The Pattern of Change

One of life's most fundamental revelations is change. Presenting the fascinating view that pattern is the manifestation of change, this unique book explores the science, mathematics, and philosophy of change and the ways in which they have come to inform our understanding of the world. Through discussions on chance and determinism, symmetry and invariance, information and entropy, quantum theory and paradox, the authors trace the history of science and bridge the gaps between mathematical, physical, and philosophical perspectives. Change as a foundational concept is deeply rooted in ancient Chinese thought, and this perspective is integrated into the narrative throughout, providing philosophical counterpoints to customary Western thought. Ultimately, this is a book about ideas. Intended for a wide audience, not so much as a book of answers, but rather an introduction to new ways of viewing the world.

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LI 理

LI — pattern (in wood or jade), inner essence, inner-pattern, principle, intrinsic order, structural coherence, science, truth

WULI 物理—innate laws of things, physics

Calligraphy of Chungliang Al Huang

The Pattern of Change

A Mathematical and Philosophical Study of How We See the World

> Robert V. Moody University of Alberta

Deng Ming-Dao



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Contents

Contents											
Pro	Preface										
1	Intro	Introduction									
	1.1	What is pattern?	1								
	1.2	Grasping the idea	4								
	1.3	Why mathematics?	5								
	1.4	Process	9								
	1.5	Founding ideas	11								
	1.6	A physical world	12								
2	The patterns of heaven										
	2.1	Patterns of the stars	16								
	2.2	The problem of the Moon	24								
	2.3	Commensurability	26								
	2.4	Conclusion	33								
3	The pendulum 3										
	3.1	Time and the pendulum	35								
	3.2	The pendulum and its state space	40								
	3.3	The state space in more detail	46								
	3.4	Galileo's conjectures and beyond	49								
	3.5	Conclusion	51								
4	Diffe	erence, change, and information	53								
	4.1	Distinction, difference, and change	53								
	4.2	Strings of bits: the shift systems	61								

vi		Contents					
		C	6				
	4.3	SHIFT SYSTEMS	62				
	4.4	SHANNON	66				
	4.5	CONCLUSION	78				
5	Cha	nce	81				
	5.1	The question of probability	81				
	5.2	A simple example of probability and law	96				
	5.3	Random walks	103				
	5.4	Probability in Markov processes	108				
	5.5	The pattern of errors	111				
	5.6	Conclusion	117				
5	Patt	ern systems defined	122				
	6.1		122				
	6.2	Pattern systems in words	124				
	6.3	The dynamics of change in a shift system	130				
	6.4	Difference and Sameness	134				
	6.5	Events and the event space	138				
	6.6	Measure and measures	148				
	6.7	Pattern systems defined	158				
	6.8	What is pattern?	159				
7	Exp	Exploring the definition of pattern 16					
	• 7.1		162				
	, 7.2	MATHEMATICS OF THE INFINITE	164				
	, 7.3	Continuous change	173				
	7.4	Measure and probability	178				
	7.5	ITINERARY PATTERN SYSTEMS	180				
	7.6	The logistic pattern system	188				
	7.7	INSIGHTS FROM ITINERARY PATTERN SYSTEMS	198				
	7.8	Versions and shadows	203				
	, 7.9	The Poincaré recursion theorem	207				
	7.10	Conclusion	209				
3	Entr	Entrony and synthesis					
	8.1	The question of synthesis	212				
	8.2	Systems	214				
	8.3	INFORMATION AND ENTROPY FOR FINITE PARTITIONS	222				
	8.4	Synthesis of pattern systems	234				
	8.5	THE BIRKHOFF ERGODIC THEOREM	261				
	2.5		-01				

		Contents	vi
	8.6	Conclusion	26
•	Sum	matry and invariance	
9	Sym		2/0
	9.1		2/
	9.2		20
	9.3		29
	9.4		30
	9.5		31
	9.0		31
10	Patt	ern systems and the brain	32
	10.1	Cognition and pattern	32
	10.2	The simplest Wilson–Cowan model and equilibrium points	33
	10.3	Aspects of the mind	33
	10.4	Conclusion	35
11	Wav	/es	35
	11.1		35
	11.2	LIGHT	35
	11.3	WAVES	36
	11.4	The unit circle $\mathbb U$	36
	11.5	The complex numbers	37
	11.6	Functions and observables	38
	11.7	TILES AND CRYSTALS	39
	11.8	EIGENSPACES AND EIGENFUNCTIONS	40
	11.9	From states to functions: how do we go further?	40
	11.10	INTEGRATION: MEASURES LIFT TO FUNCTIONS	40
	11.11	Conclusion	41
12	Retu	irn	41
	12.1	The nature of return	- 41
	12.2	Атомя	42
	12.3	THE MATHEMATICS OF DIFFRACTION	42
	12.4	Long-range order	43
	12.5	Point processes	44
	12.6	Some facts about stationary point processes	45
	12.7	Curie's principle	45
	, 12.8	Pattern out of nothing: morphogenesis	46
	12.9	Cantor dust	46
	12.10	Conclusion	47 47
			47

viii		Contents	
	0		
13	Qua		47
	13.1		4/
	13.2		40
	13.3		49
	13.4		50
	13.5		50
	13.0 13.7	Conclusion	51
14	Qua	ntum patterns	51
•	14.1	THE PERIODIC TABLE	51
	14.2	THE STRUCTURE OF ATOMS	52
	14.3	Spin	53
	14.4	AN APPROPRIATE LANGUAGE	54
	14.5	EPR AND BELL'S THEOREM	54
	14.6	Reality: what do we really know?	54
15	Afte	rword	55
	15.1	Natural philosophy	55
	15.2	Wigner's question	55
	15.3	Belonging	55
Ap	pend	lix	56
Bib	oliog	raphy	56
Ind	ex		57
I	Onl	ine Chapters (www.cambridge.org/9781009546515)	58
16	Abo	ut numbers	58
	16.1	NUMBERS	58
	16.2	The natural numbers	59
	16.3	The integers	59
	16.4	The rational numbers	59
	16.5	The real line and the real numbers	59
	16.6	The complex numbers	60
	16.6 16.7	The complex numbers	60 60

Contents				
17 Turing	624			
17.1 What does computability mean?	624			
17.2 Universal Turing machines and the halting problem \ldots .	634			
17.3 Turing	643			
More on waves				
18.1 The electro-magnetic spectrum	645			
18.2 QUASICRYSTALS: AN INTERWOVEN STORY	653			
19 Mathematics from quantum theory	663			
19.1 QUATERNIONS	663			
19.2 The hydrogen-like atom	667			
19.3 Further notes	685			
19.4 What about pattern?	689			
II Online Endnotes (www.cambridge.org/9781009546515)	699			
20 Endnotes	700			
Index	762			

Preface

The only constant in this world of change is change itself. So it is often said. Already Heraclitus living in the fifth century BCE famously emphasized the flux that underlies all things, and change is the central feature of the famous *Yijing (I Ching, The Book of Changes)* that dates back some five centuries earlier. Life leaves us with no doubt about the reality of change. But is it really true that there are no other constants than change itself? Change is not entirely random—it is not without predictable effects. Within change there arise forms and patterns, and were it not for these, which we observe, predict, and depend on, there would be no world full of life with its amazing forms that arise, thrive, and regenerate in the great cycles of things.

To say that the only constant is change itself is too blunt. What emerges from change is pattern, and it is pattern that we can recognize and depend on. Is it not the case that rivers are rivers because something about them does not change, something that supports both fish and fishermen? A closer look at what Heraclitus said reveals that he understood more than we cannot step into the same river twice:

As they step into the same rivers, other and still other waters flow upon them. [94, fragment L]

This suggests that although we cannot step into the same river *water* twice, what makes the river a river is its flowing water: no flow, no river. The river is defined as a pattern through its flow. Here we sense a deeper understanding of how sameness/difference turns out to be an important way in which we continuously reposition ourselves. The river stays the same by always consisting of new (other) water. It is the same river because it is in how it changes that it becomes itself. Thus sameness and difference are together what makes a river a river.

This is the underlying theme of this book: the world is a world of change, but change manifests itself in the form of *pattern*. It is through these patterns of change, which we

xii

Preface

recognize and act upon, that we are able to live in this world. We might even say that they are how we come to "make" our world; and so it is for all forms of life.

Still, as a word in English, pattern itself is a nebulous entity. There are weather patterns, dress patterns, holding patterns for planes, patterns of behavior, patterns of numbers, flocking patterns, and on and on *ad infinitum*. We have heard pattern described as being as information, as memory, as repetition, all of which have elements of truth, and mathematics has been called the science of patterns. Yet out of all this variety of ideas there is one that strikes us as fundamental, even universal—the one that we have already alluded to: the origin of pattern is dynamic, pattern is the signifier and identifier of change.

Put into a single sentence, our understanding is that

pattern is the means by which we experience change, change is what we are able to infer from pattern.

In effect, change and pattern are aspects of the same thing. Important to this view is that it is *systemic* in conception, in the sense that neither change nor pattern stand alone, but each supports the other, and their unity is system based. *Patterning* may be the best way to think of it.

It doesn't seem that elaborating on this should be a book-length endeavor, but our intention is to develop a formalism that encompasses this concept and to show how deeply embedded it is in the mathematics and science of our time. The ideas that lie behind patterning—change, chance, systems, information, invariance, uncertainty, measures, laws, all of which we will meet—seem common enough, but formalizing them and giving them precise meanings is decidedly modern. In many ways their story parallels the history of science, and that is how we approach it.

This not a book about philosophy, at least not explicitly so, yet in dealing with ideas about how we see and function in the world philosophical questions are bound to arise, and it is our hope that these in themselves will stimulate the reader to exploring new ways of thinking. A central idea, reflected in its title, is that we do not live in a world of "things", but rather in a world of process and relation within change. Change is foundational, pattern is how it is manifested through our senses and minds. It is neither entirely deterministic nor entirely random yet there is a deep sense of relation that entwines chance and law. Forms emerge, they persist, though changing through time, and then return to the source from which they arose. Nature seems both bound by enduring principles and free to be endlessly creative.

These ideas are not new, but they have gathered extraordinary empirical evidence over the past few hundred years. To a great extent the scientific enterprise has been fueled by foundational philosophical ideas of the West, based on belief in an externally created

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Preface

universe, teleologically directed, with humanity as a supreme achievement and capable of revealing its internal design. Strangely, the more we have learned the more science itself has undermined these ideas. We don't seem so special as to consider ourselves some supreme achievement and there is no clear evidence that the evolution of life is teleological or directed to some anthropological ideal.

For many it has left empty the meaning of life—who are we, why are we, whither our journey? At the same time it has led to an understanding that is more in tune with one that arose in ancient China during a period of war and chaos much like our own today. There, a philosophy developed that is both remarkably consistent with what we have learned from science and deeply instructive about our entwined relationships with all the transient processes of life on this planet. We have tried to introduce some of this ancient philosophical thought into our development because we think it is hugely important. Vital, even. It is subtle and hard to grasp, our minds so implicitly bound to a culture that has for millennia defined itself through anthropocentric absolutes. But there is wonderful depth and unity to it. China's artists, poets, and intellectuals were attuned to Nature and found both meaning and repose within it, witnessing themselves as expressions in its weave of ever changing pattern.

The aim of the book is to color in all these concepts and see how they fit into a consistent framework which we might consider as a basis for a theory of pattern, or better a theory of *patterning*. At the same time we want it to read as a sort of history of science and show how it raises ideas that are important to our age. Yet a third intention is to give a gentle path through some of the history of mathematics and the ways, often surprisingly metaphorical, in which it is used to give meaning to such abstractions as change, randomness, and invariance. In all of this we hope that the reader can have better access to some of the remarkable revelations of modern science. A particularly challenging goal has been to conclude with a section of quantum theory, for it is astoundingly predictive, increasingly important in modern technology, and perhaps an important ingredient in consciousness, yet it too is pattern based and must be a necessary part of our story. But it is also fascinating because its domain is a land where ordinary language and thought fail to encapsulate its strange conclusions, and where philosophical debate has always been fierce.

So this book is held together with a thread of mathematical ideas. However, there is plenty of ordinary text too, and we hope that the book can be read at many levels and will be accessible by anyone interested in how science has come to address the idea of change. Mathematics is often seen as numerical and quantitative, technical, and difficult to manipulate. But at a deeper level it is *relational*. As such it can be used to represent any situation in which those relationships appear, and it can offer neutral ground where everything other than those relations are ignored. That's the key. Much of mathematics over the centuries has been developed precisely for this reason, and by now there is a great

xiii

xiv

Preface

array of ideas and mathematical tools that are available to deepen our understanding.

The mathematics in this book has to do with the relational side of things. The difficulties that it presents are not technical. They are conceptual, and they have to do with the difficulties of finding ways in which abstract ideas can be formalized into a mathematical context. The difficulties that we encounter as readers may seem difficult, but that is not because we are no good at mathematics. The scientists and mathematicians who created these forms of expression had enormous difficulties with them too. It's like the invention of zero. We have all learned to be at ease with the concept, but its creation was a genuine step in the history of mathematics–and it did not come easily. A symbol for "nothing"? Really?

Often getting a new idea depends on grasping the right metaphor. We may not think of mathematics as having anything to do with metaphor but a great deal of it derives, not surprisingly, from attempts to model aspects of the world that we experience. We will see many examples of this.

In any case, in reading the book there is nothing wrong with breezing over the mathematics and getting some conceptual sense of what it is about. It is fine to read ahead and then go back if a concept is still vague. Perhaps the ideal reader is one with some sort of STEM background, but even a good background in high school mathematics should be sufficient. For the mathematically educated, there will be much that is familiar but, we hope, also things that are new. All the mathematics is subservient to the deeper idea that we want to present: that pattern is a fundamental attribute of change, and that it is through pattern, or patterning, that we see the world. We have tried to make the conceptual ideas clear, convincing, and readable, but at the same time to lay down a rigorous foundation for a theory of change and pattern.

The book naturally falls into three basic parts:

- Five chapters offering an expansive view of how pattern and change are related, including ideas around continuous change, discrete change, and random change and their historical development.
- Three chapters that lay down the precise mathematical concepts that are involved in formalizing pattern and change, as well as some of their consequences. Important in this part are ideas around information and what it means for pattern systems to interact and form larger integrated pattern systems: synthesis as opposed to the usual analysis.
- Seven chapters that show how universally these ideas of pattern and change apply. At the same time, starting at the chapter on waves, we see there is a new way of looking at pattern systems. We call it the *linearization* of pattern systems, and it is this that opens up the real power of the modern mathematics. It is also foundational in quantum theory.

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Preface

Beyond the physical book itself there are two sets of cross-referenced online addenda that are freely downloadable (see the Table of Contents). The first is a set of additional chapters and the second a set of endnotes that arise out of the text and elaborate on details of further interest. Some of the harder or more extended parts of the mathematics are to be found in the addenda, for those who wish to see more details. For instance the mathematical insights into the structure of the periodic table are there. Most of the details are pitched at a level compatible with the idea of the book. Online sections and chapters are referenced with the symbol ‡§ or ‡Ch. Endnotes are referenced by superscript numbers, starting from 1 at each chapter.

The unlikely combination of a mathematician and a Chinese martial artist, writer, philosopher writing this book reflects the fact that pattern and change are universal. We felt the need for seeing them from the two sides of what is often seen as an east-west divide. Human beings are human beings, but cultural history leads us into seeing things from differing points of view. Nature is not divisible into right and wrong or good and bad cultures. It cannot be reduced to a single set of facts. It is in seeing from other sides that we understand how much we implicitly assume without being aware of it, and learn thereby what we have been missing.

The Universe is both mysterious and beautiful. Looking at it from a scientific point of view can do nothing to diminish this. Rather it can do much to inspire and deepen our sense of wonder, for it reveals again and again the extraordinary coherence and unity of Nature.

Yes, we do feel that Nature deserves its capitalization!

mathematic (n.)

"mathematical science," late 14c. as singular noun, mathematik (replaced since early 17c. by mathematics, q.v.), from Old French mathematique and directly from Latin mathematica (plural), from Greek mathēmatike tekhnē "mathematical science," feminine singular of mathēmatikos (adj.) "relating to mathematics, scientific, astronomical; pertaining to learning, disposed to learn," from mathēma (genitive mathēmatos) "science, knowledge, mathematical knowledge; a lesson," literally "that which is learnt;" from manthanein "to learn," from PIE root *mendh- "to learn." (Online Etymology Dictionary) xv

xvi

Preface

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I have had a long association with Jan Zwicky that involves photography and poetry, but it has been her depth and clarity as a philosopher that has been important here, especially giving me more nuanced views into meaning and gestalt. Many hours spent with neuroscientist Mark Bodner pointed me in the right directions in starting the book. David Robertson has been an enthusiastic reader and commenter over various drafts of this book, and pleasant company for many Wednesday afternoon meetings at local cafes around Victoria. It was Marjorie Senechal who introduced me to aperiodic order and she has been an important supporter of this book. Also I want to thank my former student, and now professor, Jeong-Yup Lee who read the entire book and made many corrections.

The friendship of Yves Meyer, along with his interest in the book and his careful reading of some of its more mathematical parts, has given me enormous encouragement.

It was through the teachings of Chungliang Al Huang that I, a very much in-the-head type person, truly learned what it means to be fully embodied, both body and mind, and in a deeper sense to be free. We are indebted to him for his contribution of the cursive calligraphy of the important Chinese character Li, which is the frontispiece of the book.

I would like to give thanks to the Natural Sciences and Engineering Research Council of Canada for five decades of research support without which this book would never have been written. On the production side, we are particularly grateful to our editor Roger Astley, who did so much to promote it.

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