Theoretical Foundations of Asset Pricing

This text provides an advanced introduction to the modeling of competitive financial markets, encompassing arbitrage and equilibrium pricing of financial contracts, as well as optimal lifetime consumption and portfolio choice. Notable features include its coverage of recursive utility in discrete and continuous time and several results not previously available in book form. Each chapter concludes with a set of exercises, with solutions available to verified instructors.

Ideal as a graduate-level course text, this book can also serve as a valuable reference for researchers and finance industry practitioners. Readers with a finance focus can use the text to build analytical foundations for a significant component of the economics of financial markets, while readers with a mathematics focus will find a well-motivated introduction to basic tools of stochastic analysis and convex analysis.

COSTIS SKIADAS is the Harold L. Stuart Professor of Finance at Northwestern University, Illinois, where he has served as chairman of the Finance department. He has made research contributions on foundational aspects of the topics covered in this text. He previously authored *Asset Pricing Theory* (2009).

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To my wife, Robin

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Preface

This text is the distillation of material I teach to doctoral students at Northwestern University as part of an advanced introduction to the theoretical foundations of what is traditionally known as "asset pricing." The focus is on the modeling of competitive financial markets, encompassing arbitrage and equilibrium pricing of financial contracts, as well as optimal consumption and portfolio choice. I have relied on lectures for broader context and on this text for a self-contained statement of a theory with classical roots in Walrasian competitive analysis, extended by Arrow and Debreu to include time and uncertainty, and further developed in Finance to emphasize the role of arbitrage arguments and the tools of modern stochastic analysis. The presentation places equal emphasis on sound economics and well-motivated methodology. Readers with an economics focus can use this text to build analytical foundations for a significant component of the economics of financial markets, while readers with a mathematics focus can use the text as a well-motivated introduction to basic tools of stochastic analysis and convex analysis. Despite its introductory orientation, the book includes results I could only find in research papers, or are refined versions of original results of my last monograph.

The progression of topics can be thought of as increasing in scope and decreasing in realism. Arbitrage arguments are presented first, followed by characterizations of optimality and competitive equilibrium. Arbitrage arguments utilize the assumption that the market does not allow incremental cash flows that are desirable in the narrow sense of arbitrage. Optimality is then introduced as a refinement of the no-arbitrage assumption. The notion of a desirable cash flow is enlarged through preferences and the idea of an arbitrage is extended to allow for multiagent transactions through profitable market-making opportunities. The exclusion of such market-making opportunities refines the traditional concept of Pareto optimality and leads to a version of classical competitive welfare analysis that is better suited to financial markets, emphasizing endogenous forces for market creation. Restrictions on preferences are initially minimal, reflecting the fact that competitive equilibrium notions are robust to preference structure, and are gradually strengthened in order to express ideas of how market prices and optimal consumption–portfolio choice relate to preferences for smoothing across time and states of the world. The auxiliary Appendix A on additive utility forms and risk aversion presents some fundamental but not well-known arguments, drawing on material that, to my knowledge, has only been buried in research papers.

On the methodological side, a self-contained introduction to probabilistic methods starts with a rigorous treatment on a finite information tree and concludes with an introduction to the continuous-time theory, which omits several technical details but leverages the thorough understanding of the tools on a finite tree. In an approximate numerical sense, the continuous-time model is presented as a simplified special case of a high-frequency finite-information model. Tools like martingale representations, Girsanov change-of-measure arguments, the Ito calculus, and forward and backward stochastic differential equations are hopefully demystified in this way, providing an entry point to a literature which is typically obfuscated by the requirements of set-theoretic rigor. The optimality and equilibrium theory emphasizes a unified geometric viewpoint and convex analysis methods, making this course complementary to a macroeconomics course emphasizing dynamic programming methods. Appendix B provides a succinct rigorous statement of the background convex analysis, which can also be thought of as an introductory mini course on the functional analytic approach to optimization theory.

I have been using this text in a class of mostly second-year doctoral students taking a Finance course for the first time. I emphasize to my students that the text organizes foundational insights that are essential but far from sufficient for understanding actual financial markets. Class discussion can touch on topics such as the role of collateral, limits to arbitrage, and the inadequacy of consumption-based asset pricing. I have resisted the temptation to include such discussions in the text, which is intended only for core material in a state of completion meant to last. It should be up to the instructor and follow-up courses to offer broader perspective and avenues to research, depending on the interests of the audience. Although the material is formally self-contained, a background on basic linear algebra is essential and some prior exposure to probability theory and graduate-level introductory economics is helpful. I ask students to read Appendix B on convex analysis as preparation for my class, with emphasis on geometric understanding. I recommend two books to complement this one: Back (2017) on the more elementary side (but including topics not covered here), and Duffie (2001) for more on the continuous-time theory in a language and notation that will look familiar to readers of this book.

This text is consistent in approach with my older book (Skiadas (2009)), but differs in some significant ways (as well as in numerous smaller improvements that would take too long to list here). The treatment has been simplified around a central conceptual development, but also extended to include an introduction to continuous-time methodology, a section on preferences with habit formation or durability of consumption, as well as several other smaller additions, such as an explanation that certain multiple-prior utility forms are equivalent to more familiar recursive utility forms. The result is a more compact, panoramic and cohesive book. Some formal structures, such as those underlying the discussion of optimality, have been simplified and more-efficiently presented. Several results and proofs are improved in this text or are missing from the older book. The material is directly presented in a dynamic setting, as opposed to the traditional practice of first considering the static theory. The probabilistic foundations are pedagogically interweaved into the main material, as opposed to a disconnected appendix. Some of the older book's material that is not essential to the main narrative has been converted into exercises or omitted. The exercises are now better integrated with the main text and classroom tested, with detailed solutions available to instructors on www.cambridge.org/skiadas. Some of the peripheral theory on additive utility structures has been significantly refined and extended, and pulled into Appendix A. The convex analysis of Appendix B has also been significantly improved and extended.

Acknowledgments I was lucky to take my first steps in the area under the guidance of Darrell Duffie. I am deeply grateful for his encouragement, inspiration and friendship. Significant parts of this book's presentation are based on my subsequent research, which was always motivated by a desire to understand the fundamentals. I am grateful to my coauthors, especially Mark Schroder, who made this process so much more enjoyable. I thank Kunpeng Zhou and especially Xuning Ding for catching erroneous statements in earlier versions. Many others have helped weed out misprints, including Ariel Lanza, Huidi Lin, Nicolas Min, Junko Oguri, Raul Riva, Matheus Sampaio, Timothy Seida, Lior Shabtai, Rui Sousa, Yudan Ying, Brandon Zborowski, Mark Zhao and Weijia Zhao.