

Index

- adapted
 - market, 9
 - process, 5, 6
 - strategy, 27
- additive dynamic utility
 - inadequacy, 126, 129
- additive utility, 189
 - concave, 192
 - exponential, 195
 - power or logarithmic, 196
 - representation theorem, 190
 - scale-invariant (SI), 196
 - translation-invariant (TI), 195
 - uniqueness, 190
- aggregator, 132
 - concave, 140
 - conditional, 130
 - continuous time, 157
 - proportional, 140
 - scale invariant (SI), 136
- algebra, 2
 - generated, 2, 5
- allocation policy, 150
 - continuous time, 160
 - Markovian, 153
 - optimal, 151, 161
- arbitrage, 7
- Arrow–Debreu equilibrium, 113
- Arrow–Pratt approximation, 156
- backward stochastic differential equation (BSDE), 92, 95, 157, 161
- basis, 208
 - dynamically orthonormal, 68
 - orthonormal, 213
- Bayes’ rule, 44
- Bellman equation, 24, 185
- beta error, 101
- beta pricing, 51, 52, 58, 96, 97
 - proxy, 52
- binomial model, 38, 40, 101
 - high-frequency limit, 103
- boundary, 217
- bounded, 215
- Brownian motion, 77, 79
 - geometric, 86
 - Girsanov’s theorem, 89, 91
 - Lévy’s characterization, 80, 81
 - reflection principle, 82
 - time-changed, 82
- BSDE, 92, 95, 157, 161
- budget equation, 29, 31, 32
 - continuous time, 87
- capital asset pricing model (CAPM), 114, 179, 183
- cash flow, 7
 - Arrow, 15
 - desirable, 106
 - dominant, 21
 - generated, 12, 29
 - marketed, 8, 9
 - minimum variance frontier, 115
 - positive, 13
 - present value, 8
 - traded, 7
- Cauchy sequence, 214
- Cauchy–Schwarz inequality, 211
- central planner, 124
 - additive utilities, 126
- certainty equivalent (CE), 133, 200
 - concave, 151
 - conditional, 130
 - derivative, 134
 - expected utility, 131, 134
 - homogeneous of degree one, 136
 - scale-invariant (SI), 136, 151
- change of numeraire, 31
- change of unit of account, 32, 87, 137, 159
- closed set, 215, 217

- closure, 217
- coefficient of absolute risk aversion, 181, 205
- coefficient of relative risk aversion (CRRA), 136, 205
- compact, 215
- compensator, 48
- complementary slackness, 228
- complete set, 215
- concave function, 218
- conditional density process, 62
 - binomial model, 102
- cone, 217
- constrained optimization, 228
 - global optimality conditions, 228
 - linear constraints, 223, 231
 - local optimality conditions, 230
- consumption allocation, 107
 - optimal, 107, 123
 - optimality and utility gradients, 124
 - optimality and utility subdifferentials, 125
- consumption plan, 106
 - continuous time, 155
 - feasible, 161
 - financed, 112, 151, 161
 - optimal, 107, 119–121, 161
 - Π -optimal, 109
- consumption set, 106
- consumption-to-wealth ratio, 141
 - constant, 153, 166
- continuous function, 214
- contract, 11
 - market equilibrium, 112
 - pricing, 19, 56
 - replication in Brownian model, 93
 - synthetic, 32, 40, 93
 - traded, 12
- convenience yield, 37
- convergence, 213
- convex
 - conjugate, 143
 - dual, 143
 - function, 218
 - set, 217
- correlation coefficient, 43
- covariance, 42
- CRRA, 136
- cum-dividend price, 11
- delta hedging, 96
- density, 62
 - normal distribution, 90
- dimension, 208, 218
- directional derivative, 226
- discount process, 61
- dividend
 - process, 11
 - yield, 36, 41, 86
- dominant cash flow, 23
- Doob decomposition, 48
 - multiplicative version, 63
- Doob's optional stopping theorem, 82
- drift, 69, 83
- durability of consumption, 169
- dynamic consistency, 9
 - of a dominant choice, 23
 - of dynamic utility, 128
 - of markets, 9
 - of preferences, 110
- dynamic programming, 24, 185
- dynamic utility, 127
 - additive, 126, 129
 - homogeneous of degree one, 135
 - normalized, 127
- dynamics, 69
 - Markov, 75
- effectively complete market equilibrium, 113, 125
 - CAPM, 116, 177, 179
 - HARA preferences, 183
 - scale-invariant preferences, 118
 - translation-invariant preferences, 180
- elasticity of intertemporal substitution (EIS), 142
- elasticity of proportional aggregator, 141
- endowment, 111
 - aggregate, 111
- entropy, 202
- envelope theorems, 123
- Epstein–Zin–Weil (EZW) utility, 136, 148
 - continuous time, 158
 - multiple-prior representation, 137
- equilibrium, 111
 - Arrow–Debreu, 113
 - effectively complete market, 113
- equivalent martingale measure (EMM), 59, 61
 - forward pricing, 60, 92
- equivalent probabilities, 61

- Euclidean
 - inner product, 208
 - space, 207
- Euler equation for homogeneous functions, 139
- event, 1
- ex-dividend price, 11
- expectation, 42
 - change of measure, 62, 64
 - conditional, 45, 46
 - operator, 42
- expected discounted utility, 132
 - IMRS, 145
 - scale-invariant (SI), 136, 158
- expected utility, 197
 - concave, 203
 - more risk averse, 200, 201
 - representation theorem, 199
 - risk averse, 203
 - scale- or translation-invariant, 197
 - smooth, 204
 - uniqueness, 197
- factor pricing, 71
- Fatou's lemma, 85
- Feynman–Kac formula, 95
- filtration, 3
 - generated, 6
- finances, 112, 151, 161
- finite dimensional, 208
- forward contract, 36, 60, 91, 182
 - delivery price, 36
- forward looking preference, 127
- forward price, 37, 60, 92
 - Brownian model, 91
- full support, 42
- functional, 208
- gain process, 30
- gamma hedging, 96
- Girsanov's theorem, 89
- Girsanov–Lenglart theorem
 - for Brownian motion, 91
- gradient, 226
- gradient inequality, 225
- Gram matrix, 212
- Gram–Schmidt orthogonalization, 213
- habit formation, 169
- Hansen–Jagannathan bound, 59, 144
- hard-to-short stock, 38
- heat equation, 95
- Hilbert space, 222
- homogeneous of degree one, 119
- hyperbolic absolute risk aversion (HARA), 180
- hyperplane, 224
- IMRS, 144
- independent increments, 74
- indicator function, 4
- indifference relation, 198
- infinite-dimensional, 208
- information tree, 3
- inner product, 209
 - Euclidean, 208
 - space, 209
- integral, 30
 - stochastic, 81
- integration by parts, 84
- interior, 217
- intertemporal marginal rate of substitution (IMRS), 144
 - and consumption growth, 145, 148
 - and market return, 146, 148
- irrelevance of current consumption for risk aversion, 130
- isomorphism
 - of finite-dimensional inner product spaces, 213
 - of inner product spaces, 209
 - of vector spaces, 207
- Ito
 - calculus, 81
 - formula, 84, 103
 - integral, 83
 - lemma, 81, 84
 - process, 83
- Jensen's inequality, 66
- Kuhn–Tucker optimality conditions, 230
- Lévy process, 77
- Lagrange multiplier, 228
- Lagrangian, 228
- law of iterated expectations, 46
- law of large numbers, 103
- law of one price, 8
- leasing, 37
- Legendre transform, 143
- limit, 213
- linear
 - combination, 207
 - functional, 13, 208
 - independence, 208
 - manifold, 218

240

linear (*cont*)
 space, 206
 span, 208
 subspace, 207

marginal value of wealth, 122, 135

market, 7
 adapted, 9
 arbitrage-free, 7, 16
 clearing, 111
 complete, 8, 16
 constrained, 18
 dynamic, 9
 effectively complete, 113
 implementation, 73
 implemented, 13
 liquid, 9
 maker, 71
 return, 115, 146

market-price-of-risk, 70
 binomial model, 102
 Brownian model, 88

Markov property, 74
 change of measure, 74

martingale, 47
 continuous time, 80
 local, 80, 83
 representation, 68

matrix notation, 34

mean, 42

minimum variance frontier, 49

money-market account (MMA), 34
 rate process, 34

monotone convergence theorem, 85

more risk averse, 131, 133, 200, 201
 than risk neutral, 203

multiple-prior representation of risk
 aversion, 137, 201

norm, 210, 213

normal distribution
 characteristic function, 79
 density, 90

Novikov condition, 91

null space, 222

open set, 217

optimal consumption and portfolio
 choice, 151, 166

optimal portfolio allocation, 151, 166,
 168
 myopic, 154, 156, 167

Index

option, 22
 American, 22, 39
 American call, 22, 66, 76
 arbitrage pricing, 26
 Black–Scholes formula, 93
 dominant choice, 21
 European call, 37, 93, 95
 European put, 37
 exercise, 22
 exercise policy, 39
 noncommitment assumption, 22
 premium, 26, 37, 93
 present-value maximization, 21
 strike, 22, 37
 writing, 38

ordinal property, 128

ordinally equivalent, 118, 128

orthogonal, 210, 221
 projection, 221, 223
 subspace, 221, 223

Pareto optimality, 123

partial differential equation (PDE), 94

partition, 1
 refinement, 1

path, 3

period, 29

Π -optimality, 109, 122

Poisson process, 77

portfolio allocation, 36
 frontier, 50
 policy, 150

positive
 affine transformation, 190
 cash flow, 13
 linear functional, 13

predictable, 29
 representation, 69

preference, 187

preference correspondence, 107
 convex, 119
 scale-invariant (SI), 117, 119
 transitive, 116

preference profile, 107

preferences for the timing of resolution of
 uncertainty, 183

present value, 8

present-value function, 14
 conditional, 18
 existence, 16
 uniqueness, 16

probability (measure), 42

- process, 3
- projection, 220
 - theorem, 221
- put–call parity, 38
- Pythagorean identity, 210
- Q-martingale, 65
- quadratic variation, 79
 - high-frequency limit, 104
- Radon–Nikodym derivative, 62
- random variable, 3
 - measurable, 4
- recombining tree, 40
- recursive utility, 130, 133, 156
 - concave, 152
 - continuous time, 156
 - Duffie–Epstein form, 158
 - Epstein–Zin–Weil (EZW), 136, 148, 158
 - generalized, 129
 - gradient, 134, 141, 163
 - normalized, 131
 - scale-invariant (SI), 135
- regular consumption–portfolio policy, 176
- regular proportional aggregator, 142
- representative agent, 117, 178, 179, 182
- representative-agent pricing
 - CAPM, 114, 178, 183
 - HARA preferences, 181
 - scale-invariant preferences, 117
 - translation-invariant preferences, 179
- return, 35
 - excess, 35, 58
 - mean-variance efficient, 54
 - minimum variance, 49, 97, 99, 116
 - minimum variance frontier, 53
 - traded, 49
- Riesz representation, 211, 212, 222, 223
- risk averse, 203
- risk-neutral
 - certainty equivalent, 203
 - pricing, 60, 92
- SBM, 79
- scalar, 206
- semimartingale, 77
- separable preference, 189
- separating hyperplane theorem, 225
- sequence, 213
 - Cauchy, 214
 - convergent, 213
- Sharpe ratio, 52, 54, 100
- short-rate process, 34
- SI preference, 117, 119, 196
- solvency constraint, 87
- SPD, 55
- spot, 3
 - immediate successor, 3
 - initial, 3
 - terminal, 3
- standard deviation, 43
- state space, 1
- state-price density, 55
 - Brownian model, 88
 - and utility gradient, 120
 - and utility superdifferential, 121
- state-price process, 15
- stochastic differential equation, 94
- stochastic process, 3
 - adapted, 5, 6
 - increments, 30
 - lagged, 30
 - Markov, 74
 - strictly positive, 4
- stochastically independent, 43, 46
- stopping time, 5, 39
- strategy
 - adapted, 27
 - trading, 29
- subgraph, 225
- submartingale, 66
- superdifferential, 225, 226, 228
- supergradient, 225
- support condition, 224
- supporting hyperplane theorem, 224
- supporting vector, 220, 224
- synthetic contract, 32, 72
 - binomial model, 41
 - Brownian model, 93
- Taylor approximation, 78
- theta, 96
- time, 1
- total variation, 79
- trading strategy, 29
 - admissible, 87
 - self-financing, 87
- translation, 218
- translation-invariant preference, 179, 195
- triangle inequality, 211
- two-fund separation, 51

242

uncorrelated, 43
unit CRRA, 147, 154, 167
unit EIS, 149, 153, 166
utility function, 118
 additive, 120
 concave, 119
 gradient, 120
 homogeneous of degree one,
 119
 homothetic, 119
 normalized, 118
 superdifferential, 121

Index

utility process, 127
utility representation, 118, 187
value process, 11
variance, 43
variance averse, 115
vector, 206, 207
 space, 206
 subspace, 207
volatility, 69, 83
wealth process, 150, 169
 continuous time, 161
Wiener process, 79