

Introductory Quantum Optics

Second Edition

This established textbook provides an accessible but comprehensive introduction to the quantum nature of light and its interaction with matter. The field of quantum optics is covered with clarity and depth, from the underlying theoretical framework of field quantization, atom–field interactions, and quantum coherence theory, to important and modern applications at the forefront of current research such as quantum interferometry, squeezed light, quantum entanglement, cavity quantum electrodynamics, laser-cooled trapped ions, and quantum information processing. The text is suitable for advanced undergraduate and graduate students and would be an ideal main text for a course on quantum optics. This long-awaited second edition builds upon the success of the first edition, including many new developments in the field, particularly in the area of quantum state engineering. Additional homework problems have been added, and content from the first edition has been updated and clarified throughout.

Christopher C. Gerry is Professor of Physics at Lehman College, City University of New York. He was one of the first to exploit the use of group theoretical methods in quantum optics and is a highly regarded researcher and lecturer in the field. He has written well-regarded books, both for advanced students and researchers, and for a more general audience.

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“Quantum technology is transitioning from the research laboratory into the commercial world. Scientists and engineers are learning new languages to understand how quantum will impact applications. Beyond the math, what I love about this book are the words highlighting both the intuitive and non-intuitive science, essential understanding for progressing the transition from quantum science to quantum technology.”

Professor Miles Padgett FRS, The University of Glasgow, UK

“The book is absolutely a pleasure, with a wide coverage of the field including important developments such as optical tests of the foundational aspects of quantum mechanics, Heisenberg limited metrology, quantum gates, and decoherence. The simplicity with which deeper concepts are introduced is truly remarkable. It deserves to be on the shelf of everyone interested in the new quantum revolution of the twenty-first century.”

Professor Girish S. Agarwal FRS, Texas A&M University, USA

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Cambridge University Press & Assessment
978-1-009-41529-3 — Introductory Quantum Optics
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Frontmatter
[More Information](#)



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a department of the University of Cambridge.

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www.cambridge.org

Information on this title: www.cambridge.org/highereducation/isbn/9781009415293

DOI: 10.1017/9781139151207

First edition © C. C. Gerry and P. L. Knight 2005

Second edition © Christopher C. Gerry and Peter L. Knight 2024

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First published 2005

Third printing 2008

Second edition 2024

Printed in the United Kingdom by CPI Group Ltd, Croydon CR0 4YY, 2024

A catalogue record for this publication is available from the British Library

*A Cataloging-in-Publication data record for this book is available from the Library
of Congress*

ISBN 978-1-009-41529-3 Hardback

Additional resources for this publication at www.cambridge.org/gerry-knight2.

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C. C. G. dedicates this book to his son, Eric.
P. L. K. dedicates this book to his wife, Chris.

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Preface

The first edition of this book, published in 2005, was intended to be a readable text for students interested in the nature of light fields and their interaction with atoms at the fundamental level, focusing on concepts and ideas. At that time we noted the vibrancy of the field, and in the subsequent 20 years it has grown and flourished, with major insights being uncovered on the nature of quantum light, on entanglement, and much else. The award of the 2022 Physics Nobel Prize to Alain Aspect, John Clauser, and Anton Zeilinger for their work on quantum optical entanglement highlights the central role the subject has continued to play in modern physics. Many universities now offer courses on quantum optics, some for final-year advanced undergraduates and many more for graduate students, and it is to these that we address our book. The text is designed for students taking courses in quantum optics who have already taken a course in quantum mechanics, although we do cover some key elements that they may not have encountered before, such as the Schmidt decomposition – so useful in describing entanglement, and some key ideas in quantum measurement theory.

The presentation continues to be very much concerned with the quantized electromagnetic field. As in the first edition, topics covered include single-mode field quantization in a cavity, quantization of multimode fields, quantum phase, coherent states, quasi-probability distribution in phase space, atom–field interactions, the Jaynes–Cummings model, quantum coherence theory, beam splitters and interferometers, nonclassical field states with squeezing and so on, tests of local realism with entangled photons from down-conversion, experimental realizations of cavity quantum electrodynamics, trapped ions, decoherence, and an introduction to applications to quantum information processing, particularly quantum cryptography. We have made many updates to the text and included new sections on the quantum phase operator in Chapter 2, a detailed discussion of the connection between the photon number parity operator and the Wigner function in Chapter 3, a major expansion on beam splitters and interferometers in Chapter 6 – to include more details on how to obtain beam splitter output states for any given number-state inputs. This chapter also includes a discussion of the use of the $SU(2)$ (angular momentum) formalism in the Schwinger realization for the description of beam splitters and interferometers. The homework problems for this chapter have been significantly revised. In Chapter 7 we have added material on the distinction between Gaussian and non-Gaussian states and the significance of

that distinction, and we have added a section on the so-called pair coherent states, a particular example of a two-mode non-Gaussian state. We have added a section on entanglement generation by beam splitting, and a section on quantum state engineering by photon-level operations on Gaussian states. In Chapter 8 we have added a section on modeling losses with fictitious beam splitters, where we show how this method can be applied to number states. The section on decoherence has been reworked from this point of view. In Chapter 11 we have added sections on the no-signaling theorem and the no-cloning theorem. Also added is a section on an experiment in quantum optical interferometry where a squeezed vacuum state is mixed with a coherent state to generate a superposition of the so-called $N00N$ states. A section has been added on using photon number parity as the observable in an interferometric experiment with coherent light. In the section on quantum random number generation (QRNG) we have added a discussion on the use of measurements of the photon number parity operator in moderately intense laser light for generating random numbers and of the experiment that was performed in connection with this idea.

We took a decision to remain focused on quantum optics and resisted the temptation to extend substantially our coverage of quantum information processing, which has developed into a field of its own covered by excellent specialist texts. As theorists, we felt it would be inappropriate to go into the details of experiments, rather we concentrate on the basic theoretical ideas.

The book contains many homework problems, suggestions for further reading, and a comprehensive bibliography of the key papers that we feel students would benefit from accessing. Feedback from colleagues around the world has been invaluable as we worked on this second edition; we hope it will continue to be a useful guide to an exciting part of contemporary physics.

Acknowledgments

Acknowledgments to the First Edition

This book developed out of courses that we have given over the years at Imperial College London and the Graduate Center of the City University of New York, and we are grateful to the many students who have sat through our lectures and acted as guinea pigs for the material we have presented here.

We would like to thank our many colleagues who, over many years, have given us advice, ideas, and encouragement. We particularly thank Dr. Simon Capelin at Cambridge University Press who has had much more confidence than us that this would ever be completed. Over the years we have benefited from many discussions with our colleagues, especially Les Allen, Gabriel Barton, Janos Bergou, Keith Burnett, Vladimir Buzek, Richard Campos, Bryan Dalton, Joseph Eberly, Rainer Grobe, Edwin Hach III, Robert Hilborn, Mark Hillery, Ed Hinds, Rodney Loudon, Peter Milonni, Bill Munro, Geoffrey New, Edwin Power, George Series, Wolfgang Schleich, Bruce Shore, Carlos Stroud Jr., Stuart Swain, Dan Walls, and Krzysztof Wodkiewicz. We especially thank Adil Benmoussa for creating all the figures for this book using his expertise with Mathematica, Corel Draw, and Origin Graphics, for working through the homework problems, and for catching many errors in various drafts of the manuscript. We also thank Mrs. Ellen Calkins for typing the initial draft of several of the chapters.

We acknowledge our former students and postdocs, who have taught us so much and have gone on to become leaders themselves in this exciting subject, especially Stephen Barnett, Almut Beige, Artur Ekert, Barry Garraway, Christoph Keitel, Myungshik Kim, Gerard Milburn, Martin Plenio, Barry Sanders, Stefan Scheel, and Vlatko Vedral: they will recognize much that is here!

As this book is intended as an introduction to quantum optics, we have not attempted to be comprehensive in our citations. We apologize to those authors whose work is not cited.

C. C. G. wishes to thank the members of the Lehman College Department of Physics and Astronomy, and many other members of the

Lehman College community, for their encouragement during the writing of this book.

P. L. K. would especially like to acknowledge the support throughout of Chris Knight, who has patiently provided encouragement, chauffeuring and vast amounts of tea during the writing of this book.

Our work in quantum optics over the past four decades has been funded by many sources: for P. L. K. in particular the UK SRC, SERC, EPSRC, the Royal Society, the European Union, the Nuffield Foundation and the US Army are thanked for their support; for C. C. G. the National Science Foundation, the Research Corporation, Professional Staff Congress of the City University of New York (PSC-CUNY).

Acknowledgments to the Second Edition

We remain indebted to those named above, of course. But we would like to add our thanks to those who have given us invaluable insights since that first edition: especially Richard Birrittella, in particular for much help with graphics (Figures 3.9, 3.10, 6.6, 6.7, 6.8, 6.9, 6.10, 6.11, 6.12, 7.19, 7.20, 7.21, and 7.22) and with the student problems and Lior Cohen, for discussions on his experiments on the role of parity in quantum interferometry.

C. C. G. thanks former student and long-time collaborator Richard Birrittella for his help with various aspects of this edition of the book, and former student and long-term collaborator Ed Hach for many discussions on the contents of the book. He also thanks Lior Cohen for several discussions on the experiment he performed (with the group of H. Eisenberg) on the use of photon number parity in quantum optical interferometry. Also, he wishes to acknowledge the many conversations he has had on quantum optical interferometry over the past two decades and more with the late Jonathon Dowling. Finally, he wishes to acknowledge the summer funding he has received to support a long-term collaboration with Paul Alsing and his group at the U.S. Air Force Laboratory in Rome, New York.

P. L. K. thanks Terry Rudolph, the late Danny Segal and Richard Thompson, all at Imperial, and Miles Padgett in Glasgow, Jason Twamley in Okinawa for many discussions on the topics of this book.