MODULAR FORMS AND STRING THEORY

An indispensable resource for readers in physics and mathematics seeking a solid grasp of the mathematical tools shaping modern theoretical physics, this book comprises a practical introduction to the mathematical theory of modular forms and their application to the physics of string theory and supersymmetric Yang–Mills theory.

Suitable for adventurous undergraduates, motivated graduate students, and researchers wishing to navigate the intersection of cutting-edge research in physics and mathematics, it guides readers from the theory of elliptic functions to the fascinating mathematical world of modular forms, congruence subgroups, Hecke theory, and more. Having established a solid basis, the book proceeds to numerous applications in physics, with only minimal prior knowledge assumed. Appendixes review foundational topics, making the text accessible to a broad audience, along with exercises and detailed solutions that provide opportunities for practice. After working through the book, readers will be equipped to carry out research in the field.

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To Jody

Contents

Organization

This book is organized into three parts. Each part is preceded by a one-page organizational summary.

Part I provides an introduction to elliptic functions and modular forms and to variants such as quasi-modular forms, almost-holomorphic modular forms, non-holomorphic modular forms, mock modular forms, and quantum modular forms. Full chapters are dedicated to modular forms for congruence subgroups, vector-valued modular forms, and modular graph functions.

Part II provides various mathematical extensions and physical applications of the material of Part I. The mathematical extensions include Hecke operators, complex multiplication, and Galois theory. The physical applications include string amplitudes, T-duality of toroidal compactifications of string theories, S-duality in Type IIB string theory, dualities in Yang–Mills theories with extended supersymmetry, Seiberg–Witten theory, and two-dimensional conformal field theory.

Part III contains four appendixes of material that is central to the core chapters but may be read independently thereof, including introductions to modular arithmetic, the topology and geometry of Riemann surfaces, line bundles on Riemann surfaces, and higher rank ϑ-functions on higher-genus Riemann surfaces. A fifth appendix provides solutions to the Exercises that are formulated at the end of each one of Chapters 2–16.

Bibliographical notes are provided at the end of each chapter and appendix. For mathematics references, many excellent textbooks are available on elliptic functions, modular forms, Riemann surfaces, and Galois theory. For physics references, we shall refer as much as possible to textbooks, review papers, and lecture notes that we find useful and to research papers whenever the material is not readily available otherwise.

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