

# Contents

|  |                  |
|--|------------------|
| <i>Preface</i>   | <i>page</i> xiii |
| <i>Acknowledgments</i>   | xv               |
| <b>1 Introduction</b>  | 1                |
| 1.1 Scope and Aims of This Book  | 1                |
| 1.2 History  | 2                |
| 1.3 Contents of This Book  | 7                |
| <b>2 Field Quantization</b>  | 11               |
| 2.1 Quantization of a Single-Mode Field  | 11               |
| 2.2 Quantum Fluctuations of a Single-Mode Field                                | 17               |
| 2.3 Quadrature Operators for a Single-Mode Field                               | 18               |
| 2.4 Multimode Fields   | 20               |
| 2.5 Thermal Fields   | 28               |
| 2.6 Vacuum Fluctuations and the Zero-Point Energy                              | 32               |
| 2.7 The Quantum Phase  | 37               |
| Problems   | 45               |
| <b>3 Coherent States</b>   | 50               |
| 3.1 Eigenstates of the Annihilation Operator and Minimum<br>Uncertainty States | 50               |
| 3.2 Displaced Vacuum States  | 56               |
| 3.3 Wave Packets and Time Evolution  | 58               |
| 3.4 Generation of Coherent States  | 60               |
| 3.5 More on the Properties of Coherent States                                  | 61               |
| 3.6 Phase-Space Pictures of Coherent States                                    | 65               |
| 3.7 Density Operators and Phase-Space Probability<br>Distributions             | 68               |
| 3.8 The Photon Number Parity Operator and the Wigner<br>Function               | 75               |
| 3.9 Characteristic Functions   | 78               |
| Problems   | 85               |
| <b>4 Emission and Absorption of Radiation by Atoms</b>                         | 89               |
| 4.1 Atom–Field Interactions  | 89               |
| 4.2 Interaction of an Atom with a Classical Field                              | 91               |
| 4.3 Interaction of an Atom with a Quantized Field                              | 97               |
| 4.4 The Rabi Model   | 102              |

|          |   |     |
|----------|---|-----|
| 4.5      | Fully Quantum Mechanical Model: The Jaynes–Cummings Model                               | 105 |
| 4.6      | The Dressed States  | 115 |
| 4.7      | Density Operator Approach: Application to Thermal States                                | 119 |
| 4.8      | The Jaynes–Cummings Model with Large Detuning: A Dispersive Interaction                 | 123 |
| 4.9      | Extensions of the Jaynes–Cummings Model   | 125 |
| 4.10     | Schmidt Decomposition and Von Neumann Entropy for the Jaynes–Cummings Model             | 126 |
|          | Problems  | 129 |
| <b>5</b> | <b>Quantum Coherence Functions</b>  | 134 |
| 5.1      | Classical Coherence Functions   | 134 |
| 5.2      | Quantum Coherence Functions   | 139 |
| 5.3      | Young’s Interference  | 143 |
| 5.4      | Higher-Order Coherence Functions  | 146 |
|          | Problems  | 153 |
| <b>6</b> | <b>Beam Splitters and Interferometers</b>   | 155 |
| 6.1      | Experiments with Single Photons   | 155 |
| 6.2      | Quantum Mechanics of Beam Splitters   | 157 |
| 6.3      | Interferometry with a Single Photon   | 173 |
| 6.4      | Interaction-Free Measurement  | 175 |
| 6.5      | Interferometry with Coherent States of Light  | 177 |
| 6.6      | The SU(2) Formulation of Beam Splitting and Interferometry                              | 179 |
| 6.7      | The Beam Splitter as a Displacer  | 185 |
| 6.8      | Photons Do Not Interfere  | 186 |
| 6.9      | Are Photons Entangled?  | 186 |
|          | Problems  | 187 |
| <b>7</b> | <b>Nonclassical Light</b>   | 191 |
| 7.1      | Quadrature Squeezing  | 191 |
| 7.2      | Generation of Quadrature Squeezed Light   | 207 |
| 7.3      | Detection of Quadrature Squeezed Light  | 209 |
| 7.4      | Amplitude (or Number) Squeezed States   | 211 |
| 7.5      | Photon Antibunching   | 213 |
| 7.6      | Schrödinger-Cat States  | 216 |
| 7.7      | Two-Mode Squeezed Vacuum States   | 226 |
| 7.8      | Broadband Squeezed Light  | 232 |
| 7.9      | Pair Coherent States  | 233 |
| 7.10     | Entanglement Generation via Beam Splitting  | 238 |
| 7.11     | Quantum State Engineering: Generation of Nonclassical States by Photon-Level Operations | 239 |
|          | Problems  | 248 |

|  |     |
|--|-----|
| <b>8 Dissipative Interactions and Decoherence</b>  | 257 |
| 8.1 Introduction   | 257 |
| 8.2 Single Realizations or Ensembles?  | 258 |
| 8.3 Individual Realizations  | 262 |
| 8.4 Shelving and Telegraph Dynamics in<br>Three-Level Atoms  | 266 |
| 8.5 Modeling Losses with Fictitious Beam Splitters   | 270 |
| 8.6 Decoherence  | 272 |
| 8.7 Generation of Coherent States from Decoherence:<br>Nonlinear Optical Balance   | 275 |
| 8.8 Conclusions  | 276 |
| Problems   | 277 |
| <b>9 Optical Test of Quantum Mechanics</b>   | 280 |
| 9.1 Photon Sources: Spontaneous Parametric<br>Down-Conversion  | 281 |
| 9.2 The Hong–Ou–Mandel Interferometer  | 285 |
| 9.3 The Quantum Eraser   | 287 |
| 9.4 Induced Coherence  | 290 |
| 9.5 Superluminal Tunneling of Photons  | 293 |
| 9.6 Optical Test of Local Realistic Theories and Bell’s<br>Theorem   | 294 |
| 9.7 Franson’s Experiment   | 301 |
| 9.8 Applications of Down-Converted Light to Metrology<br>without Absolute Standards  | 303 |
| Problems   | 305 |
| <b>10 Experiments in Cavity QED and with Trapped Ions</b>  | 308 |
| 10.1 Rydberg Atoms   | 308 |
| 10.2 Rydberg Atom Interacting with a Cavity Field  | 311 |
| 10.3 Experimental Realization of the Jaynes–Cummings<br>Model  | 316 |
| 10.4 Creating Entangled Atoms in CQED  | 318 |
| 10.5 Formation of Schrödinger-Cat States with Dispersive<br>Atom–Field Interactions and Decoherence from the<br>Quantum to the Classical | 320 |
| 10.6 Quantum Non-demolition Measurement of Photon<br>Number  | 325 |
| 10.7 Quantum State Engineering in the Resonant Jaynes–<br>Cummings Model   | 326 |
| 10.8 Realization of the Jaynes–Cummings Interaction in the<br>Motion of a Trapped Ion  | 328 |
| 10.9 Concluding Remarks  | 332 |
| Problems   | 333 |

|  |     |
|--|-----|
| <b>11 Applications of Entanglement: Heisenberg-Limited Interferometry and Quantum Information Processing</b>     | 338 |
| 11.1 The Entanglement Advantage  | 340 |
| 11.2 Two No-Go Theorems: No-Signaling and No-Cloning   | 341 |
| 11.3 Entanglement and Interferometric Measurements   | 343 |
| 11.4 Quantum Teleportation   | 353 |
| 11.5 Cryptography  | 355 |
| 11.6 Private Key Crypto-systems  | 357 |
| 11.7 Public Key Crypto-systems   | 358 |
| 11.8 The Quantum Random Number Generator   | 360 |
| 11.9 Quantum Cryptography  | 364 |
| 11.10 Future Prospects for Quantum Communication   | 370 |
| 11.11 Gates for Quantum Computation  | 370 |
| 11.12 An Optical Realization of Some Quantum Gates   | 376 |
| 11.13 Decoherence and Quantum Error Correction   | 380 |
| Problems   | 381 |
| <i>Appendix A The Density Operator, Entangled States, the Schmidt Decomposition, and the Von Neumann Entropy</i> | 386 |
| A.1 The Density Operator   | 386 |
| A.2 Two-State System and the Bloch Sphere  | 389 |
| A.3 Entangled States   | 390 |
| A.4 Schmidt Decomposition  | 392 |
| A.5 Von Neumann Entropy  | 394 |
| A.6 Dynamics of the Density Operator   | 395 |
| <i>Appendix B Quantum Measurement Theory in a (Very Small) Nutshell</i>  | 398 |
| <i>Appendix C Derivation of the Effective Hamiltonian for Dispersive (Far Off-Resonant) Interactions</i>         | 402 |
| <i>Appendix D Nonlinear Optics and Spontaneous Parametric Down-Conversion</i>                                    | 406 |
| <i>Index</i>   | 408 |