

FUNDAMENTALS OF TRANSPORT PHENOMENA

Fluid mechanics, solid state diffusion and heat conduction are deeply interconnected through the mathematics and physical principles that define them. This concise and authoritative book reveals these connections, providing a detailed picture of their important applications in astrophysics, plasmas, energy systems, aeronautics, chemical engineering and materials science. This sophisticated and focused text offers an alternative to more expansive volumes on heat, mass and momentum transfer and is ideal for students and researchers working on fluid dynamics, mass transfer or phase transformations and industrial scientists seeking a rigorous understanding of chemical or materials processes. Accessible yet in depth, this modern treatment distills the essential theory and application of these closely related topics, includes numerous real world applications and can be used for teaching a range of related courses in physics, engineering and materials science departments.

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To Zoe, Adam and Alfie: each uniquely an inspiration and a guide

Contents

	<i>Preface</i>	<i>page</i> xiii
	<i>List of Symbols</i>	xv
1	Introduction	1
	1.1 Momentum, Mass and Heat Transfer	1
	1.2 Prerequisites	4
	1.3 Notation	5
2	Hydrostatics	8
	2.1 Statics Kinematics Dynamics	8
	2.2 Pressure	8
	2.2.1 Pressure under the Sea	11
	2.2.2 Pressure in the Atmosphere	12
	2.3 Forced Vortex	14
	2.3.1 Newton’s Pail	17
	2.4 Free Vortex	20
	2.5 Further Reading	24
	Exercises	24
3	Outline of Continuum Mechanics and Fluid Dynamics	26
	3.1 Introduction	26
	3.2 Deformation and Displacement	26
	3.3 Velocity and Acceleration	29
	3.4 Acceleration in Non-Inertial Frames	34
	3.4.1 Non-Inertial Frame	34
	3.4.2 Coriolis Force on Earth	38
	3.5 Displacement	40
	3.5.1 Finite Deformation	40
	3.5.2 Infinitesimal Deformation	42
	3.6 Kinematics	45

3.7	Cauchy Stress and Equilibrium	47
3.8	Generalised Hooke's Law for Isotropic Solids	50
3.9	Equations of Motion in Fluids	52
3.9.1	Kinematics	52
3.9.2	Dynamics	55
3.10	Incompressible Fluids, Navier–Stokes Equations	57
3.11	'No slip' or 'Stick-to-the-Wall' Rule	60
3.11.1	Slip Length	60
3.12	Ideal Fluids	63
3.13	Bernoulli's Equation	64
3.14	Hagen–Poiseuille Flow	66
3.15	Further Reading	69
	Exercises	70
4	Conservation and Dynamics in Fluids	71
4.1	Introduction	71
4.2	Conservations Laws	71
4.3	Reynolds's Theorem	73
4.4	Mass, Equation of Continuity	78
4.5	Momentum	81
4.5.1	Control Volume Method	81
4.5.2	Non-Inertial Frame	84
4.5.3	Differential Equation Method	85
4.6	Angular Momentum	88
4.7	Energy	90
4.8	The Infinitesimal Control Volume: Heat and Work	92
4.9	Energy Balance in Differential Form	96
4.10	Equations of Motion	98
4.10.1	Note about Pressure	101
4.11	Euler's Equations	105
4.12	Viscosity	107
4.13	Reynolds Number, Dynamical Similarity	109
4.14	Examples	113
4.14.1	Laminar Flow between Parallel Plates	113
4.14.2	Couette Flow	115
4.14.3	Fluid Flowing down a Vertical Wall	119
4.14.4	Stokes's Resistance Formula	121
4.14.5	Flow Past a Cylinder	124
4.14.6	Flow through Porous Media, the Darcy Law	126

Contents

ix

4.15	Boundary Layer	129
4.15.1	Prandtl's Equations	129
4.15.2	Boundary Layer at a Flat Plate	130
4.16	Energy and Momentum Theorems Combined	137
4.17	Further Reading	140
	Exercises	140
5	Bernoulli's Equation, Vorticity and Inviscid Flow	142
5.1	Applications of Bernoulli's Equation	142
5.1.1	Reminder of Bernoulli's Equation	142
5.1.2	Sharp Crested Weir	143
5.1.3	Sharp Enlargement of a Pipe	144
5.1.4	Venturi	146
5.2	Conservation of Circulation	149
5.3	Potential Flow	152
5.3.1	Flow Past a Ball	152
5.3.2	Flow Past a Cylinder	154
5.4	Drag, d'Alembert's Paradox	156
5.5	Caution	159
5.5.1	Solution Patching	160
5.6	Complex Variables	160
5.6.1	Free Vortex	163
5.6.2	Source and Sink	164
5.6.3	Cylinder Moving in a Stationary Fluid	166
5.6.4	Uniform Flow	166
5.6.5	Superposition	167
5.7	Further Reading	167
	Exercises	168
6	Mass Transfer in Fluids – Chemical and Materials Processing	169
6.1	Overview	169
6.2	In-Phase Transport	170
6.2.1	Heat Transfer	170
6.2.2	Mass Transfer	171
6.3	Inter-Phase Transport	172
6.4	Convection	172
6.5	Transport Equations and Transport Coefficients	172
6.6	Continuity Equations	175
6.7	Continuity of Solute Mass	178
6.8	Diffusion and Convection in Fluids	179

x	<i>Contents</i>	
	6.9 Special Cases in Diffusive and Convective Processing	183
	6.9.1 Equimolar Counter Diffusion	184
	6.9.2 Stagnant Solvent	185
	6.9.3 Ternary Mixtures	186
	6.10 Processes with Homogeneous Reaction	187
	6.11 Steady State	191
	6.11.1 Flux of Component A in a Stagnant Solvent	191
	6.12 Enthalpy Flux Density	192
	6.13 Further Reading	193
	Exercises	194
7	Diffusion in the Solid State	199
	7.1 The Diffusion Equation	199
	7.2 Diffusion from Fixed Concentration into a Semi Infinite Half Space	200
	7.2.1 Depth of Penetration	204
	7.3 A Model for Brazing	205
	7.4 Diffusion Bonding	208
	7.5 Approach to Equilibrium: Finite Spaces	210
	7.6 Diffusion Coefficients	214
	7.7 Fundamentals of Diffusion in Crystals	216
	7.7.1 Fick's First Law	216
	7.7.2 Mobility	217
	7.7.3 Binary Alloy	219
	7.7.4 Atomistic Aspects	224
	7.7.5 Rate Theory Aspects	231
	7.8 Appendix: Solution to the Finite Slab Problem	233
	7.9 Further Reading	236
	Exercises	237
8	Heat Transfer	239
	8.1 An Instance of Quenching	239
	8.2 Conduction, Convection and Radiation	240
	8.2.1 Conduction	240
	8.2.2 Convection, Newton's Law of Cooling	241
	8.2.3 Radiation	243
	8.3 Steady State Conduction	244
	8.3.1 First Type Boundary Condition	244
	8.3.2 Convection Boundary Conditions	245
	8.4 Heat Transfer across the Boundary Layer	245
	8.4.1 Reynolds Analogy	246
	8.4.2 Taylor–Prandtl Analogy	248

<i>Contents</i>		xi
8.5	Mass Transfer across the Boundary Layer	249
8.6	Further Reading	250
	Exercises	250
9	Dimensional Analysis	253
9.1	Dimensions and Units	253
9.2	Dimensionless Groups	254
9.3	The Buckingham Π -theorem	255
9.4	Drag	257
9.5	Pipe Flow	259
	9.5.1 Pipe Flow in Six Variables	259
	9.5.2 Pipe Flow in Four Variables	261
9.6	Dynamical Similarity, Open Channel Flow	264
9.7	Heat and Mass Transfer	267
	9.7.1 Heat Transfer across the Wall of a Pipe	268
9.8	Further Reading	269
	Exercises	270
	<i>Index</i>	271

Preface

In the summer of 2021, Peter Haynes asked if I would teach a course to the third year Imperial materials science students on partial differential equations. One of the greatest pleasures of academic life is being asked to teach a course in a subject that is utterly unfamiliar. I am not a mathematician, so that appeared almost too challenging, but I accepted; and later it emerged that what was really needed was a course to replace a second year module on materials processing. Looking over previous years' notes I realised that what had been attempted was what is standard in engineering: a course of heat, mass and momentum transfer. This means, the consequences in engineering, mathematics and physics of the three phenomenological laws of heat conduction by Fourier, mass diffusion by Fick and viscous laminar flow by Newton. Another expression for this is *transport phenomena*. This is a book about its *fundamentals*.

The physics of fluids has fascinated most scientists. Leonardo da Vinci observed that a river will run faster if its stream becomes narrower; he also sketched the trajectories of wine escaping from holes in the barrel at different heights from the surface. However to make such observations quantitative required mathematics that he didn't have and was left to Evangelista Torricelli, a pupil of Galileo Galilei. Further developments are associated with the names of the greatest physicists and mathematicians: Newton, Bernoulli, Euler, Cauchy, Stokes. The 'paradox' of d'Alembert resulted in a split between the engineers and the mathematicians, which was finally resolved by Prandtl in 1904. The theory of heat conduction is well known to have been founded by Joseph Fourier who was said to have been boring cannon barrels for Napoleon Bonaparte. The name of Adolf Fick is less well known, except to all metallurgy students who necessarily study solid state diffusion as the centre point of phase transformations. Indeed, the earliest fundamental equation for diffusivity was written down by Einstein in 1905. Unlike rival texts I have introduced mass diffusion as the primary function of the 'heat equation' as expected by a materials scientist, and relegated heat conduction to a later chapter. I have also ventured

beyond the continuum in my treatment of diffusion, which is a field not encountered in the standard texts.

Why have I written this book? Of course there are very many excellent textbooks already. In particular I was delighted to discover the works of the late Frank M. White, and the reader will find many a tribute to his wonderful books in these pages. There are already well established and popular texts aimed variously at engineers and applied mathematicians. Transport phenomena, like thermodynamics, while central to the education of an engineer are, sadly in my view, rapidly disappearing from the curriculum of the physics student. I hope that the present book, written by a physicist, will inspire teachers and students in physics departments to rectify this omission. My intention is to show that the subject is as beautiful as thermodynamics and electrodynamics and as essential to a scientist's education. I know of two excellent precedents of physics books that are written by authors who came upon the subject by being asked to teach it.¹ If my book reaches even close to their achievements, then I will have succeeded.

I would like to thank four cohorts of Imperial College undergraduates for persistent interruptions during lectures, queuing at the end to ask questions and detailed correspondence in their determination to clarify, to challenge and of course to correct the many errors in my lectures. They have missed the errors that remain, and for that I hold them responsible.

¹ T. E. Faber, *Fluid Dynamics for Physicists* (Cambridge University Press, Cambridge, 1995); M. G. Bowler, *Lectures on Statistical Mechanics* (Pergamon Press, Oxford, 1982).

Symbols

physical constants

k_B	Boltzmann constant
R	gas constant
g	acceleration due to gravity
M_r	relative molar mass (molecular weight)

mechanics

m	mass
ρ	mass density
$\mathbf{p}$	momentum
$\mathbf{F}$	force
$\mathbf{f}^m$	force per unit mass
$\mathbf{f}^v$	force per unit volume
$\mathbf{D}$	drag force
$\mathbf{T}$	torque
$\mathbf{L}$	angular momentum
ω	angular velocity (frequency)
μ, λ	Lamé’s elastic constants
K	solid bulk modulus ($K = \lambda + 2\mu/3$)
$\mathbf{u}$	displacement
e_{ij}	strain tensor
e	dilation (trace of the strain tensor, $e = e_{kk}$)
$\dot{e}$	rate of dilation ($\dot{e} = \text{div } \mathbf{v}$)
w_{ij}	rotation tensor
$\mathbf{t}$	traction vector
σ_{ij}	Cauchy stress tensor
$\mathbf{p}_{ij}$	pressure tensor ($\mathbf{p}_{ij} = p\delta_{ij} - \sigma_{ij}$)
Φ	viscous dissipation function

thermodynamics

p	pressure (see Section 4.10.1)
T	absolute temperature
μ_i	chemical potential of species i
c_p	specific heat at constant pressure
c_v	specific heat at constant volume may be per unit mass, mole or volume according to context
U	internal energy
u^m	internal energy per unit mass
E	total energy
e^m	total energy per unit mass
h^m	enthalpy per unit mass
h^v	enthalpy per unit volume
$\delta \dot{W}$	rate of doing work
$\delta \dot{w}^m$	rate of doing work per unit mass
$\delta \dot{w}^v$	rate of doing work per unit volume
$\delta \dot{Q}$	rate of heat input
$\delta \dot{q}^m$	rate of heat input per unit mass
$\delta \dot{q}^v$	rate of heat input per unit volume
ϕ^m	(gravitational) potential energy per unit mass ($\text{grad } \phi^m = -\mathbf{f}^m$)
ϕ^v	(gravitational) potential energy per unit volume ($\text{grad } \phi^v = -\mathbf{f}^v$)
c_i	concentration of species i (mol m^{-3})
x_i	atom or mole fraction of species i
V	volume
$\dot{V}$	discharge rate ($\text{m}^3 \text{s}^{-1}$)

transport coefficients

μ	dynamic (laminar) viscosity
λ	second coefficient of viscosity
κ	bulk viscosity ($\kappa = \lambda + 2\mu/3$)
k	thermal conductivity
h	heat transfer coefficient
α	thermal diffusivity ($\text{m}^2 \text{s}^{-1}$)
ν	kinematic viscosity ($\text{m}^2 \text{s}^{-1}$)
D_i	mass diffusivity of species i ($\text{m}^2 \text{s}^{-1}$)
$M_i, \mathcal{M}_i$	mobility of species i

velocity	
a	acceleration
a_{ni}	acceleration as measured in a non-inertial frame
a_{rel}	acceleration of a non-inertial frame relative to an inertial frame
v	velocity field
ω	vorticity ($\frac{1}{2}\text{curl } \mathbf{v}$)
φ	velocity potential ($\text{grad } \phi = -\mathbf{v}$)
ψ	stream function
$\bar{\mathbf{v}}^m$	mass average velocity
$\bar{\mathbf{v}}$	molar average velocity
$\bar{\mathbf{v}}^q$	heat-based average velocity
Reynolds’s theorem	
Ω_c	control volume
S_c	its surface
B	unspecified extensive quantity
b^m	... per unit mass
b^v	... per unit volume
flux	
j_q	heat flux
j^{m_i}	mass flux of species <i>i</i>
j_i	molar flux of species <i>i</i>
j_p	momentum flux
i^{m_i}	net mass flux of species <i>i</i>
i_i	net molar flux of species <i>i</i>
i^h	net energy flux
boundary layer	
L_s	slip length (Navier length)
δ	boundary layer thickness
δ*	deviation of the streamline
θ	momentum thickness