Introduction to Strings and Branes

Supersymmetry, strings and branes are believed to be essential ingredients in a single unified consistent theory of physics. This book gives a detailed, step-by-step introduction to the theoretical foundations required for research in strings and branes.

After a study of the different formulations of the bosonic and supersymmetric point particles, the classical and quantum bosonic and supersymmetric string theories are presented. This book contains accounts of brane dynamics including D-branes and the M5-brane as well as the duality symmetries of string theory. Several different accounts of interacting strings are presented; these include the sum over world-sheets approach and the original S-matrix approach. More advanced topics include string field theory and Kac–Moody symmetries of string theory.

The book contains pedagogical accounts of conformal quantum field theory, supergravity theories, Clifford algebras and spinors, and Lie algebras. It is essential reading for graduate students and researchers wanting to learn strings and branes.

Peter West is a Professor at King's College London and a Fellow of the Royal Society. He is a pioneer in the development of supersymmetry and its application to strings and branes.

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PETER WEST
King's College London

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Preface

If we have told lies you have told half lies. A man who tells lies merely hides the truth, but a man who tells half truths has forgotten where he put it.

The British consul to Lawrence of Arabia before he arrived with the Arab army in Damascus.

In the late 1960s a small group of theorists concluded that quantum field theory could not provide a suitable description for the main problem of the time, that is, to account for hadronic physics. As a result, they began a quest to find an S-matrix that had certain preordained properties. The search culminated in the discovery of such an S-matrix for four, and then any number of, spin-0 particles. By using physical principles and mathematical consistency it was found that these S-matrix elements were part of a larger theory that possessed an infinite number of particles. Remarkably, the early pioneers found the scattering amplitudes for any number, and any type, of these particles; they even found these results at any loop order. It was subsequently realised that this was the theory of string scattering and that the theory was more suited to describe fundamental, rather than hadronic, physics.

Supersymmetry was unearthed from the world-sheet action for the ten-dimensional string and also found by independent quantum field theory considerations in Russia. Supersymmetry is entwined with string theory, but it is an independent subject. Hopefully, it will be found at the Large Hadron Collider at CERN, but even if it is, this is unlikely to be direct evidence for string theory. Supersymmetry and string theory are believed to be essential ingredients in a unified consistent theory of all physics. It was thought initially that this theory would just be the theory of ten-dimensional strings, but we now realise that it must also include branes on an equal footing. We are quite far from having a systematic understanding of the quantum properties of branes and what the underlying theory is remains unclear. Indeed, even the concepts on which it is based may be quite different to those we know now. Ironically when string theory was first discovered it was not called string theory, but the dual model, as researchers were unaware of its stringy origins, where as nowadays all discoveries on fundamental physics involving supersymmetry and supergravity are also packaged up in the term string theory. As the subject has developed sometimes string theory, and sometimes supersymmetry, has provided the dominant insights, but it remains to be seen what the mix of ideas will be in the final theory. In this book I have tried to reflect this.

The aim of this book is to provide a systematic and, hopefully, pedagogical account of the essential topics in the subject known as string theory. Almost all of the computations are carried out explicitly. The book also contains some more advanced topics; these have been selected on the basis that I know something about them and I have a wish to explain them. There are also some pedagogical chapters such as those on Clifford algebras and Lie algebras that students should know and which could well play an even more important role in future developments. There are several very important topics that are missing: Calabi–Yau compactifications, string based black hole entropy computations and the AdS–CFT duality. However, these are rapidly developing and perhaps not yet ready for a systematic, or complete, treatment. There is also a long chapter on supergravity theories reflecting the important role they have played in the subject; this includes the methods used to construct them, their symmetries and the properties of these theories in ten and eleven dimensions. Many aspects of supersymmetric theories which are not discussed in this book can be found in my book *Introduction to Supersymmetry and Supergravity* [1.11].

This book has evolved over more than 25 years and some of the calculations were performed many years ago. Although almost certainly correct when first derived they may have developed transcription errors since then. As such, if you find a factor of 2, or a minus sign out, or some other defect in the occasional place you could be correct. Hopefully, these can be corrected in a second edition.

I have tried to reference the original papers in order to give the reader a better guide to the literature and in particular access to some of the best accounts of the material presented. I have studied quite a number of the papers that I had not read before, but I may well have missed some references. For this I apologise, and I hope to put such mistakes right in the future.

The reader who wants to get to grips with the basics of string theory in the quickest possible time could take the following path: first sections 1.1–1.2.3, then chapters 2, 3, 4, 5, 7, then sections 8.1–8.3, followed by the chapters 9 and 10, then sections 13.3–13.8.4, chapter 14, and finally sections 18.1 and 18.2.

I wish to thank Paul Cook for designing the cover, which shows the projection of the roots of E_{11} onto the Coxeter plane using the SimpLie computer programme of Teake Nutma, and Pascal Anastasopoulos for drawing and helping to construct the figures. I also wish to thank Andreas Braun, Lars Brink, Lisa Carbone, Paul Cook, Finn Gubay, Arthur Greenspoon, Joanna Knapp, Neil Lambert, Andrew Pressley, Sakura Schafer-Nameki, Duncan Steele and Arkardy Tseytlin for help with proof reading sections and references. My thanks also goes to the staff and students of the Department of Mathematics, King's College London and the Technical University of Vienna for many useful comments on my lectures which were taken from this book.