctrines and Wittgenstein's own conception. The unmistakable overall impression is that he was dissatisfied with how the subject was

[18] The debate on Wittgenstein's (non)revisionism involved Rodych (2000), who argued that Wittgenstein was a revisionist, and Maddy (1986; 1997) and Floyd (1995), whom he takes to hold the opposite view. As noted, here I understand Wittgenstein's post-1936 position as a form of nonrevisionism. Given my focus on this period, I won't regard the evidence for revisionism in the passages that Rodych found in the PG (especially those from 1931–1933) and PR (from 1929–1930) as decisive. But the 1937–1938 remarks in RFM II will have to be discussed in detail (in Chapter 7).

[19] This doctrine comes in two versions. The first, chronologically speaking, is associated with Plato, and was espoused by illustrious figures such as the mathematician G. H. Hardy and the logician Gödel (whether Plato himself would have accepted it is unclear). The second version is a more recent naturalist version, due mainly to Quine. My focus will initially be on the first version, known to Wittgenstein. In Chapter 2, I will look closely at the second version as well.

treated in the past (and by his contemporaries). For the moment, a simplified and pictorial way of conveying this attitude would be the following: these *isms* populate a two-dimensional horizontal conceptual plane, unlike his own eliminativist 'normativist' position (as I'll call it, for reasons to be soon clarified), which does *not* belong to this plane. His view is developed into a third, metadimension, so to speak; for him, to elaborate a philosophy of mathematics is to do something markedly different from what these doctrines have done – namely, to provide *explanatory theories*, meant to *solve* the philosophical puzzles raised by mathematics. He did not intend to pursue yet another explanatory project, but carried out a (meta-)eliminative-therapeutic project instead. His overall approach is exactly the opposite of the traditional approaches, in that he "do[es] away with all *explanation*" (PI §109), by showing that there is no *need* for it; as PI §126 boldly contends, "there is nothing to explain."

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The book is divided into ten chapters, including the present Introduction and the Conclusion; each chapter contains several sections. The chapters are not meant to be read in isolation, or in a random order; no chapter (let alone section) exhausts an issue and the motivation for taking up that issue is often found in previous chapters. As I said, I did my best to present a linear narrative, but sometimes returning to the same difficulty in later chapters is unavoidable with such a topic (and such an author). Here is a rough guide to how the material is distributed in chapters and sections.

Chapter 1 is mainly devoted to articulating the *eliminativist normativist* view that can, and should, be attributed to Wittgenstein. The second part of the label originates in the extremely important remark that "the sentences of mathematics . . . are *normative* sentences. And this characterizes their use" (Ms-123, 49 v; emphasis in the original).[20] This conveys what I take to be Wittgenstein's fundamental insight: that true declarative mathematical sentences primarily serve (function) to formulate norms, or rules. Understanding what this means is crucial in what follows.

The first chapter signals that the eliminativist aspect of normativism has therapeutical consequences: by eliminating problems, one achieves 'peace of mind.' This is a reading that, I submit, puts us on the right path when it comes to understanding his later approach to mathematics. Nonetheless, as far as I can tell, this reading has never been pursued systematically,

[20] From now on, when the words are emphasized in the original text, I will not draw attention to this.

although it renders Wittgenstein's take on mathematics a natural complement of his ideas espoused in the *Philosophical Investigations* – a work whose *main* aim is, in my view, eliminative-therapeutical too. Indeed, here I intend to present an account of Wittgenstein's normativist eliminativism in harmony with PI.[21] After all, PI is especially authoritative since it contains a large chunk of text (part I) that he came *very* close to publishing – with Cambridge University Press. In particular, my attribution of non-revisionism to him in the later chapters has been decisively influenced by what PI tells us – that philosophy should leave mathematics "as it is" (§124), a stance that I took to be definitive, and to supersede any revisionist-sounding pronouncements, especially those made in RFM part II.

The second chapter further articulates the normativist view, but its central task is to demonstrate its effectiveness. It illustrates the eliminativist aspect of normativism, by showing how it dissolves three central problems in the philosophy of mathematics. Section 2.1 deals with the problem of the specialness of mathematical propositions like '2 + 2 = 4.' Traditionally, they are said to be *necessarily* true (so *more* than merely true) and a priori (no empirical information is needed to assess their truth-value). In a word, they are *unfalsifiable*, incorrigible, so significantly *unlike* other propositions. I first formulate this problem – roughly, as the task of *explaining* why such propositions are (or seem) special, while other propositions (e.g., 'Kampala is the capital of Uganda') are not so, but are only contingently true and a posteriori. I will explain how this problem arises and then I will show how it can be disposed of. In Section 2.3, I will be working out another dissolution, of the so-called Benacerraf problem. Yet before that (in Section 2.2) I take a detour and clarify the notion of a pseudoproblem, a central component of the eliminativist project. In Sections 2.4–2.6, I deal with a third problem, the traditional *ontological* question 'do mathematical objects exist?', and I explain how Wittgenstein's normativism eliminates it. There I tackle three realisms: traditional platonism, Fregean logicist realism, and Quinean realism, the latter based on considerations about the indispensable role of mathematics in science and everyday life.

The next two chapters form in some respects a unit. Chapter 3 imagines, on Wittgenstein's behalf, a possible process by which the *basic* arithmetical rules emerge; Chapter 4 looks into the status of such rules (are they *unique*? Can there be *alternative* arithmetics?). Thus, in Chapter 3 I return to a series of issues raised at the end of Chapter 1. The central one is about

21 I agree with Kuusela (2008, 14) that any reading of a work by later Wittgenstein must take PI as its guide. And indeed, this could count as another methodological commitment.