A Theory of Truth

How should we treat the liar and kindred paradoxes? *A Theory of Truth* argues that we should diverge from classical logic, and presents a new formal theory of truth. The theory does not incorporate contradictions and is not substructural, but deviates from classical logic significantly, and endorses principles like ‘No sentence is both true and false’ and ‘No sentence is neither true nor false’.

The book starts with an introduction to the paradoxes, suitable for newcomers to the subject, before presenting its approach. Four versions of the theory are covered, extending the theory to a determinacy operator and to a full first-order language with quantifiers. Each includes all Tarskian biconditionals that can be formulated in its language. The author uses original methods to prove the consistency of each version and compares the theory to alternative non-classical theories, including Field’s paracomplete approach, Ripley’s nontransitive system and Zardini’s contraction-free calculus.

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PREFACE

This book presents a number of formal theories of truth, similar in spirit, which tackle the liar and other paradoxes by diverging from classical logic. Each theory includes all Tarskian biconditionals, like 'The sentence "Snow is white" is true iff snow is white', that can be formulated in its language. The theories do not incorporate contradictions (they are not dialetheic), and their logical framework is not a substructural logic, but still they diverge from classical logic significantly. They are specified in a model-theoretic, rather than proof-theoretic, manner and differ from one another in the language in which they are couched. The book develops the theories and defends them against alternative approaches to truth.

Chapter 1 is introductory. It sets out some paradoxes about truth and comments on standard treatments of them. The ground here is well trodden, but the chapter gives me the opportunity to explain how I view some central aspects of the discussion. It is written in a way that presupposes no previous familiarity with the semantic paradoxes, so it could be useful for teaching the subject.

Chapter 2 explains why I think we should treat the paradoxes by deviating from classical logic. It contains two arguments to the effect that the fault for the problem lies with classical logic, as well as some (successful or unsuccessful) variants of them. The argument in Section 2.3 is what convinced me that the culprit is our logic.

If 'true' and other semantic expressions are ambiguous or indexical, as some philosophers have suggested, then there is reason to believe that the blame for the paradoxes should be laid on biconditionals of the form 'The sentence "*p*" is true iff *p*' and not on classical logic. For the sentences that replace the letter '*p*' in the paradoxical cases contain semantic terms, and we know, independently of the paradoxes, that biconditionals of that form are problematic when '*p*' is replaced with sentences that exhibit ambiguity or indexicality. So Chapter 3 argues against the view that paradoxical sentences display those features.

The first theory of truth that is put forward in the book can be found in Chapter 4. It is formulated in a propositional language, which lacks quantifiers. The chapter presents a non-classical propositional logic, develops the theory within that logical framework and shows how the logic can be strengthened. Proving that the theory is consistent is the topic of Chapter 5.

Chapter 6 extends the propositional language by adding an operator, 'Δ', which means 'it is determinate whether …', or 'there is a fact of the matter whether …'. It also uses 'Δ' to define some more operators. There result an expansion of the non-classical propositional logic and an expansion of the previous theory of truth, and the expanded theory is again proved to be consistent. The chapter also contains a philosophical discussion of 'Δ' in Section 6.1 and a discussion, in Section 6.5, of whether the failure of classical logic can be explained in terms of indeterminacy.

Quantifiers introduce some extra complications, so they were reserved for Chapters 7–9. Chapter 7 presents a family of first-order languages, delineates a non-classical first-order logic and develops a theory of truth whose logical background is that logic. Proving the theory to be consistent is the task carried out in Chapter 8. As for Chapter 9, it develops another first-order theory of truth. It has the same logical background as its counterpart in Chapter 7, and its consistency is proved similarly, but it is couched in another language of the same family. The language is appropriate for defining a richer syntactic vocabulary, and correspondingly the theory of Chapter 9 possesses a more extensive syntactic part.

The last chapter, 10, compares the theories presented earlier in the book with those put forward by other authors who tackled the paradoxes by diverging from classical logic but without endorsing contradictions. So it comments on theories we find in the work of H. Field, N. Tennant, E. Zardini and D. Ripley.

As some proofs in the book are complicated and follow methods that cannot be found elsewhere in the literature, they have been put in separate chapters or sections. These are Chapters 5 and 8 and Sections 6.4 and 9.5. They are not presupposed in the rest of the book, so readers might skip them at first. The claims proved can be found elsewhere. Readers might also initially omit Section 9.3, which does not contain complex proofs but complex definitions; the most important concepts defined are listed later on.

In fact, there are three routes through the book. One consists in reading it all. The second is to read everything other than the chapters and sections mentioned in the preceding paragraph. The third route minimizes technicalities and is confined to Chapters 1–3, Sections 4.1, 4.3, 6.1, 6.3, 7.1.1, 7.2, 9.2 and 9.4, as well as Chapter 10. It is possible to read those chapters and sections without reading the rest of the book, and they include the sections (4.3, 6.3, 7.2 and 9.4) which present the four truth-theories that I put forward. Most philosophical points are included in the third route. There are, however, scattered philosophical remarks in the other parts of the second route; in particular, Section 6.5 is philosophically significant, but presupposes familiarity with 6.2–6.2.1 and, to some extent, with 4.2 as well.

Apart from additions and changes suggested by the referees, the book was written, at intervals, between 2007 and 2019. The logic and the theory of truth presented in Sections 4.2–4.3 were largely ready at the end of 2007, and a version of what is now Central Theorem 1 had been proved somehow. Little did I imagine then how long it would take me to complete the work. In 2012–15 my research on truth was funded by the research project Thales-UOA-APRePoSMa, headed by

Stathis Psillos; the project brought many philosophers together, and Stathis was pivotal in setting it up.

Parts of the book were delivered as talks in various venues: the staff seminar in my department, the seminar of the graduate programme "Logic and the Theory of Algorithms and Computation" in the University of Athens, the regular seminar of the Thales APRePoSMa project, the staff seminar of the Department of Philosophy in the University of Patras, the 8th and 12th Panhellenic Logic Symposia (in Ioannina and Anogeia, Crete respectively), a conference on paradox and logical revision at the Munich Center for Mathematical Philosophy, a conference at the University of Barcelona in honour of Mark Sainsbury, and the 5th workshop on philosophical logic in Sadaf in Buenos Aires. I am grateful for the questions and comments I received on those occasions.

The two anonymous referees were also helpful, since not only were their reports detailed, but they also engaged in a correspondence with me which allowed me to appreciate their points better. As a consequence of their remarks, the book was improved in various respects. Paul Larson was my point of contact with the ASL and the referees and was always ready to assist, while Thomas Piecha did the typesetting and was very efficient.

Some chapters have appeared in print. A version of Sections 2.1–2.3 was published as "Classical Logic and the Liar" [Stephanou, 2020b] in *Logic and Logical Philosophy*, vol. 29, pp. 35–56. I thank the editors of *LLP* and the Nicolaus Copernicus University in Toruń for permission to reprint. A version of Sections 4.2–4.3 and Chapter 5 was published as "A Propositional Theory of Truth" [Stephanou, 2018] in the *Notre Dame Journal of Formal Logic*, vol. 59, pp. 503-545 (copyright 2018, University of Notre Dame, all rights reserved). It is here republished by permission of the publisher, Duke University Press. I benefited from the referee reports for those papers too.

Finally, a word about use and mention. In Chapters 1–3, which do not contain many formulae, I use both quotation marks and quasi-quotation, Quine's corners. The corners are used as explained by Quine [1981, pp. 33–37]. So

$$\ulcorner \mathbf{A} \text{ or } \mathbf{B} \urcorner$$

means

the concatenation of **A**, 'or' and **B** in this order.

Bold letters are the only part that we would not enclose in quotation marks when explaining the expression that begins and ends with the corners. In Chapters 4–10, which contain a great many formulae, I omit quotation and quasi-quotation marks, since they are not frequently used in works in formal logic. But I make an exception and do use them when what they are going to enclose includes at least one ordinary English word and so does not consist entirely of symbols.