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978-1-009-49465-6 — A Prolegomenon to Differential Equations and Variational Methods
on Graphs

Yves van Gennip , Jeremy Budd

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Cambridge Elements

Elements in Non-Local Data Interactions: Foundations
and Applications

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Centre National de la Recherche Scientifique (CNRS)

A PROLEGOMENON TO DIFFERENTIAL EQUATIONS AND VARIATIONAL METHODS ON GRAPHS

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Abstract: The study of differential equations on graphs emerged about 15 years ago as a way of providing a framework for the mathematical analysis of images. Since then it has burgeoned and has expanded more broadly to encompass image processing and, more recently, machine learning. The authors have provided a bird's-eye view of both theoretical developments and concrete applications that will enable newcomers to quickly get a flavour of key results. Additionally, they provide an exhaustive, at the time of writing, bibliography which will point readers to where fuller details and other directions can be explored. This title is also available as open access on Cambridge Core.

Keywords: differential equations on graphs, variational methods on graphs, machine learning, image analysis, image processing

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