1

The pinch technique at one loop

In this chapter, we present in detail the pinch technique (PT) construction at one loop for a QCD-like theory, where there is no tree-level symmetry breaking (no Higgs mechanism). The analysis applies to any gauge group ($SU(N)$, exceptional groups, etc.); however, for concreteness, we will adopt the QCD terminology of quarks and gluons.

This introductory chapter and Chapter 2 go into both conventional technology and the pinch technique only at the one-loop level. Here, the reader will find an almost self-contained guide to the one-loop pinch technique with many calculational details plus some hints at the nonperturbative ideas used in later chapters (where nonperturbative effects will be studied by dressing the loops, i.e., using a skeleton expansion).

1.1 A brief history

Non-Abelian gauge theories (NAGTs) had been around for a long time when the pinch technique came into play [1, 2, 3, 4]. Their first use was in defining the one-loop PT gauge-boson propagator as a construct taken from some gauge-invariant object by combining parts of conventional Feynman graphs while preserving gauge invariance and other physical properties. The term *pinch technique* was introduced later [4], in a paper that extended the one-loop pinch technique to the three-gluon vertex. The name comes from a characteristic feature of the pinch technique, in which the needed parts of some Feynman graphs look as though a particular propagator line had been pinched out of existence. In all these early papers, only one-loop phenomena were studied, including a one-dressed-loop Schwinger–Dyson equation for the PT propagator. This equation showed how the infrared singularities arising because of asymptotic freedom (= infrared slavery) require dynamical gluon mass generation. Of course, the pinch technique should lead to unique results. These considerations followed from five requirements for all PT Green's functions not involving ghosts:

1. All Green's functions are independent of any gauge-fixing parameters.
2. All Green's functions are independent of the particular *S*-matrix process used to define them.
3. All Green's functions obey Ward identities of QED type, not involving ghosts.
4. All Green's functions obey dispersion relations in which there are no identifiable ghost contributions or threshholds.
5. The discontinuities (imaginary parts) of Green's functions can be calculated with the usual Cutkosky rules, consistent with unitarity for the *S*-matrix.

All these are properties of Green's functions in the background-field Feynman gauge, later shown to be equivalent to the pinch technique.

One remark concerning the imaginary parts and unitarity is in order. The photon propagator of QED satisfies a Källen–Lehmann representation with a positive spectral function, a property intimately related to the positivity of the beta function of QED. Because this beta function is negative for an asymptotically free theory, it is impossible to find a NAGT gauge-boson propagator with a positive spectral function, so unitarity holds in a generalized form, with some negative contributions to spectral functions. However, as pointed out in Section 1.7, special properties of the PT propagator allow its factorization into two terms, each obeying the Källen–Lehmann representation.[1] This factorization allows the rearrangement of PT Schwinger–Dyson equations into a form in which all necessary positivity constraints are realized.

At the beginning, how to extend the pinch technique to higher orders of perturbation theory was far from clear; the pioneering technology defined in the first papers would have been forbiddingly difficult for graphs with two or more loops. Fortunately, the problem of the all-order pinch technique has a solution that can be stated with remarkable simplicity: all that has to be done, as was shown [5, 6], is to calculate conventional Feynman graphs using the background-field methodology [7] in the Feynman gauge. The original proof was for NAGTs such as QCD, but it was extended [8] to all orders of electroweak theory. This work was inspired by remarks [9, 10, 11] to the effect that the original pinch technique and the background-field Feynman gauge gave exactly the same results at one loop in perturbation theory. This, of course, could have been a coincidence without much

[1] The product of two functions obeying the Källen–Lehmann representation need not obey it.

meaning, but the all-order proof showed constructively how the PT requirements were satisfied at all orders in the background-field Feynman gauge.[2]

In roughly the same time period, string-theory workers [12] studied the off-shell extrapolation of string-theory amplitudes in the field theory, or zero Regge slope, limit. By imposing a consistent implementation of modular invariance, these workers showed that the off-shell gauge-theory amplitudes derived from string theory were automatically given in the background-field Feynman gauge–equivalent to the pinch technique.

The results showing the equivalence of the pinch technique and the background-field Feynman gauge set the stage for nonperturbative applications of the pinch technique, including the Schwinger–Dyson equations of the pinch technique and their consequences. The output of any PT calculation is not only independence of any gauge-fixing parameter but also freedom from contamination by unphysical objects. For example, if one tries to find the contributions of gauge-invariant condensates such as $\langle \mathrm{Tr}\, G_{\mu\nu}G^{\mu\nu}\rangle$ to the usual gauge-boson propagator, one discovers that they are inextricably bound with nonphysical and gauge-dependent condensates involving the ghost fields. But for the PT propagator, only the gauge-invariant condensate, field-strength condensate emerges; there are no ghost contributions [13].

1.2 Notation and conventions

Unless explicitly stated otherwise, we adopt the conventions of Peskin and Schröeder [14]. Sometimes, such as in Chapters 7–9 and parts of Chapter 11, it is convenient to work in Euclidean space. The canonical gauge potential $A^a_\mu(x)$ is often combined in the Hermitian matrix form

$$A_\mu(x) = A^a_\mu(x)t^a, \tag{1.1}$$

where t^a are the $SU(N)$ generators satisfying the commutation relations

$$[t^a, t^b] = \mathrm{i} f^{abc} t^c, \tag{1.2}$$

with f^{abc} being the group's totally antisymmetric structure constants. The generators are normalized according to

$$\mathrm{Tr}(t^a t^b) = \frac{1}{2}\delta^{ab}. \tag{1.3}$$

In the case of QCD, the fundamental representation is given by $t^a = \lambda^a/2$, where λ^a are the Gell–Mann matrices.

[2] And in no other background-field gauge; for other than the Feynman gauge, the original PT pinching rules would have to be applied to the background-field Green's functions to get those of the PT.

In Chapters 7, 8, and 9, dealing with nonperturbative phenomena, we combine the gauge potentials in the anti-Hermitean matrix form

$$A_\mu(x) = -\mathrm{i}gA^a_\mu(x)t^a,$$

in which case the matrix potential has a unit mass dimension in all space-time dimensions. The changes in all other definitions are trivial. This definition has many advantages when we go beyond perturbation theory.

The Lagrangian density for a general $SU(N)$ non-Abelian gauge theory is given by

$$\mathcal{L} = \mathcal{L}_\mathrm{I} + \mathcal{L}_\mathrm{GF} + \mathcal{L}_\mathrm{FPG}. \tag{1.4}$$

$\mathcal{L}_\mathrm{I}$ represents the gauge invariant Lagrangian, namely,

$$\mathcal{L}_\mathrm{I} = -\frac{1}{4}G^{\mu\nu}_a G^a_{\mu\nu} + \bar{\psi}^i_\mathrm{f}\left(\mathrm{i}\gamma^\mu\mathcal{D}_\mu - m\right)_{ij}\psi^j_\mathrm{f}, \tag{1.5}$$

where $a = 1, \ldots, N^2 - 1$ (respectively, $i, j = 1, \ldots, N$) is the color index for the adjoint (respectively, fundamental) representation, and f is the flavor index. The matrix-covariant derivative and field strength are defined according to

$$\mathcal{D}_\mu = \partial_\mu - \mathrm{i}gA_\mu \tag{1.6}$$

$$\left[\mathcal{D}_\mu, \mathcal{D}_\nu\right] = -\mathrm{i}gG^a_{\mu\nu}t^a, \tag{1.7}$$

or, more explicitly,

$$(\mathcal{D}_\mu)_{ij} = \partial_\mu(I)_{ij} - \mathrm{i}gA^a_\mu(t^a)_{ij} \tag{1.8}$$

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu, \tag{1.9}$$

with g being the (strong) coupling constant. Under a local (finite) gauge transformation $V = \exp[-\mathrm{i}\theta]$,

$$A_\mu \to V\frac{\mathrm{i}}{g}\partial_\mu V^\dagger + VA_\mu V^\dagger; \qquad G_{\mu\nu} \to VG_{\mu\nu}V^\dagger; \qquad \psi \to V\psi, \tag{1.10}$$

from which the invariance of $\mathcal{L}_\mathrm{I}$ follows. In terms of infinitesimal local gauge transformations,

$$\delta A^a_\mu = -\frac{1}{g}\partial_\mu\theta^a + f^{abc}\theta^b A^c_\mu; \qquad \delta_\theta\psi^i_\mathrm{f} = -\mathrm{i}\theta^a(t^a)_{ij}\psi^j_\mathrm{f}$$
$$\delta_\theta\bar{\psi}^i_\mathrm{f} = \mathrm{i}\theta^a\bar{\psi}^j_\mathrm{f}(t^a)_{ji}, \tag{1.11}$$

where $\theta^a(x)$ are the local infinitesimal parameters corresponding to the $SU(N)$ generators t^a.

To quantize the theory, the gauge invariance needs to be broken; this breakup is achieved through a (covariant) gauge-fixing function $\mathcal{F}^a$, giving rise to the (covariant) gauge-fixing Lagrangian $\mathcal{L}_{\text{GF}}$ and its associated Faddeev–Popov ghost term $\mathcal{L}_{\text{FPG}}$. The most general way of writing these terms is through the Becchi–Rouet–Stora–Tyutin (BRST) operator s [15, 16] and the Nakanishi–Lautrup multipliers B^a [17, 18], which represent auxiliary, nondynamical fields that can be eliminated through their (trivial) equations of motion. Then, one gets

$$\mathcal{L}_{\text{GF}} = -\frac{\xi}{2}(B^a)^2 + B^a\mathcal{F}^a \tag{1.12}$$

$$\mathcal{L}_{\text{FPG}} = -\bar{c}^a s\mathcal{F}^a, \tag{1.13}$$

where

$$\delta_{\text{BRST}}\Phi = \epsilon s\Phi, \tag{1.14}$$

with ϵ being a Grassmann constant parameter and s being the BRST operator acting on the QCD fields according to

$$\begin{aligned} sA^a_\mu &= \partial_\mu c^a + gf^{abc}A^b_\mu c^c; & sc^a &= -\tfrac{1}{2}gf^{abc}c^b c^c \\ s\psi^i_{\text{f}} &= \mathrm{i}gc^a(t^a)_{ij}\psi^j_{\text{f}}; & s\bar{c}^a &= B^a \\ s\bar{\psi}^i_{\text{f}} &= -\mathrm{i}gc^a\bar{\psi}^j_{\text{f}}(t^a)_{ji}; & sB^a &= 0. \end{aligned} \tag{1.15}$$

From the preceding transformations, it is easy to show that the BRST operator is nilpotent: $s^2 = 0$. In addition, as a result, the sum of the gauge-fixing and Faddev–Popov terms can be written as a total BRST variation:

$$\mathcal{L}_{\text{GF}} + \mathcal{L}_{\text{FPG}} = s\left(\bar{c}^a\mathcal{F}^a - \frac{\xi}{2}\bar{c}^a B^a\right). \tag{1.16}$$

This, of course, is expected because of the well-known property that total BRST variations cannot appear in the physical spectrum of the theory, which in turn implies the ξ independence of the S-matrix elements and physical observables.

As far as the gauge-fixing function is concerned, there are several possible choices. The ubiquitous R_ξ gauges correspond to the covariant choice

$$\mathcal{F}^a_{R_\xi} = \partial^\mu A^a_\mu. \tag{1.17}$$

In this case, one has

$$\mathcal{L}_{\text{GF}} = \frac{1}{2\xi}(\partial^\mu A^a_\mu)^2 \tag{1.18}$$

$$\mathcal{L}_{\text{FPG}} = \partial^\mu\bar{c}^a\partial_\mu c^a + gf^{abc}(\partial^\mu\bar{c}^a)A^b_\mu c^c; \tag{1.19}$$

the Feynman rules corresponding to such a gauge are reported in the appendix. One can also consider noncovariant gauge-fixing functions such as

$$\mathcal{F}^a_n = \frac{n^\mu n^\nu}{n^2}\partial_\mu A^a_\nu, \tag{1.20}$$

where n^μ is an arbitrary but constant four vector. In general, we can classify these gauges by the different values of n^2, i.e., $n^2 < 0$ (axial gauges), $n^2 = 0$ (light-cone gauge), and finally, $n^2 > 0$ (Hamilton or time-like gauge). Clearly, the gauge-fixing form of Eq. (1.20) does not work for the light-cone gauge, which needs a separate treatment, given in Section 1.6. In the other cases,

$$\mathcal{L}_{\text{GF}} = \frac{1}{2\xi(n^2)^2}(n^\mu n^\nu \partial_\mu A^a_\nu)^2 \tag{1.21}$$

$$\mathcal{L}_{\text{FPG}} = \frac{n^\mu n^\nu}{n^2}\left[\partial_\mu \bar{c}^a \partial_\nu c^a + g f^{abc}(\partial^\mu \bar{c}^a)A^b_\nu c^c\right]. \tag{1.22}$$

Notice that these noncovariant gauges, as well as the light-cone gauge, are ghost free because the ghosts decouple completely from the S-matrix in dimensional regularization.

Finally, because of the correspondence [9, 10, 11] between the PT and the particular class of gauges known as background field gauges [7], the latter will be described in depth in Chapter 2.

We end this section observing that when dealing with loop integrals, we will use dimensional regularization and employ the shorthand notation

$$\int_k \equiv \mu^\epsilon (2\pi)^{-d}\int d^d k, \tag{1.23}$$

where $d = 4 - \epsilon$ is the dimension of space-time and μ is the 't Hooft mass scale, introduced to guarantee that the coupling constant is dimensionless in d dimensions. In addition, the standard result,

$$\int_k \frac{1}{k^2} = 0, \tag{1.24}$$

will be used often to set various terms appearing in the PT procedure to zero.

1.3 The basic one-loop pinch technique

We begin with some notation for propagators and a special decomposition for the free three-gluon vertex, a decomposition that also occurs in the background-field method.

1.3.1 Origin of the longitudinal momenta

Consider the S-matrix element for the quark-quark elastic scattering process $q(p_1)q(r_1) \to q(p_2)q(r_2)$ in QCD. We have that $p_1 + r_1 = p_2 + r_2$ and set $q = r_2 - r_1 = p_1 - p_2$, with $s = q^2$ being the square of the momentum transfer. The longitudinal momenta responsible for triggering the kinematical re-arrangements characteristic of the pinch technique stem either from the bare gluon propagator $\Delta^{(0)}_{\alpha\beta}(k)$ or from the *external* bare (tree-level) three-gluon vertices, i.e., the vertices where the physical momentum transfer q is entering.

To study the origin of the longitudinal momenta in detail, first consider the gluon propagator $\Delta_{\alpha\beta}(k)$; after factoring out the trivial color factor δ^{ab}, in the R_ξ gauges, it takes the form

$$\mathrm{i}\Delta_{\alpha\beta}(q,\xi) = P_{\alpha\beta}(q)\Delta(q^2,\xi) + \xi\frac{q_\alpha q_\beta}{q^4}, \tag{1.25}$$

with $P_{\alpha\beta}(q)$ being the dimensionless transverse projector, defined as

$$P_{\alpha\beta}(q) = g_{\alpha\beta} - \frac{q_\alpha q_\beta}{q^2}. \tag{1.26}$$

The scalar function $\Delta(q^2,\xi)$ is related to the all-order gluon, self-energy

$$\Pi_{\alpha\beta}(q,\xi) = P_{\alpha\beta}(q)\Pi(q^2,\xi), \tag{1.27}$$

through

$$\Delta(q^2,\xi) = \frac{1}{q^2 + \mathrm{i}\Pi(q^2,\xi)}. \tag{1.28}$$

Because $\Pi_{\alpha\beta}$ has been defined in Eq. (1.28) with the imaginary factor i factored out in front, it is simply given by the corresponding Feynman diagrams in Minkowski space. The inverse of $\Delta_{\alpha\beta}$ can be found by requiring that

$$\Delta^{am}_{\alpha\mu}(q,\xi)(\Delta^{-1})^{\mu\beta}_{mb}(q,\xi) = \delta^{ab} g^{\beta}_{\alpha}, \tag{1.29}$$

and it is given by

$$-\mathrm{i}\Delta^{-1}_{\alpha\beta}(q,\xi) = P_{\alpha\beta}(q)\Delta^{-1}(q^2,\xi) + \frac{1}{\xi}q_\alpha q_\beta. \tag{1.30}$$

At tree level,

$$\mathrm{i}\Delta^{(0)}_{\alpha\beta}(q,\xi) = d(q^2)\left[g_{\alpha\beta} - (1-\xi)\frac{q_\alpha q_\beta}{q^2}\right] \tag{1.31}$$

$$d(q^2) = \frac{1}{q^2}. \tag{1.32}$$

Evidently, the longitudinal (pinching) momenta are proportional to the combination $\lambda = 1 - \xi$ and vanish for the particular choice $\xi = 1$ (Feynman gauge) so that the free propagator is simply proportional to $g_{\alpha\beta} d(q^2)$. This is a particularly important feature of the Feynman gauge, which, as we will see, makes PT computations much easier. In this gauge, only longitudinal momenta from vertices can contribute to pinching at the one-loop level. The popular case $\xi = 0$ (Landau gauge) gives rise to a transverse $\Delta^{(0)}_{\alpha\beta}(k)$, which may have its advantages but really complicates the PT procedure at this level.

Next, we consider the conventional three-gluon vertex, to be denoted by $\Gamma^{amn}_{\alpha\mu\nu}(q, k_1, k_2)$, given by the following manifestly Bose-symmetric expression (all momenta are incoming, i.e., $q + k_1 + k_2 = 0$):

$$
\begin{aligned}
\mathrm{i}\Gamma^{amn}_{\alpha\mu\nu}(q, k_1, k_2) &= g f^{amn} \Gamma_{\alpha\mu\nu}(q, k_1, k_2) \\
\Gamma_{\alpha\mu\nu}(q, k_1, k_2) &= g_{\mu\nu}(k_1 - k_2)_\alpha + g_{\alpha\nu}(k_2 - q)_\mu + g_{\alpha\mu}(q - k_1)_\nu .
\end{aligned}
\tag{1.33}
$$

This vertex satisfies the standard Ward identities:

$$
q^\alpha \Gamma_{\alpha\mu\nu}(q, k_1, k_2) = k_2^2 P_{\mu\nu}(k_2) - k_1^2 P_{\mu\nu}(k_1) \tag{1.34}
$$

$$
k_1^\mu \Gamma_{\alpha\mu\nu}(q, k_1, k_2) = q^2 P_{\alpha\nu}(q) - k_2^2 P_{\alpha\nu}(k_2) \tag{1.35}
$$

$$
k_2^\nu \Gamma_{\alpha\mu\nu}(q, k_1, k_2) = k_1^2 P_{\alpha\mu}(k_1) - q^2 P_{\alpha\mu}(q). \tag{1.36}
$$

Unfortunately, the right-hand side is not the difference of inverse propagators, a defect that shows up in higher orders as the appearance of ghost terms in the identities, now called the Slavnov–Taylor identities.

But it is possible to decompose the vertex in a special way into two pieces, one of which satisfies a Ward identity of an elementary (ghost-free) type and the other contains the only longitudinal momenta capable of generating pinches [1, 19]. In the general ξ gauge, this decomposition, as applied to the vertex of Figure 1.1(b), is

$$
\Gamma_{\mu\nu\alpha}(q, k_1, k_2) = \Gamma^{\xi}_{\mu\nu\alpha} + \Gamma^{\mathrm{P}\xi}_{\mu\nu\alpha}, \tag{1.37}
$$

where

$$
\begin{aligned}
\Gamma^{\xi}_{\mu\nu\alpha}(q, k_1, k_2) = (k_1 - k_2)_\alpha g_{\mu\nu} - 2q_\mu g_{\nu\alpha} + 2q_\nu g_{\mu\alpha} \\
+ \left(1 - \frac{1}{\xi}\right)[k_{2\nu} g_{\alpha\mu} - k_{1\mu} g_{\alpha\nu}],
\end{aligned}
\tag{1.38}
$$

and

$$
\Gamma^{\mathrm{P}\xi}_{\mu\nu\alpha}(q, k_1, k_2) = \frac{1}{\xi}[k_{2\nu} g_{\alpha\mu} - k_{1\mu} g_{\alpha\nu}]. \tag{1.39}
$$

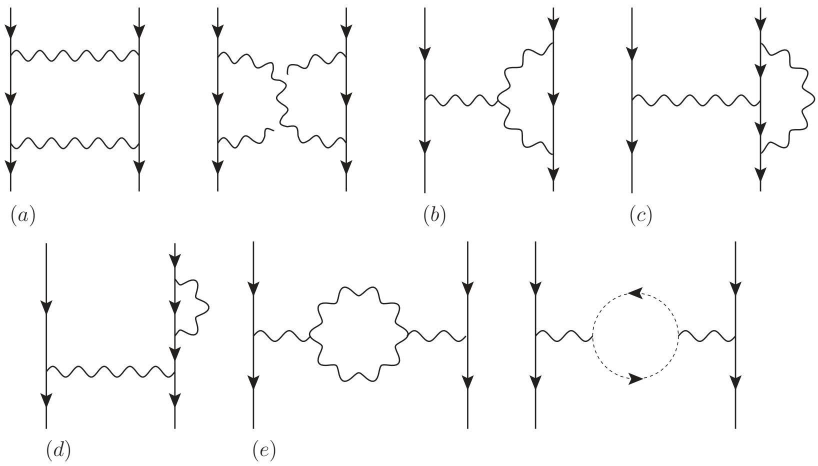

Figure 1.1. The diagrams contributing to the one-loop quark elastic scattering S-matrix element. (a) box contributions, (b) non-Abelian and (c) Abelian vertex contributions (two similar diagrams omitted), (d) quark self-energy corrections (three similar diagrams omitted), and (e) gluon self-energy contributions.

It is easy to check that Γ^{ξ} obeys the elementary Ward identity:

$$q^{\alpha}\Gamma^{\xi}_{\mu\nu\alpha}(q,k_1,k_2) = \Delta^{-1}_{\mu\nu}(k_2,\xi) - \Delta^{-1}_{\mu\nu}(k_1,\xi), \tag{1.40}$$

and that $\Gamma^{\mathrm{P}\xi}$ is the only part of the vertex that triggers pinches. In the pinch technique, (a trivial modification of) this ghost-free Ward identity holds to all orders and has, as a consequence, as in QED, the equality of the gluon wave function and vertex renormalization constants – a relation of great importance for further developments. Note that the vertex $\Gamma^{\xi}_{\alpha\mu\nu}(q,k_1,k_2)$ is Bose symmetric only with respect to the μ and ν legs. Evidently, the preceding decomposition assigns a special role to the q-leg, which is attached to two on-shell lines. In fact, this vertex Γ^{ξ} also occurs in the background-field method (see the appendix).[3]

It would be possible to carry out the (one-loop) PT manipulations with this vertex decomposition for any ξ, but, just as for the propagator, things simplify in the Feynman gauge, where a substantial part of Γ^{ξ} vanishes. Because we will use this gauge extensively, we record its vertex decomposition using the notation $\Gamma^{\mathrm{F}} = \Gamma^{\xi=1}$, $\Gamma^{\mathrm{P}\xi=1} = \Gamma^{\mathrm{P}}$. Then,

$$\Gamma_{\alpha\mu\nu}(q,k_1,k_2) = \Gamma^{\mathrm{F}}_{\alpha\mu\nu}(q,k_1,k_2) + \Gamma^{\mathrm{P}}_{\alpha\mu\nu}(q,k_1,k_2), \tag{1.41}$$

[3] Actually, in both the pinch technique and the background-field method, there are two kinds of vertices; at the one-loop level, only the one used here matters.

with

$$\Gamma^{\mathrm{F}}_{\alpha\mu\nu}(q,k_1,k_2) = (k_1-k_2)_\alpha g_{\mu\nu} + 2q_\nu g_{\alpha\mu} - 2q_\mu g_{\alpha\nu}, \tag{1.42}$$

$$\Gamma^{\mathrm{P}}_{\alpha\mu\nu}(q,k_1,k_2) = k_{2\nu} g_{\alpha\mu} - k_{1\mu} g_{\alpha\nu}, \tag{1.43}$$

and this allows $\Gamma^{\mathrm{F}}_{\alpha\mu\nu}(q,k_1,k_2)$ to satisfy the Ward identity

$$q^\alpha \Gamma^{\mathrm{F}}_{\alpha\mu\nu}(q,k_1,k_2) = (k_2^2 - k_1^2) g_{\mu\nu}, \tag{1.44}$$

where the right-hand side is the difference of two inverse propagators in the Feynman gauge.

1.3.2 The basic pinch operation

The term *pinch* arises from the operation of longitudinal momenta, such as in Γ^{P}, on vertices, which triggers Ward identities that lead to the cancellation of a preexisting propagator by an inverse propagator coming from the Ward identity. The resulting graph looks like a Feynman graph from which one line has been removed, as if it had been pinched out.

Whether acting on a vertex or a box diagram, as in Figure 1.1, the effect of the pinching momenta, regardless of their origin (gluon propagator or three-gluon vertex), is to trigger the elementary Ward identity

$$k_\nu \gamma^\nu = (\not{k} + \not{p} - m) - (\not{p} - m), \tag{1.45}$$

where the right-hand side (rhs) is the difference of two inverse tree-level quark propagators. The first of these terms cancels (pinches out) the internal tree-level fermion propagator $S^{(0)}(k+p)$, and the second term on the rhs vanishes when hitting the on-shell external leg. Diagrammatically speaking, an unphysical effective vertex appears in the place where $S^{(0)}(k+p)$ was, i.e., a vertex that does not appear in the original Lagrangian; as we will see, all such vertices cancel in the full, gauge-invariant amplitude.

First of all, it is immediate to verify the cancellation of the ξ-dependent terms at tree level. After extracting a kinematic factor of the form

$$\mathrm{i}\mathcal{V}^{a\alpha}(p_1,p_2) = \bar{u}(p_1)\mathrm{i}g t^a \gamma^\alpha u(p_2), \tag{1.46}$$

the tree-level amplitude reads

$$\mathcal{T}^{(0)} = \mathrm{i}\mathcal{V}^{a\alpha}(r_1,r_2)\mathrm{i}\Delta^{(0)}_{\alpha\beta}(q)\mathrm{i}\mathcal{V}^{a\beta}(p_1,p_2). \tag{1.47}$$

Then, because the on-shell spinors satisfy the equations of motion

$$\bar{u}(p)(\not{p}-m) = 0 = (\not{p}-m)u(p), \tag{1.48}$$