FUNDAMENTALS OF TRANSPORT PHENOMENA

Fluid mechanics, solid state diffusion and heat conduction are deeply interconnected through the mathematics and physical principles that define them. This concise and authoritative book reveals these connections, providing a detailed picture of their important applications in astrophysics, plasmas, energy systems, aeronautics, chemical engineering and materials science. This sophisticated and focused text offers an alternative to more expansive volumes on heat, mass and momentum transfer and is ideal for students and researchers working on fluid dynamics, mass transfer or phase transformations and industrial scientists seeking a rigorous understanding of chemical or materials processes. Accessible yet in depth, this modern treatment distills the essential theory and application of these closely related topics, includes numerous real world applications and can be used for teaching a range of related courses in physics, engineering and materials science departments.

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To Zoe, Adam and Alfie: each uniquely an inspiration and a guide

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Preface

In the summer of 2021, Peter Haynes asked if I would teach a course to the third year Imperial materials science students on partial differential equations. One of the greatest pleasures of academic life is being asked to teach a course in a subject that is utterly unfamiliar. I am not a mathematician, so that appeared almost too challenging, but I accepted; and later it emerged that what was really needed was a course to replace a second year module on materials processing. Looking over previous years' notes I realised that what had been attempted was what is standard in engineering: a course of heat, mass and momentum transfer. This means, the consequences in engineering, mathematics and physics of the three phenomenological laws of heat conduction by Fourier, mass diffusion by Fick and viscous laminar flow by Newton. Another expression for this is *transport phenomena.* This is a book about its *fundamentals*.

The physics of fluids has fascinated most scientists. Leonardo da Vinci observed that a river will run faster if its stream becomes narrower; he also sketched the trajectories of wine escaping from holes in the barrel at different heights from the surface. However to make such observations quantitative required mathematics that he didn't have and was left to Evangelista Torricelli, a pupil of Galileo Galilei. Further developments are associated with the names of the greatest physicists and mathematicians: Newton, Bernoulli, Euler, Cauchy, Stokes. The 'paradox' of d'Alembert resulted in a split between the engineers and the mathematicians, which was finally resolved by Prandtl in 1904. The theory of heat conduction is well known to have been founded by Joseph Fourier who was said to have been boring cannon barrels for Napoleon Bonaparte. The name of Adolf Fick is less well known, except to all metallurgy students who necessarily study solid state diffusion as the centre point of phase transformations. Indeed, the earliest fundamental equation for diffusivity was written down by Einstein in 1905. Unlike rival texts I have introduced mass diffusion as the primary function of the 'heat equation' as expected by a materials scientist, and relegated heat conduction to a later chapter. I have also ventured beyond the continuum in my treatment of diffusion, which is a field not encountered in the standard texts.

Why have I written this book? Of course there are very many excellent textbooks already. In particular I was delighted to discover the works of the late Frank M. White, and the reader will find many a tribute to his wonderful books in these pages. There are already well established and popular texts aimed variously at engineers and applied mathematicians. Transport phenomena, like thermodynamics, while central to the education of an engineer are, sadly in my view, rapidly disappearing from the curriculum of the physics student. I hope that the present book, written by a physicist, will inspire teachers and students in physics departments to rectify this omission. My intention is to show that the subject is as beautiful as thermodynamics and electrodynamics and as essential to a scientist's education. I know of two excellent precedents of physics books that are written by authors who came upon the subject by being asked to teach it.[1] If my book reaches even close to their achievements, then I will have succeeded.

I would like to thank four cohorts of Imperial College undergraduates for persistent interruptions during lectures, queuing at the end to ask questions and detailed correspondence in their determination to clarify, to challenge and of course to correct the many errors in my lectures. They have missed the errors that remain, and for that I hold them responsible.

[1] T. E. Faber, *Fluid Dynamics for Physicists* (Cambridge University Press, Cambridge, 1995); M. G. Bowler, *Lectures on Statistical Mechanics* (Pergamon Press, Oxford, 1982).

Symbols

physical constants	
k_{B}	Boltzmann constant
R	gas constant
g	acceleration due to gravity
M_{r}	relative molar mass (molecular weight)
mechanics	
m	mass
ρ	mass density
$\mathbf{p}$	momentum
$\mathbf{F}$	force
$\mathbf{f}^m$	force per unit mass
$\mathbf{f}^v$	force per unit volume
$\mathbf{D}$	drag force
$\mathbf{T}$	torque
$\mathbf{L}$	angular momentum
$\boldsymbol{\omega}$	angular velocity (frequency)
μ, λ	Lamé's elastic constants
K	solid bulk modulus ($K = \lambda + 2\mu/3$)
$\mathbf{u}$	displacement
e_{ij}	strain tensor
e	dilation (trace of the strain tensor, $e = e_{kk}$)
$\dot{e}$	rate of dilation ($\dot{e} = \operatorname{div} \mathbf{v}$)
w_{ij}	rotation tensor
$\mathbf{t}$	traction vector
σ_{ij}	Cauchy stress tensor
p_{ij}	pressure tensor ($\mathsf{p}_{ij} = p\delta_{ij} - \sigma_{ij}$)
Φ	viscous dissipation function

thermodynamics	
p	pressure (see Section 4.10.1)
T	absolute temperature
μ_i	chemical potential of species i
c_p	specific heat at constant pressure
c_v	specific heat at constant volume
	may be per unit mass, mole or volume according to context
U	internal energy
u^m	internal energy per unit mass
E	total energy
e^m	total energy per unit mass
h^m	enthalpy per unit mass
h^v	enthalpy per unit volume
$\delta\dot{W}$	rate of doing work
$\delta\dot{w}^m$	rate of doing work per unit mass
$\delta\dot{w}^v$	rate of doing work per unit volume
$\delta\dot{Q}$	rate of heat input
$\delta\dot{q}^m$	rate of heat input per unit mass
$\delta\dot{q}^v$	rate of heat input per unit volume
ϕ^m	(gravitational) potential energy per unit mass (grad $\phi^m = -\mathbf{f}^m$)
ϕ^v	(gravitational) potential energy per unit volume (grad $\phi^v = -\mathbf{f}^v$)
c_i	concentration of species i (mol m^{-3})
x_i	atom or mole fraction of species i
V	volume
$\dot{V}$	discharge rate (m^3 s^{-1})
transport coefficients	
μ	dynamic (laminar) viscosity
λ	second coeffient of viscosity
κ	bulk viscosity ($\kappa = \lambda + 2\mu/3$)
k	thermal conductivity
h	heat transfer coefficient
α	thermal diffusivity (m^2 s^{-1})
ν	kinematic viscosity (m^2 s^{-1})
D_i	mass diffusivity of species i (m^2 s^{-1})
M_i, $\mathcal{M}_i$	mobility of species i

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velocity	
$\mathbf{a}$	acceleration
$\mathbf{a}_{\mathrm{ni}}$	acceleration as measured in a non-inertial frame
$\mathbf{a}_{\mathrm{rel}}$	acceleration of a non-inertial frame relative to an inertial frame
$\mathbf{v}$	velocity field
$\boldsymbol{\omega}$	vorticity ($\frac{1}{2}$curl $\mathbf{v}$)
ϕ	velocity potential (grad $\phi = -\mathbf{v}$)
ψ	stream function
$\bar{\mathbf{v}}^m$	mass average velocity
$\bar{\mathbf{v}}$	molar average velocity
$\bar{\mathbf{v}}^q$	heat-based average velocity
Reynolds's theorem	
Ω_{c}	control volume
$\mathcal{S}_{\mathrm{c}}$	its surface
$\mathfrak{B}$	unspecified extensive quantity
$\mathfrak{b}^m$	. . . per unit mass
$\mathfrak{b}^v$	. . . per unit volume
flux	
$\mathbf{j}_q$	heat flux
$\mathbf{j}_i^m$	mass flux of species i
$\mathbf{j}_i$	molar flux of species i
$\mathbf{j}_p$	momentum flux
$\mathbf{i}_i^m$	net mass flux of species i
$\mathbf{i}_i$	net molar flux of species i
$\mathbf{i}^h$	net energy flux
boundary layer	
L_s	slip length (Navier length)
δ	boundary layer thickness
δ^*	deviation of the streamline
θ	momentum thickness