

CHAPTER 1

Introduction

1.1 What is pattern?

Pattern is the manifestation of change, it is by pattern that we know the world, and it is only by pattern that we know the world. This applies not only to human life, but to all life. Indeed pattern underlies all aspects of the physical world, at least as we can describe them in conceptual terms.

Making such a grandiose statement involves considerable justification, not the least being exactly what we mean by pattern. In some sense we all know what pattern is, and can recognize it effortlessly. But if we ask around, or even look the word up in a dictionary, we realize that it has a diverse array of potential meanings. Over the last seven hundred years, it has developed at least ten major meanings, including ideas of design, recurrence, configuration of natural or chance events, discernible coherence or interrelationship of component parts, and that which serves as a model for imitation. It is the last of these that comes closest to the etymology of the word, deriving from the fourteenth-century Middle English *patron* and ultimately from *pater* meaning father, and by implication someone to follow. However, common usage goes well beyond that. We are quite happy talking about weather patterns, dress patterns, wallpaper patterns, patterns of behavior, cyclical patterns, and flight patterns, rhythmic patterns, rhyming patterns, migration patterns, cultural patterns, and pattern recognition both natural and artificial. We talk about a familiar pattern, a disturbing pattern, and an evolving pattern. Even the difficult concept of information is often described as being pattern. In all of these senses of pattern there is something that everyone understands and everyone can recognize. Yet no simple definition seems to embrace them all.

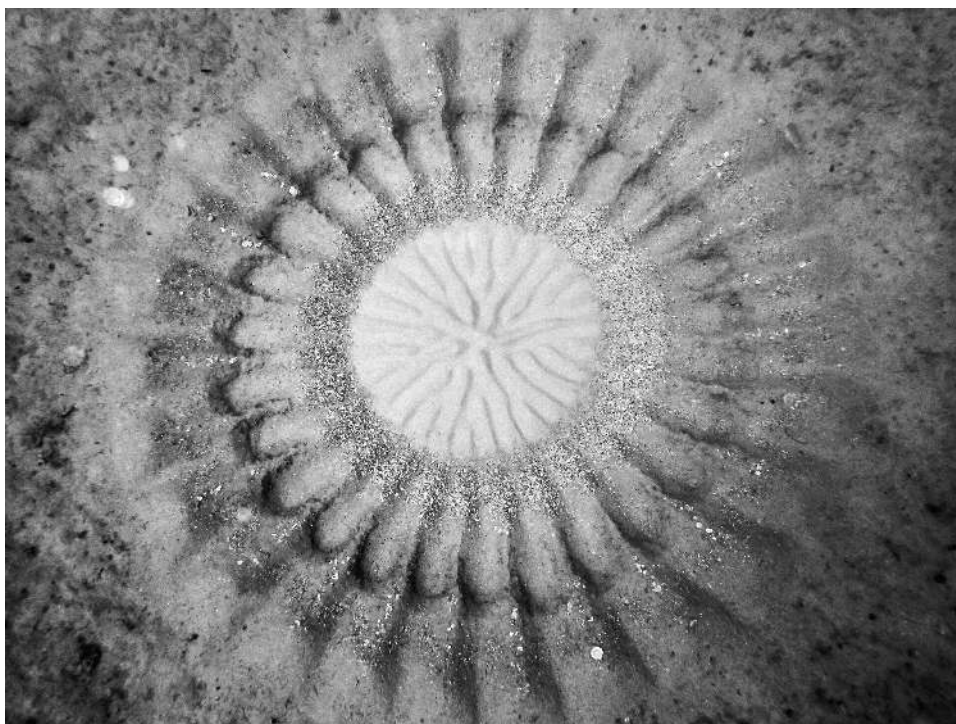
Nonetheless, the incredible number of phenomena that we can recognize and describe as instances of pattern seems to suggest that there really is something fundamental and even universal in our appreciation of it. This goes far beyond the human sphere: it is

evident that all animals recognize and react to pattern. Think of their reactions to light, fire, human presence, thunder, or seasonal changes, and think of their mating rituals, nesting habits, predators, and prey. This is not restricted to conscious recognition of pattern either. The antennae of lunar moths contain sensilla, sense organs, which can detect individual molecules which are chemical signals or pheromones. Notably these are sex cues that are dispersed in minute quantities into the air by receptive females. These molecules are, in effect, patterns that are recognized as such by the sensilla of males and then transported within through the insect’s neural system to effect an appropriate physical response. It has been reported that Indian lunar moths can respond to just one or two of these molecules, and at distances measured in miles. The induced response in the moth is then to fly into the wind, searching for further pheromones that will gradually lead it to the source. As we open our minds to the possibilities, we begin to recognize pattern everywhere. But that only opens the question wider. What is it that makes pattern pattern?

This book tries to give some answers to this question.

At the outset we should state that we think that pattern is really not about things, but about process. We can get some inkling of this from the fact that pattern can hardly be separated from recognition, and recognition itself is process. Part of what pattern involves is the concept of re-cognition, literally “knowing again”, and knowing again involves process, typically a flow of time and engagement with pre-existing experience of this pattern. In other words memory. The pheromone that stirs a lunar moth might by itself be considered as bare pattern, and it is well established that similarly shaped molecules can be designed to have the same effect on the insect. Looking further though, we sense that the real pattern here lies in a much larger context. The molecular shape of the pheromone is pattern only because there is a complementary shape in the receptor that can accept it, and this acceptance, along with the subsequent conditioned response to it, is process. That process itself is based on genetic memory which itself has been handed down generation to generation by process. Most pattern is open-ended like that. In reality a better word for pattern is *patterning*, and that is how we shall often refer to it.

The little male pufferfish, only some 6 cm in length, will spend a week, nonstop, making the magnificent pattern shown in Fig. 1.1 in the sandy seafloor. There is something profoundly about this. A tiny creature far removed from the human evolutionary tree deliberately creates a pattern of extraordinary symmetry, some 2m in diameter, that is immediately accessible to our own human sensibilities. But of course its efforts must be seen in light of its own existence. When a male goes to all that trouble to make this apparently unproductive piece of art, one can be sure that in the background there is a female to impress. Indeed this is the case—it is there to attract a female pufferfish. If she is sufficiently impressed she will swim into the pattern, whereupon mating, release



**Figure 1.1:** The amazing nesting circle worked into the sandy seabed by the tiny puffer fish, *Torquigener* sp. (Tetraodontidae). Kawase, H., Okata, Y. & Ito, K. With the kind permission of Hiroshi Kawase [95].

of her eggs, and spawning will take place. The sand pattern, having served its purpose, is left to disappear under the ceaseless motion of the sea.

Extraordinary though it is in its own right, it is even more enlightening to think of this beautiful pattern in the sand as part of the far larger pattern which is the dynamical life cycle of this species of puffer fish. Beyond its physical form, it is also a piece of information, ready to attract a female and stimulate her into releasing her eggs within its bounds. We soon become aware of a complex of overlapping pattern systems: the neural patterning in each fish; how their mating ritual is replicated in each subsequent generation; the sea-patterning of the sand itself; the lifecycle of both fish; and the continuity of the species. We could even consider how and why the local aquatic environment supports the existence and life of this unique species of puffer fish, which has only been found in the coastal waters off southern Amami-Oshima, Japan. That such patterning can be generated from the neural systems of such a small fish and beyond that, the fact that we, members of a species seemingly remote from puffer fish, are instantly drawn to it, suggests the profound ways in which pattern and pattern recognition must be woven into the fabric of reality.

The little puffer fish is a gentle reminder that pattern is not the private domain of humankind or of a species endowed with special mental powers. It is rooted in the motions of the planets, in the genetic code of all life, in the stature of trees, the shape of molecules, the spider’s web, and the ice on the winter pond. With all this in mind, it is easy to augment our list without end, for pattern can be found everywhere. Yet, with all these forms, how can we articulate what pattern itself is? How do we move beyond this general gut feeling and turn it into a concept, or perhaps a family of related concepts, through which we can speak about pattern with precision and clarity? Is that even possible?

Our initial interest in pattern began from precisely this question, for we saw the many occasions in which scientists and philosophers, puzzling about information, complexity, the workings of the brain, the natural world, or social problems, would return to our inherent sense of pattern, taking that as a given that everyone understands. In fact, while there are many books written around the highly practical and extraordinarily difficult subject of pattern recognition, one is hard pressed to find within them a definition of pattern itself. We are hard-wired to detect it, and we are so wonderfully good at doing this that it is hard to gain perspective. That’s part of the problem. We are too good at it to even realize that at every moment we are in the midst of a swirling ocean of pattern.

In trying to clarify what pattern is and to locate founding principles, we have often found ourselves looking more backwards than forwards. The marvels of patterning are wonderful, and we want to explore them, but what we are looking for are the fundamental ideas that lie at the basis of all our patterning. In doing this we will find inspiration from the natural sciences, the life sciences, mathematics, and more generally from the insights of philosophy, both from the West and from the East.

1.2 Grasping the idea

In writing this book, we have come to think about pattern in terms of a few fundamental ideas — words — that we think underlie it: context, difference and sameness, change and invariance, chance and determinism, order and disorder, meaning and metaphor, reductionism and synthesis, equilibrium and evolution, repetition and randomness, emergence and return. In a single word, we have come to see pattern as expressions or manifestations of *systems*: the nature of pattern is *systemic*.

We have begun to understand how much our patterning not only allows us to “see”, but how much it actually *determines* what we see, simultaneously opening horizons while hiding others from view. The more we appreciate the ways in which pattern informs our conceptions of reality at all levels, the more we realize the freedoms and limitations that the underlying assumptions of subject/object, self/other, man/beast relationships so often bring to it.

There have always been two broad approaches to pattern. The first is reductive, seeking to account for pattern by looking minutely at the parts out of which it arises: know the parts and you will know the whole. The second is integrative, tending to avoid detailed study of parts in isolation and caring more for over-all function, movement, and inter-relationship. These two approaches are often seen as antagonists, especially in the field of medicine where traditional medicine with its extreme specialization is held in contrast to integrative medicine which tries to see sickness in terms of the whole being. Traditionally Western thought has been seen to favor the first route, largely assuming that the integrative part will follow from the details. Eastern cultures, notably those derived from Chinese thought, have favored the second, based on ideas of emergence from and return to an encompassing whole. There is still a tendency to see the two approaches as being exclusive to each other.

In actuality we need both. The parts can only be understood in terms of the patterning of the whole, and the whole cannot be understood without taking into account the patterning of the parts. The rise of Western science has been dominated by reductionism, and to incredible effect. Yet, over the course of our own lives, we have seen how modern science is increasingly aware of this two-way road. Disciplines that for ages have been separated, jealously guarding their own territory and even mutually antagonistic, are now seen to deeply inform each other. Consider climate change and ecology, or the vast neural hierarchical complexity of the brain and its profound intimacy with the physical world. The reductionist/integrative dichotomy is less and less helpful. The deeper aim of this book is to show just how integrative the idea of patterning is, and how intimately connected to planet Earth and the encompassing Cosmos we are. We are creatures of this cosmos, and patterning is the conceptual basis of our understanding of it and ourselves.

### 1.3 Why mathematics?

One of the first issues we have to deal with is that in any exploration of pattern or patterning we are inevitably going to use our own innate and learned systems of patterning in doing so.<sup>1</sup> To free ourselves as much as possible from dragging our preconceived ideas and prejudices into the process requires a large element of abstraction. Whenever there is a collection of things or events in which we seek or find a commonality, that commonality will be an abstraction, of which the individual things or events are then representations or examples. Man, dog, animal, transportation, house, verb, word, emotion, these and thousands of other words that we know, are abstractions, each of which can be instantiated in many ways. Inevitably though, these abstractions, though easy to exemplify through particular examples, are hard to define.

One of our teachers, the famous twentieth-century geometer H. S. M. Coxeter, used to talk about a simple game he called *vish*. This consists of looking up a word in a dictionary,

and then following the trail: that word is defined in the dictionary by using words, and those words are defined by other words, and so it goes. Inevitably, of course, we end up in a *vicious circle* (hence the name “vish”), finding the very word we were looking up being ultimately used to define itself. We try to define words with words, and end up defining a word in terms of itself. When we look up the word “dog” we are told that it is a type of “wolf”. Looking up “wolf” tells us that it is a “canid”. Looking up “canid” gives us dogs and wolves (plus jackals and foxes). And that’s as far as we are going to get with the word “dog”. The point of this is, of course, that we cannot define everything. In the sciences, we have to start somewhere, and these are primitive concepts that we don’t try to define.

One of the more suggestive definitions of mathematics is that *mathematics is the science of patterns* [159]. There is a good deal of merit in this definition, but it does rely upon us knowing what patterns are! This book puts things the other way around: how can we use mathematics to give meaning to the elusive word “pattern”. We seem to be right in the middle of the game of vish. Following *The Princeton Companion to Mathematics* [67], we choose not to try and define mathematics, but are content to allow the book itself to answer that question. Ultimately we may well conclude that pattern is something that lies beyond mathematics, but we will certainly see that it gives us genuine ways to elucidate pattern and amazing tools to explore it.

Pattern as an idea is an abstraction, and it is a particularly vicious one from the point of view of definition. It quickly becomes clear that anything we can say, words or pictures, or anything involving the physical senses is ultimately itself an expression of pattern. Whatever the basic principles that underlie patterning may be, they have to be principles that can appear in the countless different forms that pattern can assume. This is where mathematics comes in, largely because it is so context free. Think for instance of the ordinary numbers 1, 2, 3, 4, 5, ... These are abstractions, free from any particular physical context, but capable of being exemplified in thousands of ways.

The fact that all of us use numbers in all sorts of ways, and so effortlessly, is a good indication of how good we are at abstractions. In fact language itself is immensely abstract, and the letters that you are reading right now are even more abstract. Mathematics is often considered as difficult, but in some ways it is no different from any other form of language. It too has to deal with definitions and cope with the problem of vicious circles. It does so by assuming a few “primitives”—concepts for which it makes no attempt at definition. The primitives are simply assumed to be understood or accepted as givens. Most often, and in particular in our case, they have strong metaphorical associations to things we know very well, so we feel comfortable with them. After that there follow the axioms, or we might say principles, that spell out assumed *relationships* between the primitives. The mathematical theory then arises out of this in the form of accumulative outcomes of these concepts and relations, and new definitions and concepts derived from them.



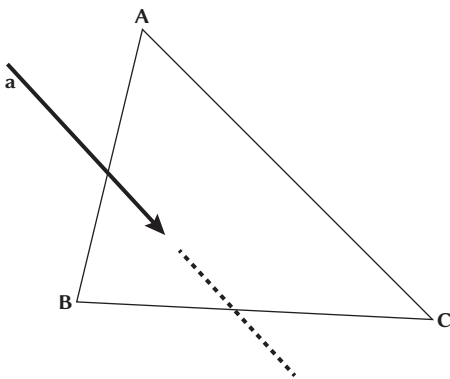
The most famous example of an axiomatic system in all its grandeur is Euclid's *Elements* (c. 300 BCE). This is one of the great achievements of the ancient Greeks, and by extension of the Western world. In the *Elements*, points and lines are primitives. Based on the primitives are the axioms: for instance, there is the axiom that for any two distinct points there is a line on which both of them lie. Beyond that there are definitions, for instance isosceles (literally equal legs) triangles, and after that theorems like “all isosceles triangles have two equal angles”, and later the famous theorem that “the interior angles of all triangles add up to two right-angles” ( $180^\circ$ ).

The mathematics of Greek geometry, and much of mathematics in general, is based on metaphor. The points, lines, circles, of Euclid's *Elements*, and their mutual relationships defined by axioms, are based on what we already have deep experience with. But now they are made into abstractions, and it is the metaphor (for example, that we call the mathematical abstraction a “line” thereby bringing to it our long-standing familiarity with lines) that provides us with the meaning and intuition to understand it. What is sometimes difficult, and we will become increasingly aware of this as we go on, is that abstracting an idea and using a familiar name in a new abstract sense can bring about unexpected confusions. We will see many words, used in new senses—events, measures, partitions, information, randomness, maps, operators, and so on—that derive from common ideas but have more specific meanings in mathematical and scientific contexts. Using metaphor to extrapolate meaning out of these special words has to be done with awareness and care.

A famous example of this problem actually occurs with Euclid's axioms. As stated by Euclid it is not possible to prove that if a line cuts across one side of a triangle and enters into the triangle then it must exit by cutting across one of the other two sides. The idea of betweenness, as in the points *between* two points on a line, is so obvious from experience that it was never included in the axioms. This lacuna is filled in with Pasch's axiom, see Fig. 1.2. David Hilbert, who appears several times in other situations in the book, was one of several people who presented complete lists of axioms that characterize our intuitive picture of Euclidean geometry.

What mathematics does is to produce conclusions out of its definitions and axioms by using the rules of inference (logical deduction). The results are often impressive, revealing structure implicit in the definitions and axioms that we might not otherwise have realized. Pythagoras' theorem is a famous example: in any right-angled triangle, the square on the hypotenuse (the longest side) is in area equal to the sum of the areas of the squares on the other two sides. This and other such theorems are truths about this system, consequences of logical reasoning from the assumed axioms, yet far from intuition.

Often these conclusions can be brought back into the physical world by means of the metaphor, now used in reverse. Returned to the physical world, Pythagoras theorem



**Figure 1.2:** Pasch’s axiom (1882): *let  $A, B, C$  be three points that do not lie on a line and let  $a$  be a line in the plane  $ABC$  which does not meet any of the points  $A, B, C$ . If the line  $a$  passes through a point of the segment  $AB$ , it also passes through a point of the segment  $AC$ , or through a point of segment  $BC$ , Moritz Pasch (1843-1930).*

becomes an important tool for determining distance. For if  $h, x, y$  are the lengths of the hypotenuse, and the two sides, then  $h^2 = x^2 + y^2$  gives us a way to find  $h$  given  $x$  and  $y$ . Returning again to the mathematical world, this becomes a foundational tool in coordinate geometry for determining distance. It is used endlessly in the monitor displays of our computing devices, which are laid out in rectangular grids.

This back and forth transfer of concept from experience to abstraction is why mathematics is so extraordinarily successful in science. Rafael Nuñez, a scholar who with George Lakoff has thought a lot about this, writes:

The effectiveness of mathematics in the world is a tribute to evolution and to culture. Evolution has shaped our bodies and brains so that we have inherited neural capacities for the basics of number and for primitive spatial relations. Culture made it possible for millions of astute observers of nature, through millennia of trial and error, to develop and pass on more and more sophisticated mathematical tools—tools shaped to describe what they have observed. There is no mystery about the effectiveness of mathematics for characterizing the world as we experience it: That effectiveness results from a combination of mathematical knowledge and connectedness to the world. The connection between mathematical ideas and the world as human beings experience it occurs within human minds [122].

Our development of pattern and patterning in this book works in exactly the same way. Our *undefined premises* are essentially just two:

- simple set theory (the idea of sets and elements that make up sets);
- the normal ideas that we have of two- and three-dimensional space.<sup>2</sup>



Admittedly, the latter is not especially primitive, but it is very intuitive, and going back any further would not enlighten us any further about the ideas of pattern that we wish to develop.

1.4 Process

We have already suggested that pattern is more about process than it is about things. Process already seems to imply some sort of *system* in which change takes place. It’s not about things or even things changing in time, but about entities which are perceived and make sense only when seen in relation to other entities, and only within a dynamical context that unifies them all. The idea that seeing the world from a systems point of view might give us new insights into meaning (making sense of the world) seems natural enough, but it is hard to take it from this vague statement into something truly convincing. Often we tend to ascribe our thoughts and emotions to transcendental realms of mind or soul, beyond the physical world. Western philosophy has a long tradition of doing just this. But it does not necessarily have to be so. A suggestion of what a systems point of view might signify can be seen through a little vignette.

The Tang Dynasty (618–907) is widely considered to be the golden age of Chinese poetry. There was a long tradition of thought that the physical world and its changes are fundamental aspects of reality, and it was common for poets was to express their thoughts and emotions within the larger context of the physical world around them. Consider this example, “At the Mountain House of My Teacher, Waiting in Vain for Elder Ding” by the poet Meng Haoran (689/691–740):

The sun angles below the western mountain range;  
valley after valley plunges abruptly into dark.  
The moon rises through pines, the night grows cold;  
I listen to the wind and the stream, full and pure.  
The woodsmen trudge homeward, wanting rest;  
birds search through cooking smoke for perches to settle.  
You promised to come and stay for a time:  
I’m alone with my qin at this vine-draped path.<sup>3</sup>

The first three lines give us scenes from nature. There is no mention of who is making the description. We are plunged into the landscape as if we are there ourselves. The inexorable passage of time is apparent. In the next two lines, the yearning of woodsmen and birds for a home within the vast wilderness is clear. Only in the fourth line does the poet begin to speak placing himself in the sounds of wind and the stream. Finally the human element enters: there’s a promise, one that parallels the return of woodmen and birds. The word “alone” in the last line, is tied to a *qin* (a seven-stringed zither) and a vine-

draped path. Since the qin is a solo instrument, it heightens the fact that his friend is not be there to hear the song. The poet's melancholy and isolation are immediate because we are already beside him. The world is seen as a physical system, imbued with meaning through the poet's relationship within it. We share his feeling at the exact moment that the poet realizes his own sadness.

The idea of systems and how they can deepen our insight into our relationships to the world of which we are a living part, is what this book is about. We treat it in terms of pattern because we think pattern is the basic element of cognition. But our approach to pattern is system based and it is this line of thought that leads to the principal mathematical concept lying behind our development of patterning : the *dynamical system*.

Dynamical systems arose initially in the study of the dynamics of a system of several planetary bodies moving around a sun under the mutual influences of gravity. In time this idea morphed into a mathematical theory that could frame and address any sort of question that involved temporal change. Later it morphed again to be a theory (or better, a collection of theories) that could address any form of change whatever, whether it be temporal, spatial, or otherwise; framed either in discrete or continuous forms; with or without aspects of chance and randomness. We will see that it is an idea that can express not only change, but also the nature of change in the forms of recurrence, equilibrium, evolution, emergence, chaos, and chance.

Although the idea of a system in which things are changing is quite natural and not in itself necessarily mathematical, the mathematical conceptualization of dynamical systems has proven to be important. As we will see, it offers extraordinary insights into what pattern is, as well as a rich vocabulary with which to express and develop those insights. This mathematics is not much about technical or computational skills, or the ability to slug through complicated looking equations and long chains of logical reasoning. It is really about concepts. And although these concepts can seem difficult at first, they are not difficult because they are complicated. They are difficult because they are unfamiliar.

Innovations in science are often inspired by questioning the conventional ways in which we have been conditioned to see things, and learning to see them in a different way. The framework of dynamical systems is like that. The forms in which we develop it here are the outcome of a great deal of very hard conceptual thinking by a large number of people over a period of several hundred years. Yet once we see them we wonder why they were so difficult to see. It is like the concept of heat. For a long time people thought that heat must be a material thing (it was called *caloric*.) This seemed like a reasonable idea, yet upon analysis the behavior of heat described in such terms was incomprehensible and there was no scientific progress in understanding it. Once it was realized that heat is nothing but the motion of atoms and molecules, the whole thing became clear.<sup>4</sup>

Fortunately the origins of dynamics, as Nuñez suggests, are deeply connected to our experience with the world. For sure the theory goes far beyond metaphor, but its roots