

Contents

Contents	v
Preface	xi
1 Introduction	1
1.1 WHAT IS PATTERN?	1
1.2 GRASPING THE IDEA	4
1.3 WHY MATHEMATICS?	5
1.4 PROCESS	9
1.5 FOUNDING IDEAS	11
1.6 A PHYSICAL WORLD	12
2 The patterns of heaven	16
2.1 PATTERNS OF THE STARS	16
2.2 THE PROBLEM OF THE MOON	24
2.3 COMMENSURABILITY	26
2.4 CONCLUSION	33
3 The pendulum	35
3.1 TIME AND THE PENDULUM	35
3.2 THE PENDULUM AND ITS STATE SPACE	40
3.3 THE STATE SPACE IN MORE DETAIL	46
3.4 GALILEO’S CONJECTURES AND BEYOND	49
3.5 CONCLUSION	51
4 Difference, change, and information	53
4.1 DISTINCTION, DIFFERENCE, AND CHANGE	53
4.2 STRINGS OF BITS: THE SHIFT SYSTEMS	61

vi	CONTENTS
4.3	SHIFT SYSTEMS 62
4.4	SHANNON 66
4.5	CONCLUSION 78
5	Chance 81
5.1	THE QUESTION OF PROBABILITY 81
5.2	A SIMPLE EXAMPLE OF PROBABILITY AND LAW 96
5.3	RANDOM WALKS 103
5.4	PROBABILITY IN MARKOV PROCESSES 108
5.5	THE PATTERN OF ERRORS 111
5.6	CONCLUSION 117
6	Pattern systems defined 122
6.1	INTRODUCTION 122
6.2	PATTERN SYSTEMS IN WORDS 124
6.3	THE DYNAMICS OF CHANGE IN A SHIFT SYSTEM 130
6.4	DIFFERENCE AND SAMENESS 134
6.5	EVENTS AND THE EVENT SPACE 138
6.6	MEASURE AND MEASURES 148
6.7	PATTERN SYSTEMS DEFINED 158
6.8	WHAT IS PATTERN? 159
7	Exploring the definition of pattern 162
7.1	INTRODUCTION 162
7.2	MATHEMATICS OF THE INFINITE 164
7.3	CONTINUOUS CHANGE 173
7.4	MEASURE AND PROBABILITY 178
7.5	ITINERARY PATTERN SYSTEMS 180
7.6	THE LOGISTIC PATTERN SYSTEM 188
7.7	INSIGHTS FROM ITINERARY PATTERN SYSTEMS 198
7.8	VERSIONS AND SHADOWS 203
7.9	THE POINCARÉ RECURSION THEOREM 207
7.10	CONCLUSION 209
8	Entropy and synthesis 212
8.1	THE QUESTION OF SYNTHESIS 212
8.2	SYSTEMS 214
8.3	INFORMATION AND ENTROPY FOR FINITE PARTITIONS 222
8.4	SYNTHESIS OF PATTERN SYSTEMS 234
8.5	THE BIRKHOFF ERGODIC THEOREM 261

CONTENTS		vii
8.6	CONCLUSION	267
9	Symmetry and invariance	270
9.1	WHAT IS SYMMETRY?	270
9.2	THE GROUP OF ISOMETRIES	283
9.3	THE FINITE SYMMETRY GROUPS IN THREE-DIMENSIONAL SPACE	291
9.4	PERIODIC SYMMETRY	303
9.5	THE PATTERN IDEA AND ORBITS	310
9.6	CONCLUSION	319
10	Pattern systems and the brain	322
10.1	COGNITION AND PATTERN	322
10.2	THE SIMPLEST WILSON–COWAN MODEL AND EQUILIBRIUM POINTS	330
10.3	ASPECTS OF THE MIND	337
10.4	CONCLUSION	355
11	Waves	356
11.1	INTRODUCTION	356
11.2	LIGHT	359
11.3	WAVES	364
11.4	THE UNIT CIRCLE $\mathbb{U}$	367
11.5	THE COMPLEX NUMBERS	375
11.6	FUNCTIONS AND OBSERVABLES	385
11.7	TILES AND CRYSTALS	393
11.8	EIGENSPPACES AND EIGENFUNCTIONS	400
11.9	FROM STATES TO FUNCTIONS: HOW DO WE GO FURTHER?	404
11.10	INTEGRATION: MEASURES LIFT TO FUNCTIONS	406
11.11	CONCLUSION	414
12	Return	418
12.1	THE NATURE OF RETURN	418
12.2	ATOMS	420
12.3	THE MATHEMATICS OF DIFFRACTION	426
12.4	LONG-RANGE ORDER	439
12.5	POINT PROCESSES	446
12.6	SOME FACTS ABOUT STATIONARY POINT PROCESSES	455
12.7	CURIE’S PRINCIPLE	458
12.8	PATTERN OUT OF NOTHING: MORPHOGENESIS	461
12.9	CANTOR DUST	466
12.10	CONCLUSION	474

viii	CONTENTS
13 Quantum	478
13.1 INTRODUCING A STRANGE WORLD	478
13.2 WHAT THE MATHEMATICS LOOKS LIKE	482
13.3 INDETERMINACY	490
13.4 THE NATURE OF OBSERVABLES	501
13.5 POLARIZATION	505
13.6 THE UNCERTAINTY RELATION	511
13.7 CONCLUSION	515
14 Quantum patterns	519
14.1 THE PERIODIC TABLE	519
14.2 THE STRUCTURE OF ATOMS	526
14.3 SPIN	532
14.4 AN APPROPRIATE LANGUAGE	541
14.5 EPR AND BELL’S THEOREM	542
14.6 REALITY: WHAT DO WE REALLY KNOW?	549
15 Afterword	552
15.1 NATURAL PHILOSOPHY	552
15.2 WIGNER’S QUESTION	557
15.3 BELONGING	558
Appendix	562
Bibliography	566
Index	576
I Online Chapters (www.cambridge.org/9781009546515)	587
16 About numbers	588
16.1 NUMBERS	588
16.2 THE NATURAL NUMBERS	590
16.3 THE INTEGERS	591
16.4 THE RATIONAL NUMBERS	592
16.5 THE REAL LINE AND THE REAL NUMBERS	594
16.6 THE COMPLEX NUMBERS	601
16.7 THE MYSTERIES OF THE NATURAL NUMBERS	605
16.8 CONCLUSION	621

CONTENTS	ix
17 Turing	624
17.1 WHAT DOES COMPUTABILITY MEAN?	624
17.2 UNIVERSAL TURING MACHINES AND THE HALTING PROBLEM	634
17.3 TURING	643
18 More on waves	645
18.1 THE ELECTRO-MAGNETIC SPECTRUM	645
18.2 QUASICRYSTALS: AN INTERWOVEN STORY	653
19 Mathematics from quantum theory	663
19.1 QUATERNIONS	663
19.2 THE HYDROGEN-LIKE ATOM	667
19.3 FURTHER NOTES	685
19.4 WHAT ABOUT PATTERN?	689
II Online Endnotes (www.cambridge.org/9781009546515)	699
20 Endnotes	700
Index	762