Optimization Models in Electricity Markets

Get up-to-speed with the fundamentals of how electricity markets are structured and operated with this comprehensive textbook.

Key features

• Coverage of key topics in electricity markets, including power system and power market operations, transmission, unit commitment, reserves, demand response, hydrothermal planning, investment in generation capacity and risk management.
• Over 140 practical examples, inspired by industry applications, connecting key theoretical concepts to practical scenarios in electricity market design.
• Over 100 coding-based examples and exercises with mathematical programming models, and selected solutions for readers.

Without requiring an advanced background in power systems or energy economics, this is the ideal introduction to electricity markets for senior undergraduate and graduate students in electrical engineering, economics, and operations research, and a robust introduction to the field for professionals in utilities, energy system operations, and energy regulation. Accompanied online by datasets, AMPL code, supporting videos, and full solutions and lecture slides for instructors.

Anthony Papavasiliou is an Assistant Professor of Electrical and Computer Engineering at the National Technical University of Athens, and a former Associate Professor and holder of the ENGIE Chair at UCLouvain. He obtained his PhD and conducted post-doctoral research at the department of Industrial Engineering and Operations Research at UC Berkeley. He is the recipient of the Francqui Foundation research professorship, an ERC Starting Grant, and the Bodossaki Foundation Distinguished Young Scientist award. He is a former Associate Editor for Operations Research and IEEE Transactions on Power Systems.
“Strong interactions in electricity networks require coordination to support competition. The key to successful electricity market design exploits the convex case of equivalence between economic dispatch and market equilibrium. This places optimization theory and models as essential ingredients for the great variety of applications explored in this vital book.”

William Hogan, *Harvard University*

“This book highlights the important role of optimization models in modern power systems planning, operations and markets. It provides an excellent introduction to the application of optimization models for power systems students who do not have an operations research background, and introduces operations research students to the variety of problems arising in the context of power systems.”

Shmuel S. Oren, *University of California, Berkeley*
Optimization Models in Electricity Markets

Anthony Papavasiliou
National Technical University of Athens
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Preface

**Intended readers**  This book is intended to be used by students, researchers, and industry professionals with a background in operations research or power systems who are interested in learning about or conducting research in electricity markets. Readers would benefit from an elementary background in operations research. A background in economics or power systems engineering is not required. The learning of the material is enhanced by solving problems on a computer solver using a mathematical programming language (e.g. AMPL, GAMS, JuMP, Pyomo). Basic familiarity with a mathematical programming language is therefore especially useful.

**Summary of content**  Chapter 1 provides an overview of mathematical programming models that are used in the energy industry. The chapter then continues with an introductory discussion about the economic interpretation of mathematical programming models for electricity markets in the specific context of the capacity expansion planning problem. The link is made between the capacity expansion model and the missing money problem in order to motivate the relevance of modeling for policy analysis.

Chapter 2 covers introductory concepts in duality theory, Karush–Kuhn–Tucker (KKT) conditions, and sensitivity. Supplementary introductory material on linear programming is provided in appendix A. The material on duality is developed as a self-contained learning unit. KKT conditions, which are used extensively throughout the text, are derived from duality theory.

Chapter 3 provides an overview of power system operations and power market operations. A minimum engineering background is provided regarding the operation of electric power systems, ranging from production, to transmission and distribution, down to the consumption of electricity. The timing of power system operations is classified between short-term, medium-term, and long-term operations. This classification guides the organization of the material in subsequent chapters. Electricity markets, which are tightly linked with power system operations, are introduced. Important design dilemmas related to electricity markets are discussed, including the exchange/pool dilemma and the design of auctions. A generic blueprint of a typical electricity market is then outlined, which is analyzed in increasing detail in subsequent chapters.

The next four chapters cover short-term electricity market operations, ranging from real-time to hour-ahead operations. Chapter 4 introduces the simplest possible model of an electricity market, where only energy is traded. After defining competitive market equilibrium, the interpretation of the economic dispatch model as a competitive market equilibrium is discussed in detail. This interpretation is gener-
Preface

alized in the last section of the chapter, where the equivalence between competitive market equilibrium and optimization is established. This result is invoked repeatedly throughout the text for providing economic interpretations to models of increasing complexity.

Chapter 5 adds one layer of detail to the economic dispatch model by introducing the transmission network, leading to the optimal power flow model. This chapter presents various linear approximations of the physical equations that govern the flow of electric power. The economic interpretation of the optimal power flow problem is discussed and is linked to the so-called locational marginal pricing market design, which is compared to the zonal pricing market design.

Chapter 6 further refines the economic dispatch model by introducing reserves, which are used for operating the system securely under uncertain conditions. The simultaneous and sequential procurement of reserves are discussed and compared. The activation of reserves is discussed in the context of balancing markets.

Chapter 7 concludes the modeling of short-term operations by introducing unit commitment, the task of scheduling generators in the day-ahead and intra-day time frame. The binary (on/off) nature of this scheduling task presents challenges in the economic interpretation of the model and the design of mechanisms for rewarding units that incur fixed scheduling costs. The pool-versus-exchange dilemma is discussed on the basis of the unit commitment model.

The next two chapters focus on medium-term (month-ahead and year-ahead) aspects of power market operations, specifically hydrothermal planning and financial hedging. Chapter 8 focuses on the hydrothermal planning problem. The chapter commences with a discussion of modeling decision making under uncertainty in two and in multiple periods, as well as various ways for representing uncertainty that include scenario trees, lattices, and stagewise independence. These ideas are then applied to the hydrothermal planning problem, and the concept of dynamic programming is introduced, along with a discussion of the value of water. The chapter concludes with metrics for evaluating the performance of a stochastic programming model relative to alternatives such as perfect foresight or deterministic policies.

Various instruments that are widely used in electricity markets are defined in chapter 9, including forward contracts, futures contracts, financial transmission rights, call options, and bundles of the above. The use of these instruments for hedging risk in the production, transmission, and consumption of electricity are demonstrated through examples.

The next two chapters focus on long-term (multi-year ahead) decision problems. Chapter 10 focuses on the design of mechanisms which can be used for mobilizing demand response. This is classified as a long-term topic because the focus is on long-term (e.g. multi-year) contracts that can be used for enlisting flexible consumers to participate actively in supporting system operation. Two specific approaches are discussed, time of use pricing and priority service pricing.

Chapter 11 focuses on the capacity expansion planning model and its economic interpretation in terms of how energy markets can be used for rewarding investment
costs. This chapter then discusses various alternative designs that can be used for remunerating investment cost, including capacity mechanisms and adaptations to ancillary services markets.

Chapter 12 expands the analytical methodology of the text to models beyond electricity markets, with a specific focus on hydrocarbons (oil and natural gas), biofuels, and nonrenewable resources. Rather than focusing on a detailed analysis of the energy sectors themselves, the goal of this chapter is to illuminate how KKT analysis, which is an overarching theme in the textbook, can be used for the analysis of diverse phenomena in energy sectors beyond electricity markets.

Background knowledge and exercise solutions are provided in the appendices. Appendix A covers basic knowledge in linear programming, and appendix B introduces the direct current power flow. Appendix C provides brief solutions to exercises.

Use of the material in courses I have used the material in this book for a number of courses spanning various subjects (power system economics, energy economics, mathematical programming), background training (operations research, energy engineering), and levels of expertise (undergraduate, graduate).

The core material was developed for a graduate course in quantitative models for electricity markets at UCLouvain, which targets students in mathematical engineering with an optimization background but no exposure to power systems. For this purpose, I followed the same structure as of the book by selecting topics across chapters 1 to 11.

The material has also been used for undergraduate and graduate courses in power system economics at the department of electrical and computer engineering at the National Technical University of Athens. The main difference here is that chapter 2 is replaced by appendix A, and certain parts of the main content that rely more on duality (for instance, discussions on pricing in unit commitment models, chapter 7) are dropped from the lectures. The remainder of the structure can be followed straightforwardly (chapter 1 and choices from chapters 3–11), with less of an emphasis on background engineering knowledge in chapter 3.

A course on power system economics such as the one described in the previous paragraph can be supplemented in a subsequent semester by topics related to reliability and specialized topics in electricity market design. For instance, an undergraduate course on economic and reliable system operations can cover topics related to reserves, unit commitment, demand response, and expansion planning. An indicative sequence of lectures includes a review of chapter 1, appendix A, and chapters 3, and 4, followed by a detailed treatment of chapters 6, 7, 10, and 11.

The material has further been used for undergraduate and graduate courses in energy economics at the National Technical University of Athens, where students are not assumed to have prior exposure to operations research. Apart from replacing chapter 2 with appendix A, topics that are more specific to the electricity sector can be skipped. An indicative sequence of lectures includes chapter 1, appendix A, and chapters 3, 4, 9, 11, and 12.
Table 0.1 Possible use of the book material in courses.

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<td>Reliable</td>
<td>1, A, 3–11</td>
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<tr>
<td>undergraduate</td>
<td>system operations</td>
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<td>Energy markets</td>
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<tr>
<td>Operations research/energy</td>
<td>Math programming models</td>
<td>A, 2, 4.1, 7.1, B, 5.1, 6.2, 11.1</td>
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Finally, parts of the textbook have been used in an undergraduate course on mathematical programming models. Here, students are not assumed to have any exposure to energy or power systems, and the emphasis is rather on implementing mathematical programming models with industry relevance. An indicative sequence of lectures that can be used for this purpose is appendix A, chapter 2, section 4.1, section 7.1, sections 5.1 and 6.2, chapter 8, and section 11.1.

This information is summarized in table 0.1.

Exercises Exercises are provided at the end of each chapter. Terse solutions are provided in appendix C. Additional resources are available to instructors, including a detailed solutions manual for the textbook exercises, as well as detailed solutions to additional exercises beyond the textbook. Certain exercises involve programming, and the available data and codes are available on the textbook website: www.cambridge.org/Papavasiliou. Exercises include theoretical proofs of results that are presented or alluded to in the textbook (with varying difficulty), the formulation of mathematical programs on paper and their analysis or implementation in code, simple applications of concepts presented in the text on numerical examples, or more elaborate numerical examples that require code. There is no specific indication of the level of difficulty of any given exercise.

Notation and terminology Mathematical notation in the textbook varies from chapter to chapter, nevertheless I do try to follow some standard notation that is used in the scientific literature, and notation is anyway explained every time a new model is introduced. English lowercase letters are often used for decision variables, English uppercase letters are often used for parameters, and Greek lowercase letters are often used for dual variables. Market clearing prices are typically denoted as $\lambda$ or $\rho$, quantities produced are often denoted as $p$, and quantities consumed are often denoted as $d$. Binary commitment decisions are often denoted as $u$. There is no bold, uppercase, or otherwise specific notation for vectors. Whenever introduced, vectors typically correspond to column vectors, and their transpose is typically denoted with a $T$ superscript.
Part of what makes the energy industry and research area fascinating, but also challenging, is the speed and uncertainty of its evolution. The book uses numerous examples throughout, and refers to technologies such as “gas,” “coal,” “oil,” or “nuclear.” The cost figures that are used for these technologies, and their presence in the fuel mix, are representative of their cost and role at the time of writing of this book. Nevertheless, and as recent evolutions in the petroleum and gas markets suggest, things can change very rapidly in energy markets, in terms of both costs and the role of various technologies in future energy mixes. For this reason, these technology names should be understood as labels rather than literal references to a specific technology.
I am grateful to numerous people for the development and evolution of this book. Firstly, I wish to thank the numerous students who have approached me with suggestions for and corrections to content. In addition, I wish to single out certain people who have influenced my thinking about our scientific area and therefore the content of this book. Shmuel Oren, my PhD advisor, has been a constant source of inspiration. And Yves Smeers, my informal advisor during my time at UCLouvain, has greatly shaped my understanding of electricity market models. Special thanks to William Hogan for making himself available during my visits to Harvard University during my sabbatical under the Francqui Foundation professorship. I am also especially grateful to my students for constant corrections and enrichments of the text, with special thanks to Ignacio Aravena for kicking off the solutions manual, and Quentin Lété for spotting important corrections in the text. Many of the examples and remarks in the book have been inspired by discussions with various colleagues and former students, including (in alphabetical order) Gilles Bertrand, Mette Björndal, Pantelis Capros, Jacques Cartuyvels, Philippe Chevalier, Jehum Cho, Athanasios Dagoumas, Gauthier de Maere d’Aertrycke, Gerard Doorman, Andreas Ehrenmann, Céline Gerard, Kory Hedman, Yves Langer, Mehdi Madani, Marijn Maenhoudt, Alain Marien, Yuting Mou, Alex Papalexopoulos, Nicolas Stevens, Mathieu Van Vyve. Special thanks to Stephen Boyd for granting permission for the notation and organization of the material in chapter 2, which is inspired by his own material. Also special thanks to Christos Karydas, whose teaching material largely inspired the content of chapter 12. I am especially grateful to the Francqui Foundation and the European Research Council for important financial support, which created resources (including time) that made this project possible.