Introduction

Logical pluralism is the view that there is more than one correct logic. A logic is a formal system that gives an account of valid arguments or proofs. There are many *interesting* logics, to be sure, but we might expect just one of them to be *correct* at the end of the day. Pluralism upends this expectation with its endorsement of many logics.

Ideally, a pluralist view will offer us a satisfying path between unity and plurality. On the one hand, the pluralist should draw a meaningful distinction between *logic* and *other* subject matters. Doing so bestows a significant unity to the phenomenon. It tells us how we know that we are talking about different instances of one and the same thing. On the other hand, this distinction should be drawn in a way that allows more than one thing to be correctly categorized on the side of logic. It is a useful exercise to reflect on which versions of logical pluralism successfully navigate this path.

There are a number of well-known sources of diversity and uncertainty in logic that do not rise to the level of pluralism. For one thing, the field of logic is largely unified by its tools. Modern logicians heavily rely on mathematical systems called *formal logics* and these systems are quite flexible. Formal logics are widely used in technical applications such as circuit design and computer program verification, but these applications are somewhat orthogonal to the traditional philosophical aims of logic. It is completely uncontroversial to say that there is more than one useful formal logic for technical applications. This is not open to debate and it is not what logical pluralists care about.

When philosophers discuss the subject matter of logic, they often use value terms like "good reasoning" and "rationality," but these normative concepts raise all sorts of challenges for philosophy and logic. An agent who prefers efficient solutions to problems might value different methods of reasoning than an agent who prefers more accurate solutions. Both types of reasoning are good in their own way. An expressivist about normative appraisals might even say that statements about good reasoning and rational belief do not have determinate content. If logic is entwined with these issues, it may be impossible for us to *justify* one logic over another. Still, diffuse skepticism about rationality and the justification of logic are also not what logical pluralists care about.

Logical pluralism stakes out a different position altogether. It says that there is a many-one relationship between logics *qua* formal theories, and the phenomena that these theories are intended to study or codify. We cannot isolate one logic that triumphs over all others. Pluralists have a number of different reasons for holding this view.

Some pluralists consider logics to either be creative devices or at best only indirectly conducive to an understanding of their target phenomena. On this type of view, logics play an active role in regimenting the concept of validity and more than one logic successfully fulfills this task. Another strand of pluralism contends that our preexisting concept of validity is underspecified in a distinctive way. On this type of view, logics target an interesting property, but we discover that this really bifurcates into more than one property of the same kind. Pluralists claim, for various reasons, that it is futile to search for a single, uniquely correct logic. More than one logic satisfies the criteria we care about and there is “no further fact of the matter” about which logic is correct (Restall, 2002, p. 426).

Obviously, there is much more to say about this research program. What is validity, how do we theorize this phenomenon, and why do these factors allegedly conspire to produce pluralism? Does pluralism imply that all logics are equal? When logicians disagree about the correct logic, does logical pluralism imply that these logicians are talking past one other or stuck in a pointless muddle? Is logical pluralism a stable and coherent view in the first place? Does it have any interesting consequences for how we think about the foundations of science and mathematics? This Element addresses such issues.

The earliest seeds of logical pluralism are found in the work of Alfred Tarski and Rudolph Carnap from the 1930s onward. It is unclear whether they would have accepted modern versions of the view, but their ideas were a great source of inspiration for logical pluralists. Susan Haack first used the term “logical pluralism” in roughly the modern sense, relating it to several other points of view on logical systems. Haack tentatively defends logical pluralism by drawing on ideas about scientific model-building.

The current debate on logical pluralism kicked off in the early 2000s with the ambitious work of Jc Beall and Greg Restall. They advanced an interesting pluralist view of logic that sparked dozens of responses from both critical and sympathetic points of view. It is fair to say that the vast majority of the recent literature on pluralism is connected to this particular theory in some way; but as the literature matured, it also extended well beyond the idiosyncrasies of Beall and Restall’s view. Another excellent inroad to this debate is the Stanford Encyclopedia’s entry “Logical Pluralism” (Russell, 2021).

Logical pluralism has its detractors. There is some pushback from the perspective of the history of logic, while other problems are alleged to be internal to the pluralist view itself. As we will see, there are quite a few versions of logical pluralism. It often seems as if different pluralists advocate for views quite unrelated to one another. My greatest hope for this Element is that it tells a convincing story about the natural evolution of pluralist thought from its early roots to its current state.

This Element is written in the hope that it will be accessible to a wide swath of readers. It does not rely on extensive prior knowledge. To that end, the Element has three parts. Section 1 offers a mini-textbook treatment of the modern practices of formal logic. This clarifies the sort of thing that philosophers are concerned with when they raise questions about correct logic(s). Section 2 covers the history of logical pluralism, different approaches to pluralism, as well as some discussion of how these views contrast with each other. Section 3 reviews some of the prominent worries that have been lodged against logical pluralism and how pluralists may handle these worries.

1 Doing Logic

1.1 Validity

Logic is concerned with good arguments and, most especially, with *proofs* (arguments that are important in mathematics and abstract theory-building). It is good for arguments to be clear and economical, but logic is not concerned with those stylistic features. Logic is concerned with a standard of *correctness* for arguments, known as *validity*.

Here is a typical example of a valid argument.

Example 1

(1.1) All dogs are mammals.
(1.2) All mammals are animals.
$\therefore$ All dogs are animals.

This argument starts with premises (1.1) and (1.2) and ends with the concluding sentence “All dogs are animals.” The conclusion is marked with the special symbol “$\therefore$.” We can read this symbol as expressing the phrase “Therefore….” To check whether this argument is valid we need to *evaluate* the connection between its premises and conclusion.

When we say that an argument is valid (correct), it means that the right kind of connection is really there; if the connection is missing, the argument is invalid (incorrect). Standard textbook logic says that Example 1 is valid; the premises of the argument really do support its conclusion; the conclusion follows from the premises. These are three different ways of expressing that the argument has the right logical properties.

The roots of this field of study in Western philosophy trace back to Aristotle: “A deduction is speech (logos) in which, certain things having been supposed, something different from those supposed results of necessity because of their being so" (*Prior Analytics* I.2, 24b18–20, trans. Smith 1989).

The translation of this passage uses the word "deduction" to refer to roughly the same thing as a valid argument. Aristotle suggests that when we make certain assumptions, lo and behold, something else results *of necessity*. A deduction or valid argument expresses this kind of airtight connection between its premises and conclusion.

Good arguments and reasonable conclusions are just as important in empirical science as they are in logic. However, there is a difference in emphasis. Science is much more concerned with actual, observable facts. For example, a physicist might ask: Is it actually true that dark matter is composed of axion particles? This is a question about how things work in the physical world. Reasoning and argument are indirectly involved only because they can help scientists find good answers to their questions.

Logic directly concerns itself with reasoning and argument themselves. We can see logical questions as *hypotheticals*. A typical question of logic is something like this: assuming that everything in the universe is composed of axion particles, does it follow that dark matter is composed of axion particles? We do not need to know real physics to answer this question. It is not a question about matter or particles. The logical question is about *what follows from what*. Hypothetical thinking along these lines has an important role in all walks of life, but logic makes this into its primary topic of study.

If the conclusion of an argument really does follow from the premises by an airtight, necessary connection, then the argument is valid.

Concept. (Valid Argument) An argument whose conclusion follows from its premises.

It would be nice to know which arguments are valid. In modern logic, we build systematic accounts of validity. There are different accounts, hence different logics.

The main job of logic is to classify valid arguments. Logical pluralism says that there is more than one correct way to do this. In order to discuss this philosophical view, it might help to know something about the methods of modern logic.

The rest of this section introduces the methods of modern, formal logic. The goal is to showcase how minor variations and choices in these methods produce a variety of different logical systems. This will make certain aspects of logical pluralism much clearer. If you already know the difference between first-order logic and second-order logic, or that between classical logic and non-classical logic, you can skip ahead to the summary of key results in Section 1.5. For everyone else: read on.

1.2 Forms, Rules, Models

Perhaps the most important idea in modern logic is this: Validity is often determined by the internal structure or *logical form* of arguments. Some examples should help to make the idea of logical form clearer. Here is an example of an argument.

Example 2

(2.1) Fish swim.
(2.2) Birds fly.
$\therefore$ Fish swim and birds fly.

Here is another argument with the same form.

Example 3

(3.1) Water is wet.
(3.2) Fire is hot.
$\therefore$ Water is wet and fire is hot.

When we say that Examples 2 and 3 have the same form, what are we talking about? It probably seems quite obvious, but let's spell it out. Each argument has two premises. The conclusion is a more complex sentence that joins the premise sentences together with the word "and" between them. That pattern is exactly the same between both arguments. Formal logic uses such patterns to analyze which arguments are valid.

We use *formal languages* to make this sort of analysis clear. A formal language represents sentence parts with symbols. In this Element, we will use the lowercase letters p, q, r to stand for whole, declarative sentences. We also have special *logical* symbols:

- The negation connective $\neg$ is read as the English word "not."
- The conjunction connective $\wedge$ is read as the English word "and."
- The disjunction connective $\vee$ is read as the English word "or."
- The conditional connective $\rightarrow$ is read as the English phrase "if…, then…."

The rules for putting these symbols together create *formulas*. Each formula represents an underlying structure that occurs in many sentences, based on the parts of the sentence and how those parts are put together. An argument form is simply the collection of formulas that represent the logical forms of the sentences in the argument.

For example, here is the argument form shared by Examples 2 and 3.

Example 4

(4.1) p
(4.2) q
$\therefore$ $p \wedge q$

This should be easy enough to understand. The formula $p \wedge q$ represents a sentence in which the simpler sentence p is joined with q by putting the word "and" between them, just like we saw in Examples 2 and 3.

The type of logic that draws on this formal language is called *propositional* logic. It only focuses on the aspects of argument form that involve the placement of whole, declarative sentences and the logical connectives. We could imagine various ways to expand this formal language to represent other operators on sentences. For example, we could include symbols for *modalities.* In the formal language of modal logic, $\Box p$ says "p is necessary" and $\Diamond p$ says "p is possible." This allows us to represent a wider range of argument forms than is possible with just the basic language of propositional logic.

Here is another pair of examples to think about.

Example 5

(5.1) Alytus is small.
(5.2) Vilnius is large.
$\therefore$ Something is small and something is large.

The following argument has the same form as the preceding one.

Example 6

(6.1) Chocolate is delicious.
(6.2) Vegemite is disgusting.
$\therefore$ Something is delicious and something is disgusting.

When we say that Examples 5 and 6 have the same form, what are we talking about? Each argument has two premises that ascribe a property to some specific, named object. The conclusion sentence has the word "and" in the middle.

Each half of the conclusion sentence talks about one of the same properties from the premises, but instead of ascribing that property to a specific, named object, it says more generically that *something* has the salient property. Again, this pattern is exactly the same between both arguments.

To represent this kind of pattern we need a formal language that can represent the internal parts of sentences. We use the lowercase letters a, b, c to stand for names of specific people, places, or things. We use the uppercase letters F, G, H to stand for predicates, which express properties of things. We also need more *logical* symbols:

- The existential quantifier $\exists$ is read as the English phrase "there is…."
- The universal quantifier $\forall$ is read as the English phrase "for all…."
- The variables x, y, z make clear how the quantifiers are used.

Here is the argument form shared by Examples 5 and 6.

Example 7

(7.1) Fa
(7.2) Gc
$\therefore$ $(\exists xFx \wedge \exists yGy)$

We read this as follows. Fa represents the form of a sentence that applies predicate F to name a, so it ascribes a property to some specific, named object. Likewise for Gc. This is just like the premises in Examples 5 and 6. $\exists xFx$ attaches the quantifier phrase to a variable, which you can think about as a sort of "test" to see whether any chosen object has property F. In other words, $\exists xFx$ represents the form of a sentence that says there is at least one thing that satisfies the Fx "test," or in more natural English, it just says *something has property F*. Likewise, $\exists yGy$ says that *something has property G*.

The type of logic that draws on this formal language is called *first-order* logic. It focuses not only on sentences and logical connectives, but other aspects of argument form that involve the placement of names, predicates, and objectual quantifiers. We could imagine various ways to expand this formal language to represent other types of quantifiers. For example, we could add *second-order* quantifiers to generalize over predicate position. In the language of second-order logic, $\forall XXa$ says "All properties apply to a" and $\exists XXc$ says "Some property applies to c." This allows us to represent a wider range of argument forms than is possible with just the basic language of first-order logic.

Formal languages draw our attention to specific aspects of sentence structure, which is extremely helpful for logical analysis, but it also involves a choice. Should our logical tools pay attention to modal operators or second-order quantifiers? These are not part of standard, introductory logic books.

Are modal operators or second-order quantifiers inappropriate for logical study? We will revisit this question about the *logical vocabulary* in Section 2.2 and consider whether it may be connected with logical pluralism.

Creating a formal language is only the first step into formal logic. The next natural question is: What do we do with this structural information? How can we use argument forms to classify valid arguments? There are two prominent techniques for doing this. One of them uses *proof rules*, the other uses *semantic models*.

Outside of formal logic, a proof is just a piece of conclusive reasoning. Most of us encounter proofs in mathematics. Just think about examples you have seen, like the proof that $\sqrt{2}$ is irrational or the proof of the Pythagorean theorem which states that $a^2 + b^2 = c^2$ for the edges of a right triangle. You may recall that these proofs involve a lot of steps. This is a key idea in *proof theory*, an important method for classifying valid arguments.

A formal analysis of proofs, also known as a *derivation*, is built up from individual steps. Each step involves a transition from input formulas to output formulas. The essence of proof theory is given by *proof rules* that tell us what steps are allowed. In some sense, our goal is to choose rules that are so secure that we can blindly follow them without ever making a mistake. The derivable arguments are all valid.

There is a second important technique for working with argument forms. Recall how Aristotle said that the conclusion of a valid argument results *of necessity*. This may suggest the idea that validity comes from an external relationship between sentences and the circumstances that make sentences true. A semantic model is a kind of precise representation or a mathematical abstraction of such a circumstance. This is the key idea in *model theory*, which is yet another method for classifying valid arguments.

In broad outline, models assign truth-values to formulas. A formula might be true in some models and not in others. We think of each model as a *logically possible* circumstance. Now, we can look for relationships between arguments and models. If every model that makes the premises true also makes the conclusion true, we say that such an argument is truth-preserving. The truth-preserving arguments are all valid.

In the next few sections, we will look at some details of proof theory and model theory to understand how these techniques are used to define different logics. Before moving on, there are a few more useful preliminary remarks to make.

In principle, we can take *any* formulas $p_1, p_2, \ldots$ and *any* single formula q and think of them, respectively, as the premises and conclusion of an argument. This determines a class of argument forms. A specific formal technique divides that class into two parts: The valid arguments and the invalid arguments. Simply put, the result of using proof theory or model theory is to produce a demarcation of the set of valid argument forms. This is what we call an *extensional definition* of validity. It sometimes happens that we can use two different techniques to get exactly the same extensional definition. One of the most famous mathematical results in logic is called the Completeness Theorem, which basically shows that a proof theory and a model theory are extensionally equivalent.

In the next section, we will compare some famous formal logics known as classical, intuitionistic, and relevant logics. These "named logics" are not identified with just one specific proof theory or one specific model theory. When we talk about a logical system by name, what matters is the extensional definition of validity that goes along with that type of logic. There are different ways of doing proof theory or model theory that produce exactly the same logic in the extensional sense. When we find two different tools that define the same logic, like in the Completeness Theorem, then we have at least two ways of understanding the nature of that single notion of validity.

Logics are used to analyze arguments in the first place, but they also have repercussions for how we think about theories. A theory is a bunch of sentences, so we can think about what would happen if we treated those sentences as the premises of an argument. If conclusion C follows from a theory, we say that C is one of the *commitments* of that theory. These commitments are part of the implicit worldview of the theory. A theory can only be true if all of its commitments are true.

Adding information to a theory often causes its commitments to expand. If we add too much information, we might end up with a theory that overgenerates commitments. The most extreme outcome is a *trivial* theory that is committed to everything whatsoever.

Concept. (Trivial Theory) A theory that validly implies everything.

Triviality is extremely bad. Since a trivial theory has indiscriminate implications, it cannot be true (on pain of everything being true). The source of this defect is the internal structure of the theory itself. For this reason, trivial theories are widely considered to be the paradigm examples of nonsensical or uninterpretable theories.

The boundary between trivial and non-trivial theories indicates a difference between minimally acceptable theories and useless, nonsensical theories. Since this depends on logic, we can compare how different logics categorize trivial theories. This is one more way to compare and contrast different logical systems.