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Introduction

Groups St Andrews 2022 was the eleventh conference of the Groups St Andrews series. It was originally planned as “Groups St Andrews 2021” and scheduled for August 2021 at Newcastle University in the United Kingdom. In common with everyone on the planet, the organizers had to revise their plans due to the global COVID pandemic and so the conference actually took place in 2022. By this point, some semblance of normality had returned and academics were able to travel to attend conferences again and sit in lecture theatres together learning mathematics. Even so, a number of colleagues unfortunately still dropped out at short notice due to contracting COVID just before they intended to travel to Newcastle. Despite these disruptions, there were 70 mathematicians able to attend the meeting. The members of the Organising Committee of Groups St Andrews 2022 were Colin Campbell, Martyn Quick, Edmund Robertson and Colva Roney-Dougal (all from St Andrews) and David Stewart (Newcastle).

The academic business of the conference ran for seven days from Sunday 31st July to Saturday 6th August. Four main speakers delivered four talks each, surveying areas of contemporary development in group theory and related areas: Michel Brion (Institut Fourier, Université Grenoble Alpes), Fanny Kassel (Institut des Hautes Études Scientifiques), Denis Osin (Vanderbilt University) and Pham Huu Tiep (Rutgers University). There were also invited speakers delivering one-hour plenary talks: Miklos Abert (Alfréd Rényi Institute of Mathematics), Bettina Eick (Technische Universität Braunschweig), Scott Harper (University of Bristol), Julia Pevtsova (University of Washington) and Simon Smith (University of Lincoln). In addition, there were 40 contributed short talks, from which Alexander Hulpke (Colorado State University) and Péter Pál Pálffy

(Alfréd Rényi Institute of Mathematics) were invited to give one-hour versions.

A lively social programme ran through the conference. A wine reception was held on the Monday evening shortly after the final lecture of the day and the conference dinner took place on the Tuesday evening. There was no academic business on the afternoon of Wednesday 3rd August and many delegates took advantage of the conference excursion to Hadrian's Wall. They had the opportunity to walk along the Wall and to explore some of the Roman Forts along it. As with previous Groups St Andrews conferences, **The Daily Group Theorist** ran throughout, publishing each day's activities together with additional contributions from the delegates. We thank the various editors of this fine, and by now traditional, publication.

The Organisers are grateful to a number of organisations whose support ensured the success of the conference. Groups St Andrews 2022 was organised in partnership with the Clay Mathematics Institute. It was supported by the Heilbronn Institute for Mathematical Research (HIMR), the UKRI/EPSRC Additional Funding Programme for Mathematical Sciences, and by the London Mathematical Society and the Edinburgh Mathematical Society. As well as supporting the expenses of the invited speakers, the grants obtained were used to support postgraduate research students and participants from Scheme 5 countries.

These proceedings contain substantial surveys written by some of the invited speakers together with some other papers, also of a survey nature, contributed by delegates.

Finally, the St Andrews-based members of the Organising Committee would like to thank the local organiser, David Stewart, for his herculean efforts to ensure that the conference ran smoothly. As the first paragraph of this introduction indicates, the COVID pandemic required us to be flexible and make considerable adjustments from the original plans. Without David's efforts, this would not have been possible.

Colin M. Campbell
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Edmund F. Robertson
Colva M. Roney-Dougal
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