

Index

- $\mathbf{1}_A$, xii
 2^S , 652
 $a \vee b$, 38
 $a \wedge b$, 38
 $A(\mathbb{D})$, 85, 571
 $A \subseteq B$, 353
 $A \subset B$, 314
 $A^2(\mathbb{C})$, 111
 $A^2(\mathbb{D})$, 110
 $A^\perp$, 93, 134
 ${}^\perp A$, 134
 $B_b(\Omega)$, 287
 $BMO(\mathbb{R}^d)$, 86
 B_X , 3
 $\bar{B}_X$, 3
 $B(x; r)$, 639
 $\bar{B}(x; r)$, 639
 $\mathcal{B}(X)$, 650
 $\mathbb{C}$, xii
 c_0 , 33
 $\text{co}(F)$, 22
 $C([0, T]; \mathbb{K}^d)$, 43
 $C(K)$, 35
 $C(K; X)$, 78
 $C_0(X)$, 118
 $C_b(X)$, 35
 $C^\infty(D)$, 350
 $C^\infty(\bar{D})$, 350
 $C_c^\infty(D)$, 350
 $C^k(D)$, 350
 $C_c^k(D)$, 350
 $C_c^k(\bar{D})$, 350
 Δ_{Dir} , 415
 Δ_{Neum} , 416
 δ_{ij} , 98
 ∇ , 360
 $\partial(x)$, 505
 ∂^α , 350
 ∂_j , 350
 $D(A)$, xii
 $\mathbb{D}$, xii
 $\mathcal{E}(H)$, 562
 $\mathbb{E}$, 37
 ϕ_h , 601
 $f \otimes x$, 26
 $\mathcal{F}$, 180
 $\hat{f}$, 180
 $\Gamma(T)$, 610
 $\Gamma^n(V)$, 627
 $g \otimes h$, 250, 283
 γ , 104, 487
 $G(T)$, 176
 $H(\Omega)$, 218
 $H^2(\mathbb{D})$, 244
 $\mathcal{H}_n$, 601
 $|h\rangle$, 554
 $H^k(D)$, 375
 $H^s(\mathbb{R}^d)$, 372
 $H_0^1(D)$, 375
 $H_1 \oplus H_2$, 94
 $H_{\text{av}}^1(D)$, 381
 $\mathbb{H}^d$, 591
 id , 290
 $\text{Im } z$, xii
 J_n , 605
 K_h , 601
 $\Gamma^n(\mathbb{K}^d)$, 607
 $\mathcal{K}(X, Y)$, 228
 $\mathbb{K}$, xii
 $(\mathbb{K}^d)^{\otimes n}$, 606
 $\Lambda^n(V)$, 627
 $\mathcal{L}(X)$, 11

704

$\mathcal{L}(X, Y)$, 11
 $\mathcal{L}_1(H)$, 516
 $\mathcal{L}_2(H)$, 509
 ℓ^∞ , 33
 ℓ^p , 34
 $\ell^p(S)$, 84
 $L^1(0, T; X)$, 449
 $L^1_{\text{loc}}(D)$, 350
 $L^1_{\text{loc}}(\mathbb{R}^d)$, 60
 $L^\infty(\Omega, P)$, 290
 $L^p(\Omega)$, 48
 $L^p(\Omega) \otimes X$, 83
 $L^p(\Omega; X)$, 83
 $\text{Lip}[0, 1]$, 85
 $\mu^\pm$, 67
 Mf , 61
 $M(\Omega)$, 64
 $M_1^+(\Omega)$, 564
 $N(A)$, xii
 $NBV[0, 1]$, 85
 $\mathbb{N}$, xii
 P_Σ , 607
 $\mathcal{P}(H)$, 544
 $\mathcal{P}_{\text{fin}}(H)$, 546
 P_T , 627
 $\mathbb{P}$, 37
 $R(\lambda, T)$, 210
 $\text{Re } z$, xii
 $R(A)$, xii
 $\rho(T)$, 209
 $R(\lambda, A)$, 320
 $\mathbb{R}_+^d$, 366
 S° , xii
 Σ_ω , 457
 $\mathcal{S}(H)$, 550
 $\overline{S}$, xii
 $\sigma(T)$, 210
 $\sigma(\mathcal{C})$, 649
 $\sigma_p(T)$, 210
 $S \otimes T$, 627
 $|T|$, 271
 T^* , 138
 T^* , 141
 $\mathbb{T}$, xii
 U_g , 582
 $V \otimes W$, 625
 $V^{\otimes n}$, 627
 $V^{\odot n}$, 627
 V_γ , 582
 $W^{k,p}(D)$, 361
 $W_0^{1,p}(D)$, 363

Index

$|x|$, xii
 $(x|y)$, 87
 ξ^α , 373
 X^* , 12
 $X_0 \oplus X_1$, 6
 $\bar{z}$, xii
 absolutely
 continuous, function, 84
 continuous, measure, 69
 convergent, 3
 accretive
 form, 402
 maximal, 465
 operator, 464
 accretive form, 387
 adjoint
 of a bounded Hilbert space operator, 141
 of a bounded operator, 138
 of a densely defined Hilbert space operator, 319
 of a densely defined operator, 316
 semigroup, 501
 admissible contour, 218
 affine, 546
 algebra
 C^* , 303, 678
 Banach, 677
 von Neumann, 303
 algebraic
 basis, 113
 multiplicity, 234
 tensor product, 625
 analytic C_0 -semigroup, 457
 bounded, 457
 angle, 588
 angular momentum, 588
 annihilation operator, 614
 annihilator, 134
 pre-, 134
 antisymmetric tensor product, 627
 antiunitary, 578
 approximate
 eigensequence, 215
 eigenvalue, 214
 Arveson spectrum, 440
 atomic, 146, 544
 Axiom
 of Choice, 623
 of Determinacy, 674
 ball
 closed, 639
 open, 639

- Banach
 - algebra, 677
 - lattice, 76
 - limit, 168
 - space, 3
- basis
 - algebraic, 113
 - orthonormal, 98
 - Schauder, 202
- Bernstein polynomials, 36
- Bessel potential space, 372
- bicommutant, 302
- bit, 558
- Bloch
 - sphere, 558
 - vector, 558
- Bochner
 - integrable, 27
 - integral, 27
- boost, 582
- Borel
 - σ -algebra, 650
 - measurable, 663, 664, 666
 - measure, 651
 - topology, 640
- boson, 600
- boundary conditions
 - Dirichlet, 375
 - Neumann, 380
- bounded
 - analytic C_0 -semigroup, 457
 - form, 387
 - mean oscillation, 86
 - operator, 10
 - pointwise, 40
 - totally, 645
 - uniformly, 14
 - variation, 85
 - weakly, 173
- C^* -algebra, 303, 678
- C_0 -group, 437
- C_0 -semigroup, 430
 - adjoint, 501
 - analytic, 457
 - analytic contraction, 457
 - bounded analytic, 457
 - compact, 446
 - uniformly exponentially stable, 504
 - weakly continuous, 436
- C^k -boundary, 367
- Calderón–Zygmund theory, 676
- Calkin algebra, 241
- cartesian product, 636
- Cauchy problem
 - abstract, 429
 - inhomogeneous, 449
 - semilinear, 452
- Cauchy sequence, 642
- chain rule, 362
- chaos
 - Gaussian, 601
- character, 582
- closable
 - operator, 315
- closed
 - ball, 639
 - convex hull, 22
 - linear operator, 176, 312
 - set, 629, 639
 - subspace, 5
- closure, xii, 629, 640
 - of a closable operator, 315
 - of a form, 412
- cocycle identity, 580
- codimension, 133
- coercive form, 387, 402
- commutant, 302
- commutation
 - of selfadjoint operators, 308
- commutation relation
 - Heisenberg, 586
 - Weyl, 582
- commutator, 534
- compact, 631
 - operator, 227
 - relatively, 631
 - sequentially, 645
 - weak*, 152
- compactness
 - and extreme points, 147
 - in $C(K)$, 40
 - in $L^p(\Omega)$, 58
 - of the closed convex hull, 22
 - weak, 155
 - weak*, 152
- complement, xii
- complementarity, 584
- complemented, 132
- complete, 642
- completely positive, 688
- completion
 - of a Banach space, 4

706

of a Hilbert space, 92
 of a metric space, 644
 complex conjugate, xii
 conditional expectation, 111
 conjugate dual, 405
 conjugate exponents, 51
 conjugate-linear, 87
 connected, 360
 conserved, 581
 continuous, 630, 644
 at a point, 630, 644
 embedding, 404
 form, 403
 sequentially, 644
 uniformly, 645
 contraction, 14
 as a convex combination of unitaries, 520
 dilation to a unitary, 276
 represented by a POVM, 571
 von Neumann theorem, 299
 convergence
 strong, of operators, 14
 uniform, of operators, 14
 weak, of operators, 14
 convergent, 2, 640
 convex
 function, 50
 hull, 21
 set, 93
 convolution, 57
 multiplicative, 82
 twisted, 594
 core, 433
 countably generated, 574
 covariant, 581
 cover
 open, 631
 creation operator, 614
 decomposition
 Hahn, 67
 Jordan, 67
 orthogonal, 94
 Wold, 279
 decomposition property
 of a vector lattice, 127
 defect operator, 275
 dense, 629
 densely defined, 311
 density
 argument, 12
 operator, 552

Index

derivative
 weak, 352
 diagonal argument, 96
 dilation, 191
 of a bounded operator, 280
 unitary, 276
 direct sum
 orthogonal, 94
 Dirichlet
 boundary conditions, 375
 kernel, 203
 Laplacian, 415
 problem, 397
 Dirichlet-to-Neumann operator, 682
 disc algebra, 571
 disjoint
 projections, 546
 dispersive, 685
 distance function, 639
 distribution, 663
 domain
 of a form, 402
 of a linear operator, 311
 doubly submarkovian, 481
 dual
 conjugate, 405
 of a normed space, 12
 Pontryagin, 582
 duality, 115
 trace, 521
 dyadic, 633
 Dynkin system, 651
 effect, 562
 eigensequence
 approximate, 215
 eigenspace, 232
 eigenvalue, 210
 approximate, 214
 eigenvector, 210
 elementary observable, 555
 elliptic problem, 386
 embedding
 continuous, 404
 energy
 ground state, 492
 energy functional
 associated with a coercive form, 428
 for the Dirichlet problem, 397
 for the elliptic problem, 386
 for the Poisson problem, 379, 385
 for the wave equation, 497

- entangled state, 560
- equation
 - Schrödinger, 492
 - Schrödinger, abstract, 580
 - wave, 494
- equicontinuous, 40
- equivalent norm, 18
- essential
 - range, 224
 - range, P -, 340
 - supremum, 47
- essentially
 - bounded, 47
 - selfadjoint, 324
- euclidean norm, 7
- Euler's identity $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$, 102, 539
- exact sequence, 250
- existence and uniqueness of solutions
 - under a Lipschitz assumption, 42
- existence of solutions
 - under a continuity assumption, 45
- expected value, 37
 - of an observable, 556
- extension
 - Friedrichs, 412
 - of a functional, 128
 - of a linear operator, 314
 - operator, 370
- extension, vector-valued, 83, 674
- exterior product, 527, 627
- extreme point, 146
- $\star$ -field, 687
- face, 147
- Fejér kernel, 101
- fermion, 600
- finite element method, 398
- finite intersection property, 631
- finite rank operator, 228
- finitely additive, 563
- first quantisation, 600
- fixed point, 42
 - argument, 43, 455
- form, 387, 402
 - accretive, 387, 402
 - bounded, 387
 - closed, 403, 409
 - coercive, 387, 402
 - continuous, 403
 - Hermitian, 687
 - sectorial, 466
- formula
 - MacMahon, 528
 - Stone, 308
- Fourier
 - coefficient, 101
 - multiplier, 187
 - series, 102
 - transform, 180, 186
- Fourier–Plancherel transform, 185
- Fredholm
 - alternative, 235
 - alternative, for integral equations, 237
 - determinant, 528
- Friedrichs extension, 412
- functional, 12
 - normal, 550
 - positive, Hilbertian, 550
- functional calculus
 - bounded, 287
 - continuous, 266, 268
 - entire, 217
 - holomorphic, 218
 - measurable, 329
- Gaussian
 - chaos, 601
 - measure, standard, 104, 487
- Gelfand triple, 404
- generator, 430
- geometric multiplicity, 233
- gradient, 360
- Gram–Schmidt orthogonalisation, 99
- graph
 - norm, 312
 - of a bounded operator, 176
 - of a linear operator, 312
- Green function, 376
- ground state energy, 492
- group
 - free Schrödinger, 492
 - Heisenberg, 591
 - Schrödinger, 492
 - translation, 476
 - unitary, 474
 - wave, on $\mathbb{R}^d$, 495
 - wave, on domains, 494
- Haar
 - measure, 582
 - property, 165
- Hahn decomposition, 67
- Hardy space, 244
- Hardy–Littlewood maximal function, 61
- harmonic, 206

708

heat
 equation, linear, 477
 equation, positivity, 480
 equation, semilinear, 455
 kernel, 477
 semigroup, on $\mathbb{R}^d$, 477
 semigroup, on domains, 479
 Heisenberg
 commutation relation, 586
 group, 591
 Hermite
 operator, 491
 polynomials, 104
 Hermitian
 form, 687
 vector space, 687
 hidden variables, 574
 Hilbert space, 91
 characterisation via category theory, 688
 characterisation via complementation, 674
 characterisation via Hermitian forms, 687
 Hilbert transform, 191
 L^p -boundedness, 197
 and the Poisson semigroup, 486
 characterisation, 192
 holomorphic
 X -valued function, 163
 weakly, 201
 homotopy, 246
 ideal property
 of compact operators, 228
 of Hilbert–Schmidt operators, 510
 of trace class operators, 520
 identity
 Parseval, 97
 image measure, 663
 imaginary part, xii
 index, 239, 420
 indicator function, xii
 indiscernible, 574
 inequality
 Bessel, 107
 Cauchy–Schwarz, 88
 Clarkson, 207
 Hölder, 50
 Hölder, converse to, 52, 177
 Jensen, 81
 Minkowski, 48
 Minkowski, continuous, 83
 Poincaré, 376
 Poincaré–Wirtinger, 383

Index

triangle, 2, 639
 Young, 56
 infinite-dimensional, 9
 infinitesimal generator, 430
 inner product, 87
 space, 87
 integrable, 668
 Bochner, 27
 uniformly, 169
 integral
 Bochner, 25, 27
 Lebesgue, 665
 Pettis, 205
 Riemann, 23
 stochastic, 114
 interior, xii, 629, 640
 involution, 678
 irreducible, 591
 isometric isomorphism, 14
 isometry, 4
 partial, 271
 spectrum of, 214
 isomorphism, 14
 Itô isometry, 114
 Jordan
 decomposition, 67
 normal form, 234
 Kato square root problem, 686
 kernel
 Dirichlet, 203
 Fejér, 101
 heat, 477
 Poisson, 483
 Poisson, for the disc, 109
 ket, 554
 Laguerre polynomials, 108
 Laplace operator
 Dirichlet, 415
 Neumann, 416
 on $\mathbb{R}^d$, 414
 weak, 372
 Laplace transform, 435
 inverse, 458
 of a measure, 537
 lattice, 74
 Banach, 76
 normed vector, 75
 vector, 74
 law of large numbers, 37
 Lebesgue

- measure, 656
- measure, normalised, 180
- point, 63
- set, 83
- Leibniz formula, 360
- lemma
 - Céa, 398
 - Cotlar, 198
 - Dynkin, 651
 - Fatou, 668
 - Riemann–Lebesgue, 180
 - Riesz, 20
 - three lines, 193
 - Urysohn, 633
 - Vitali covering, 61
 - Zorn, 623
- limit, 2, 640
 - Banach, 168
- linear operator, 311
- Lipschitz continuous, 42
- locally compact, 118
- locally integrable, 60, 350
- lower bound, 74
- m -accretive, 465
- Malliavin calculus, 487, 601
- maximal element, 623
- maximal regularity
 - for bounded analytic semigroups, 468
 - for the Poisson problem on $\mathbb{R}^d$, 418
- measurable, 663
 - Borel, 663
 - set, 649
 - strongly, 26
 - weakly, 136
- measurable space, 649
- measure, 650
 - $\mathbb{K}$ -valued, 64
 - σ -finite, 651
 - outer regular, 659
 - atomic, 146, 544
 - Borel, 651
 - finite, 651
 - inner regular, 659
 - Lebesgue, 656
 - outer, 652
 - probability, 651
 - Radon, 661
 - Radon, $\mathbb{K}$ -valued, 118
 - regular, 659
 - standard Gaussian, 104, 487
- measure space, 650
- Mehler kernel, 489
- metric, 639
- metric space, 639
- metrisable, 154
- minimiser, 93, 379, 385, 386, 397, 428
- Minkowski functional, 143
- mixed state, 554
- modulus, xii
 - of an operator, 271
- mollification, 57
- momentum operator, 325, 584, 617
- multi-index, 349
- multiplication
 - semigroup, 475
- multiplicity, 264, 283
 - algebraic, 234
 - geometric, 233
- multiplier
 - Fourier, 187
 - pointwise, 188
- negation, 545
- Nemytskii mapping, 456
- Neumann
 - boundary conditions, 380
 - Laplacian, 416
 - series, 211
- norm, 2
 - equivalent, 18
 - euclidean, 7
 - nonequivalent, 113
 - product, 6
- normal, 632
 - functional, 550
 - operator, 256, 328
- normed space, 2
- null space, xii, 13
- numerical
 - radius, 277
 - range, 277
- observable, 544, 555
 - elementary, 544, 555
 - sharp, 555
 - unsharp, 564
- open
 - ball, 639
 - cover, 631
 - set, 629, 639
- operator
 - adjoint, Hilbertian, 141
 - adjoint, of a bounded operator, 138
 - adjoint, of a densely defined operator, 316

710

angular momentum, 620
 annihilation, 614
 antiunitary, 578
 bounded, 10
 boundedly invertible, 209
 closable, 315
 closed, 176, 312
 compact, 227
 creation, 614
 densely defined, 311
 Dirichlet-to-Neumann, 682
 divergence form, 417
 doubly submarkovian, 481
 essentially selfadjoint, 324
 extension, 370
 finite rank, 228
 Fourier multiplier, 187
 Fredholm, 239, 420
 Hermite, 491
 Hilbert–Schmidt, 507
 linear, 311
 momentum, 325, 617
 normal, 256, 328
 Ornstein–Uhlenbeck, 488
 position, 296, 617
 positive, 324
 positive, Hilbertian, 256
 positivity preserving, 76
 quotient, 15
 selfadjoint, 256, 324
 symmetric, 324
 Toeplitz, 245
 trace, 680
 trace class, 511
 translation invariant, on $L^1(\mathbb{R}^d)$, 157
 translation invariant, on $L^2(\mathbb{R}^d)$, 189
 unbounded, 311
 uniformly elliptic, 466
 unitary, 256, 591
 Volterra, 17

order
 of a multi-index, 349

Ornstein–Uhlenbeck
 operator, 488
 semigroup, 487

orthogonal, 92
 complement, 93
 decomposition, 94
 polynomials, 305
 projection, 94, 258

orthomodular, 687

Index

orthonormal
 basis, 98
 system, 98
 system, maximal, 100

outer measure, 652

parallelogram identity, 89

partial
 isometry, 271
 order, 623
 trace, 524

partition of unity, 634
 smooth, 351

Pauli matrices, 559

perturbation, 442

Pettis integral, 205

phase, 572

point
 evaluation, 16
 spectrum, 210, 447

pointwise
 bounded, 40
 convergence, vs. norm convergence, 202
 multiplier, 16, 188

Poisson
 kernel, 2-dimensional, 206
 kernel, d -dimensional, 483
 kernel, for the disc, 109
 problem, Dirichlet boundary conditions, 375
 problem, inhomogeneous boundary conditions, 396
 problem, Neumann boundary conditions, 380
 semigroup, 483

polarisation, 256

polynomials
 Bernstein, 36
 Hermite, 104
 Laguerre, 108

Pontryagin dual, 582

position operator, 296, 584, 617

positive
 cone, 76
 definite, 272, 569
 functional, Hilbertian, 550
 operator, 324
 operator, Hilbertian, 256

positive definite, 532

positivity preserving, 76

potential, 491

POVM, 563

power bounded, 226

power set, 652

- pre-annihilator, 134
- principle of local reflexivity, 675
- probability space, 651
- problem
 - abstract Cauchy, inhomogeneous, 448
 - abstract Cauchy, linear, 429
 - abstract Cauchy, semilinear, 452
 - Dirichlet, 397
 - elliptic, 386
 - Poisson, 374
 - Sturm–Liouville, 389
- product
 - σ -algebra, 650
 - cartesian, 636
 - measure, 659
 - metric, 648
 - norm, 6
 - rule, 359, 434
- projection, 132
 - orthogonal, 94, 258
 - spectral, 221
- projection-valued measure, 285
 - of a bounded normal operator, 293
 - of a normal operator, 337
- pure state, 543, 553
- quantisation
 - first, 600
 - second, 600
- quantum harmonic oscillator, 491
- quasi-optimality estimate, 398
- quaternions, 687
- quotient
 - Banach spaces, 5
 - map, 15
 - operator, 15
- Radon measure, 661
- random variable, 663
- range, xii, 14
- real part, xii
 - of a form, 402
- recurrence
 - three point, 305
- reflexive, 155
- relation, 623
- relatively compact, 631
- representation, 272
 - irreducible, 591
 - projective, 590
 - Schrödinger, 591
 - unitary, 272
- resolvent
 - identity, 212, 320
 - operator, 210
 - set, 209
 - set, of a linear operator, 320
- Riemann
 - hypothesis, 675
 - integral, 24
- Riesz projection, 245
- ring, 653
- σ -algebra, 649
 - Borel, 650
 - generated by $\mathcal{C}$, 649
 - generated by f , 650
- scalar homogeneity, 2
- Schauder basis, 202
- Schrödinger
 - equation, 492
 - equation, abstract, 580
 - group, 492
 - representation, 591
- Schur
 - property, 169
- Schur estimate, 82
- second quantisation, 600
- sector, 457
- sectorial, 466
- Segal–Plancherel transform, 612
- selfadjoint
 - essentially, 324
 - operator, 256, 324
- semigroup
 - heat, on $\mathbb{R}^d$, 477
 - heat, on domains, 479
 - multiplication, 475
 - Ornstein–Uhlenbeck, 487
 - Poisson, 483
- seminorm, 129
- separable
 - measure space, 510
 - metric space, 648
 - normed space, 9
- sequence
 - convergent, 2
 - singular value, 512
- sequentially
 - closed, 640
 - compact, 645
 - continuous, 644
- sesquilinear, 87
- sharp observable, 555
- simple function, 664

712

Index

- μ -, 27
 - vector-valued, 26
- singular integrals, 676
- singular value, 512
 - decomposition, 283, 512
 - sequence, 512
- Sobolev space, 361
- solution
 - classical, of the linear Cauchy problem, 449
 - classical, of the semilinear Cauchy problem, 452
 - global, 42
 - local, 44
 - mild, of the inhomogeneous Cauchy problem, 450
 - mild, of the semilinear Cauchy problem, 452
 - strong, 449
 - weak, of the elliptic problem, 386
 - weak, of the inhomogeneous Cauchy problem, 503
 - weak, of the Poisson problem, 375, 381, 396
- spectral
 - inclusion formula, 502
 - mapping theorem, for normal operators, 340
 - mapping, for compact semigroups, 446
 - mapping, the holomorphic calculus, 220
 - projection, 221
 - radius, 222
- spectral flow, 682
- spectrum, 210
 - approximate point, 215
 - Arveson, 440
 - of a compact operator, 232
 - of a linear operator, 320
 - point, 210, 447
- spin, 689
- state, 543, 551
 - entangled, 560
 - mixed, 554
 - pure, 543, 553
 - spin, 559, 689
 - vector, 553
- stiffness matrix, 400
- stochastic integral, 114
- Stone's formula, 308
- Strichartz estimate, 685
- strictly convex, 107, 165, 208
- strong convergence, of operators, 14
- strong operator topology, 302
- strongly measurable, 26
- Sturm–Liouville problem, 389
- subadditive mapping, 199
- subalgebra, 216
 - $\star$ -, 265
- subcover, 631
- subdifferential, 505
- sublinear, 128
- substitution formula, 668
- substitution rule, 362
- superposition, 554
- support, 633
 - of a continuous function, 54
- supremum norm, 35
- symbol
 - of a Fourier multiplier operator, 187
 - of a Toeplitz operator, 245
- symmetric
 - Fock space, 610
 - operator, 324
 - part, 402
 - second quantisation, 610
 - sesquilinear mapping, 388
 - tensor product, 607, 627
- symmetry, 580
- tensor basis, 106
- tensor product
 - algebraic, 625
 - antisymmetric, 627
 - of Hilbert spaces, 524
 - of vector spaces, 625
 - symmetric, 607, 627
- test function, 350
- theorem
 - Arzelà–Ascoli, 40
 - Atkinson, 239
 - Baire, 171
 - Banach fixed point, 42
 - Banach–Alaoglu, 152
 - Beckner, 676
 - bicommutant, 302
 - Bolzano–Weierstrass, 647
 - Busch, 563
 - Cauchy, for Banach space-valued functions, 163
 - Chernoff, 225
 - closed graph, 177
 - closed range, 178
 - Coburn, 249
 - continuous functional calculus, 266, 268
 - Courant–Fischer, 422
 - Datko–Pazy, 504
 - Dieudonné, 242
 - Dini, 77
 - dominated convergence, 669
 - Euler formula, 444

- Fedosov, 534
 Fréchet–Kolmogorov, 58
 Fredholm alternative, 235
 Friedrichs, 412
 Fubini, 670, 671
 Fuglede–Putnam–Rosenblum, 264
 Gearhart–Prüss, 505
 Gelfand–Naimark, 678
 Goldstine, 154
 Green, 380
 Hahn–Banach extension, for Banach spaces, 130
 Hahn–Banach extension, for vector spaces, 128, 129
 Hahn–Banach separation, 143
 Hahn–Jordan, 67
 Hardy–Littlewood maximal, 61
 Hartman–Wintner, 248
 Hausdorff–Young, 197
 Hellinger–Toeplitz, 277
 Helton–Howe, 535
 Hille, 347
 Hille–Yosida, 441
 injectivity of the Fourier transform, 186, 187
 inversion of the Fourier transform, 182
 Krein–Milman, 147
 L^p -boundedness of the Hilbert transform, 197
 Lax–Milgram, 388
 Lebesgue differentiation, 63
 Lidskii, 527
 Lumer–Phillips, 463, 506
 Lusin, 81
 Marcinkiewicz, 199
 Mercer, 532
 min-max, 284, 422
 monotone convergence, 667
 Naimark, 568
 Noether–Gohberg–Krein, 246
 Nyman & Baez–Duarte, 675
 open mapping, 175
 Parseval, 97
 Peano, 45
 Pettis measurability, 26, 136
 Phillips, 436
 Picard–Lindelöf, 42
 Plancherel, 185
 Portmanteau, 159
 Prokhorov, 159
 Radon–Nikodým, 69, 112
 Rellich–Kondrachov, 381
 Riesz representation, for Hilbert spaces, 95
 Riesz–Schauder, 232
 Riesz–Thorin, 192
 Segal, 613
 singular value decomposition, 283, 512
 Sobolev embedding, 394
 Solèr, 687
 spectral mapping, for bounded normal operators, 269
 spectral mapping, for normal operators, 340
 spectral mapping, for the holomorphic calculus, 220
 spectral mapping, for the point spectrum, 447
 spectral radius formula, 223
 spectral, for bounded normal operators, 293
 spectral, for compact normal operators, 281
 spectral, for normal operators, 337
 Stinespring, 689
 Stone, 474, 500
 Stone–von Neumann, 591
 Stone–Weierstrass, 38
 Sz.-Nagy, 276, 502
 three-point recurrence, 306
 Tietze extension, 635
 Toeplitz–Hausdorff, 278
 trace duality, 522
 Tychonov, 636
 uniform boundedness, 172
 von Neumann, bicommutant, 302
 von Neumann, bicommutant of a selfadjoint operator, 304
 von Neumann, on commuting selfadjoint operators, 300
 von Neumann, on contractions, 299
 Weierstrass approximation, 36
 Weyl, 424
 Wiener’s Tauberian, 183
 Wiener–Itô isometry, 607
 Wigner, 579
 Wintner, 619
 three point recurrence, 305
 tight, 660
 uniformly, 158
 Toeplitz operator, 245
 topological space, 629
 countably generated, 574
 Hausdorff, 630
 locally compact, 118
 normal, 632
 topology, 629
 Borel, 640
 generated by a collection of sets, 629
 strong operator, 302

714

weak, 149
 weak operator, 302
 weak*, 149
 totally
 bounded, 645
 ordered, 623, 636
 trace
 class, 511
 duality, 521
 of a positive operator, 510
 of a trace class operator, 515
 operator, 680
 partial, 524
 trace formula
 and Euler's identity, 539
 and Fredholm operators, 534
 and Toeplitz operators, 535
 for the Dirichlet heat semigroup, 537
 transform
 Fourier, 180, 186
 Fourier–Plancherel, 185
 Hilbert, 191
 Segal–Plancherel, 612
 transition probability, 578
 translation, 582
 adjoint of, 139
 continuity of, in L^p , 55
 invariant, operators on $L^1(\mathbb{R}^d)$, 157
 Wiener's Tauberian theorem, 183
 triangle inequality, 2, 639
 reverse, 2
 trigonometric polynomial, 40
 twisted convolution, 594
 UMD property, 674
 unbounded operator, 311
 uncertainty
 of an observable, 557
 principle, 557
 uniform convergence
 of functions, 8
 of operators, 14
 uniformly
 bounded, 14
 continuous, 645
 contractive, 42
 exponentially stable, 504
 integrable, 169
 tight, 158
 unit ball, 3
 unital, 678
 unitary

Index

dilation, 276
 operator, 256, 591
 representation, 272
 unsharp
 observable, 564
 upper bound, 74, 623
 vanishing at infinity, 118
 variance
 of an observable of A in the state $|h\rangle$, 556
 variation, 64
 variational method
 for the Poisson problem, 379, 385
 vector lattice, 74
 normed, 75
 vector state, 553
 Volterra operator
 norm of, 260
 spectrum of, 279
 von Neumann algebra, 303
 wave
 equation, 494
 function, 554
 group, on $\mathbb{R}^d$, 495
 group, on domains, 494
 weak
 compactness, of B_X , 155
 convergence, 150
 convergence, of measures, 158
 convergence, of operators, 14
 derivative, 352
 Laplacian, 372
 operator topology, 302
 topology, 149
 weak L^1 -bound, 61
 weak*
 compactness, of B_{X^*} , 152
 continuous, functional, 150
 convergence, 150
 topology, 149
 weakly
 bounded, 173
 closed, 150
 continuous, functional, 150
 continuous, semigroup, 436
 holomorphic, 201
 well posed, 451, 454, 531
 Weyl commutation relation, 582
 winding number, 217, 246
 Wold decomposition, 279